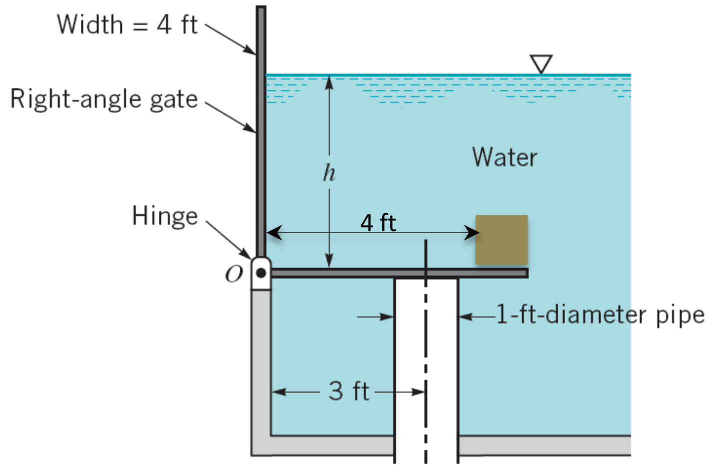
1. A thin 4-ft wide right angle gate with negligible mass is free to pivot about a frictionless hinge at point O, as shown in the figure. The horizontal portion of the gate covers a 1-ft diameter drain pipe, which contains air at atmospheric pressure. A 1x1x1 ft cube with specific gravity of 2.0 rests on the tip of the gate as shown.



Determine the minimum water depth, *h*, at which the gate will pivot to allow water to flow in to the pipe.

2. A liquid of constant density ρ flows steadily through a reducing elbow, which is attached at its upstream end to a section of pipe of inner radius R. The liquid enters the elbow at Section **1** with a fully-developed turbulent flow profile

$$V_1(r) = U\left(\frac{R-r}{R}\right)^{1/6}$$

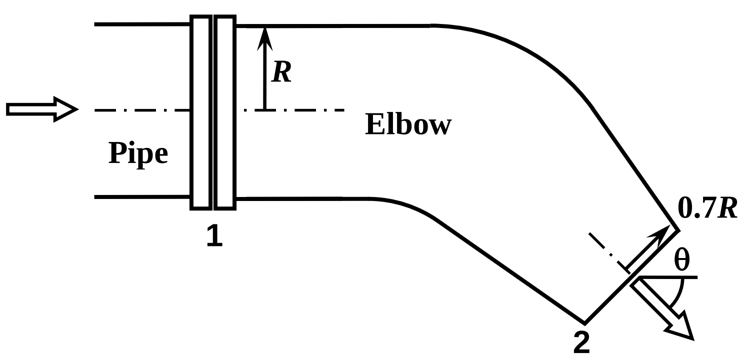
(where r is the radial coordinate measured from the centerline) and gage pressure p_1, and exits the elbow at Section **2** as a jet into air at zero gage pressure. The inlet and exit radii of the elbow are R and $0.7R$, respectively, and the elbow turns the flow by an angle θ, as shown. Neglect gravitational effects.

Please define your control volume.

a) What is the velocity profile V_2 just downstream of the exit of the elbow?

b) What is the loss L experienced by a fluid particle as it travels along the centerline of the elbow from Section **1** to Section **2**?

c) What is the force $\vec{\mathbf{F}}$ exerted by the elbow on the pipe?



3. Kolmogorov-Obukhov argued that for high Reynolds number, in terms of Fourier analysis, the turbulence energy spectrum contains an inertial subrange. They further argued that in the inertial subrange, the turbulence-energy spectrum function, $E(\kappa)$ (dimensions L^3/T^2), depends only upon the dissipation rate, ε (dimensions L^2/T^3), and the wave number, κ (dimensions $1/L$).

Using dimensional analysis, develop a formula for $E(\kappa)$ as a function of ε and κ.

4. A fully-developed, laminar steady flow of constant density Newtonian fluid (density ρ and viscosity μ) within the annular gap between two long concentric tubes of radii R_i and R_o (as shown in the figure) is used to apply a controlled shear stress on the surface of the inner tube. Determine the velocity distribution within the gap (as a function of the given parameters) by deriving an equation that describes the balance of forces on a small annular fluid element as shown in Figure 1 (sketch all the relevant forces) and using the appropriate boundary conditions when the flow is driven by:

a) Constant pressure drop per unit length Δp (stationary tubes),

and

b) Moving the inner tube to the right with constant velocity U_o only (i.e., without external pressure).

c) Is it possible to achieve the *same* shear stress on the surface of the inner tube by using these two different flow approaches? Explain briefly.