COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – SPRING 2018

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- All questions in this exam have a common theme: ***Winter Olympics***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

As a product engineer, you are given a task to design a new snowboard helmet, as illustrated in the following picture.

You are asked to use Bézier surface patches to model the major portion of the helmet.

a) You received the design of the first patch with the control points as

Patch 1:

(1 17 2)	(3 17 6)	(6 17 5)
(1 14 1)	(3 14 7)	(6 14 5)
(1 11 2)	(3 11 4)	(6 11 4)

To start the design of the second patch, you are given some of the control points as:

Patch 2:

$\mathbf{P}_1$	$\mathbf{P}_3$	(11 17 1)
$\mathbf{P}_2$	$\mathbf{P}_4$	(11 14 2)
(6 11 4)	$\mathbf{P}_5$	(11 11 1)

To ensure the C^0 and C^1 continuity between the two patches, what are the coordinate values of $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, and $\mathbf{P}_5$? Explain how you decide the values in detail and show calculations.

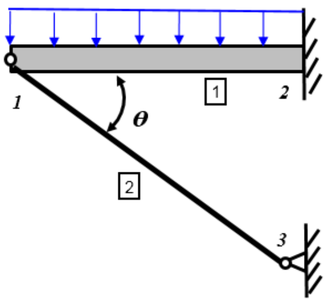
b) Derive the equation of the Bézier surface patch in a matrix form.

c) Calculate the unit normal vector of Patch 1 at control point **(1, 17, 2)**.

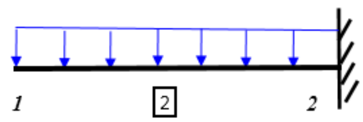
d) If control point **(1, 17, 2)** in Patch 1 is moved to a new position (1,18, 2), how will this change affect the shapes of the two surface patches?

2) Finite-Element Analysis

There are two types of ski jumping events (jumping off a normal hill or a large hill) in 2018 PyeongChang Winter Olympics. You are asked to design a ramp for normal hill jumping events. The ramp should support the distributed load of $\boldsymbol{q}$. In the 2D simplified model, you decide to use two different types of elements. The Type 2 element is used to stiffen the Type 1 element, as shown in Figure (a). For the Type 1 element, the area, moment of inertia, and length are given as A_1, I, and L. For the Type 2 element, the area is given as A_2. The young's modulus of both elements is E. Also, the angle between the two elements is θ°.



(a) Question 1



(b) Question 3

1. Determine the deflection at node 1 of Figure (a) by using finite element formulation. In your calculations: a) State all of your assumptions clearly. Show the boundary conditions and loading conditions. b) State how the example stiffness matrices given below should be modified for the two elements. c) Determine individual element matrix sizes and assembly matrix size. d) Show all steps to determine the deflection at node 1. You need not solve.
2. In Question 1, will the deflection at node 1 be the same as the exact solution? Fully explain your answer. What about the accuracy of the deflection in the middle of the element?
3. If you were to use only one Type 2 element as shown in Figure (b), what potential issues would come up in your solution process?

<u>Examples of Stiffness Matrix</u>

$$[k_A] = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix} \qquad [k_B] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where E, I, A, and L are the Modulus of Elasticity, Moment of inertia, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element.

Question 3: Numerical Methods

The sport of curling has experienced a resurgence of public interest, since it was reintroduced as a full Winter Olympic sport in 1998. Scientific investigations of curling dynamics has attracted much attention in the past two decades. Assume that you are carrying out a research project aiming to develop a quantitative model using numerical methods to predict the dynamics of a curling stone, which could be of practical value to coaches, curlers in training, and developers of curling game simulators.

Specifically, you need to develop a numerical model based on some initial kinematic measurements. Your task is to predict the sliding time (T) based on experiment measurements of the translational speed (V) and the rotational speed (ω). You have obtained the following data:

Experiment #	V (m/s)	ω (rad/s)	T (s)
1	2.26	1.60	21.9
2	2.35	1.67	23.3
3	2.21	1.61	24.6
4	2.28	1.64	25.3

(3.1) From the perspective of numerical methods, what type of problem is this?

(3.2) Describe the equations or algorithm you would use to predict T, given V and ω.
Note: It is not necessary to perform the actual computations, but you should clearly show your solution procedures with the given data. Describing the algorithm using either pseudo code or Matlab code would be an acceptable answer.

(3.3) Explain any assumption that you have made in your approach and also any potential numerical issue in the given data.

(3.4) Can you check the model adequacy from solution? If so, do you have any concerns on your model adequacy checking procedure? If the estimated accuracy is not acceptable, what would be your next strategy?