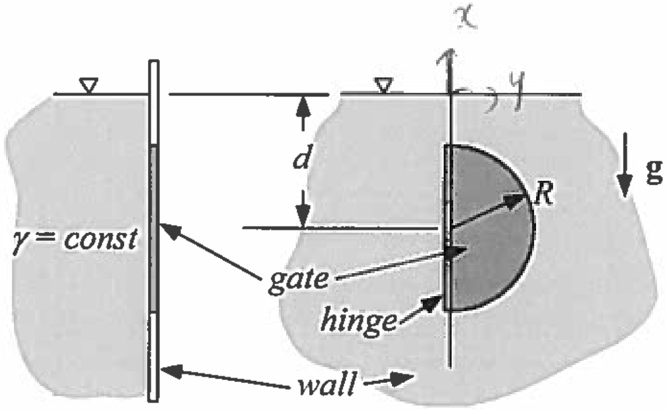
1.

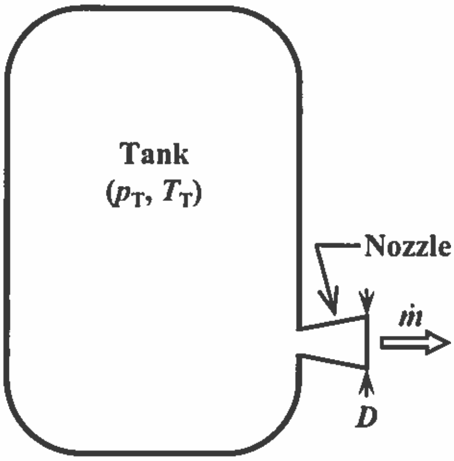


A semicircular gate of radius R, hinged along its vertical, straight edge, is located in a vertical wall bounded on one side by a liquid of constant specific weight γ. The center of the gate is at a depth d below the liquid free-surface.

a) What *moment* would a spring in the hinge need to provide in order to keep the gate closed?

b) Set up the integral necessary to compute the *force* exerted by the water on the gate.

2. A large pressurized tank is filled with a gas with gas constant R and constant-pressure specific heat c_p at pressure p_T and temperature T_T. The tank is attached to a nozzle with an exit diameter D.

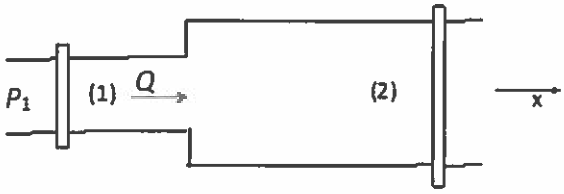


a) Use dimensional analysis to express the mass flow rate of the gas exiting the nozzle $\dot{m}$ as a function of the other parameters if $\dot{m}$ depends upon R, c_p, p_T, T_T and D.

b) Based on your dimensional analysis, how will the mass flow rate of the gas exiting the tank change if the original nozzle is replaced with a new nozzle that has half the exit diameter?

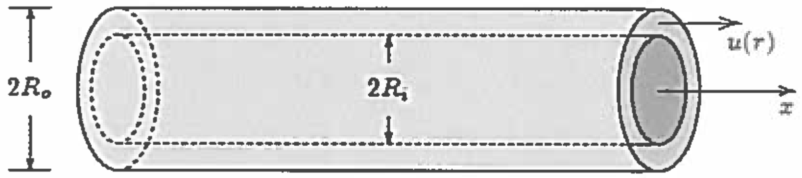
c) How will the mass flow rate change if the tank is filled with more of the same gas, and the tank pressure and the tank temperature both double?

3. Water flows through a sudden expansion transition in a pipe held by flanges, as shown. The diameter of the pipes, D_1 and D_2, the pressure at inlet, P_1, and volumetric flow rate, Q, are known. You can assume that the kinetic energy coefficient is approximately 1. You cannot neglect minor losses.



Derive the axial force, R_x, required to hold the transition in place in terms of known quantities.

4.



We wish to solve for incompressible, viscous flow between two long concentric rotating cylinders. The inner cylinder rotates at angular velocity Ω_i, while the outer cylinder has angular velocity Ω_o. The radii of the cylinders are R_i and R_o. There is no flow in the direction parallel to the centerline, x. All motion is in the circumferential direction.

- **a)** Simplify the continuity and Navier-Stokes equations taking advantage of the axial symmetry, and the fact that the flow is independent of x. State the appropriate boundary conditions.
- **b)** Verify that the radial velocity is zero.
- **c)** Solve for the circumferential velocity, u_θ.
- **d)** Determine the limiting form of the solution for an unbounded fluid, $R_o \to \infty$, with $\Omega_o = 0$. What inviscid flow does this resemble?

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial(u_\theta)}{\partial\theta}+\frac{\partial(u_z)}{\partial z}=0$$

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial^2 u_r}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r u_\theta}{r}+u_z\frac{\partial u_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\rho g_\theta+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial\theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$