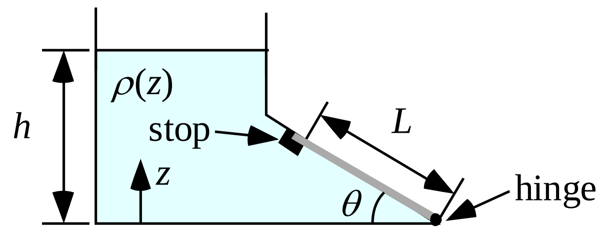
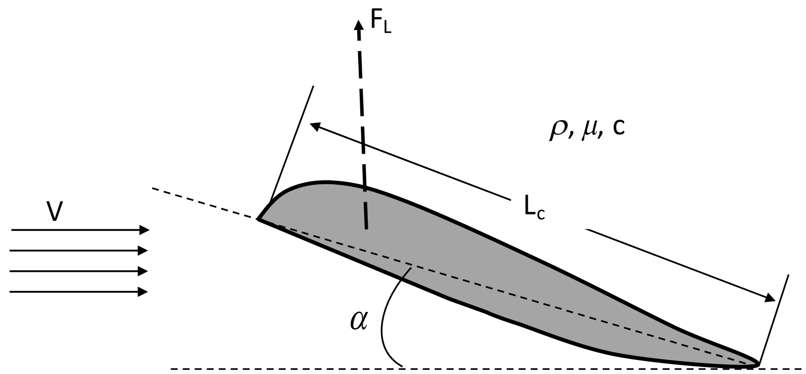
1. A static fluid of variable density with depth h is contained in the tank as shown below. A uniform rectangular gate is hinged at the bottom and rests against a stop. The gate has an inclination angle of θ with respect to the horizontal, a width w, a length L, and weighs W. The density of the fluid decreases linearly from the bottom of the tank to the top according to the equation $\rho(z) = \rho_b - \alpha z$, where ρ_b is the density at the bottom of the tank and α is a constant.



a) Sketch the pressure distribution along the internal gate surface.

b) Determine an expression for the minimum weight of the gate $W_{\min}$ that will prevent the gate from opening.

2. As part of a team designing a new airplane, you are assigned to predict the lift, F_L, produced by the new wing design. The cord length L_c of the wing in 1.12 m. The prototype is to fly at V=50 m/s close to the ground where T=25°C and pressure, P=1 atm. Consider a model wing for wind tunnel experiments that is ten times smaller in scale than the prototype. The wind tunnel can be pressurized to a maximum of 5 atm.



Assumptions:

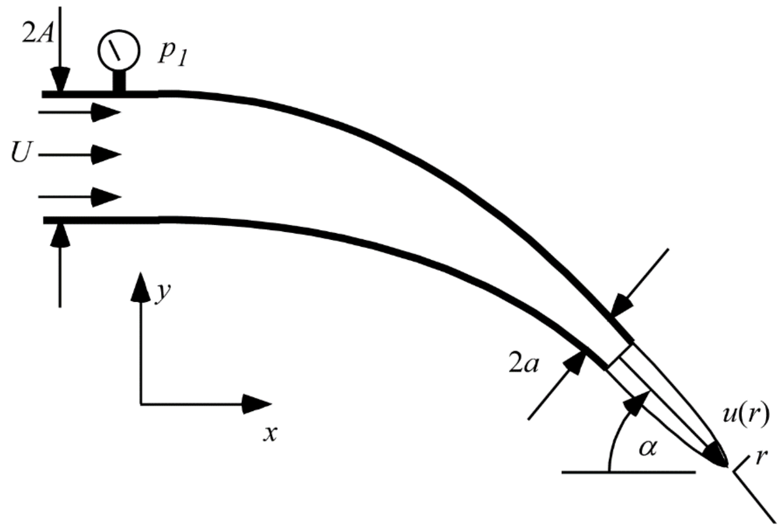
- The prototype wing flies through air at standard atmospheric pressure of 1.0 atm.
- Viscosity, μ, and speed of sound, c=350 m/s, do not change with pressure, P.
- Air can be treated as ideal gas at constant temperature where density is proportional to pressure, that is P/ρ = constant.
- As long as the Mach number (V/c) Ma<0.33, flow is incompressible and the results are independent of Ma.
- Angle of attach, α, is a non-dimensional number.

Determine:

a) All of the nondimensional parameters (Pi groups).

b) At what speed and pressure should you run the wind tunnel in order to achieve dynamic similarity.

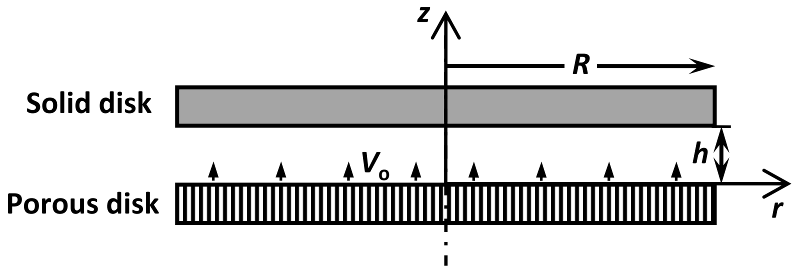
3. A reducing elbow accepts a uniform flow of water at speed U and gage pressure p_1 from a circular conduit of radius A, turns it through a horizontal angle α, where it exits to the atmosphere through a smaller opening of diameter a with a parabolic velocity profile given by $u(r) = C(1 - r^2/a^2)$, where r is a local coordinate indicated and C is a constant to be determined.



a) Determine the constant C in terms of given quantities.

b) Determine the horizontal (x and y) components of the force exerted *by the flow upon the elbow*.

4. A viscous oil of constant density ρ and constant viscosity μ flows steadily through a porous disk into a thin gap of height H (where $H << R$) between the porous disk and a solid disk, both of radius R. The gap is completely filled with oil, and the flow of oil at the surface of the porous disk is uniform and of constant speed V_o. The pressure p is only a function of r (because the gap is thin).
The known parameters are ρ, μ, H, R, and V_o.



a) What are the boundary conditions on the radial velocity component of the viscous oil in the gap V_r? Identify the type(s) of boundary condition (*e.g.* no flux).

b) Determine V_r in terms of $p(r)$ and the known parameters if: inertia is negligible, edge effects (at the edge of the disks) are negligible, and there is no azimuthal velocity, *i.e.*, $V_\theta = 0$. *Please list any additional assumptions.*

c) Determine $p(r)$ in terms of the known parameters if the boundary condition on the pressure is $p(R) = p_o$.

For your reference, the Navier-Stokes equations in cylindrical polar coordinates are:

$$\rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rV_r)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial^2 V_r}{\partial z^2}\right\} \quad (r)$$

$$\rho\left(\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial z^2}\right\} \quad (\theta)$$

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right] \quad (z)$$