**PhD Qualifying Exam**
**Heat Transfer, Written Exam**

## Problem 1

An experiment is being conducted in a combustion chamber with visual access through a quartz sight glass. The inner surface (x = 0) of the window receives net radiation from the combustion process as a uniform heat flux $q"_o$. A portion of this, β, is absorbed at the inner surface, while the remaining radiation is partially absorbed as it passes through the window. This leads to volumetric heat generation in the window that can be expressed as follows:

$$\dot{q}(x) = (1-\beta)q"_o \alpha e^{-\alpha x}$$

Heat is lost by convection to the ambient from the outer surface (x = L) of the window to ambient air at $T_\infty$, with a convective heat transfer coefficient *h*. You may neglect convection at the inner surface of the window as well as emission of radiation from that surface.

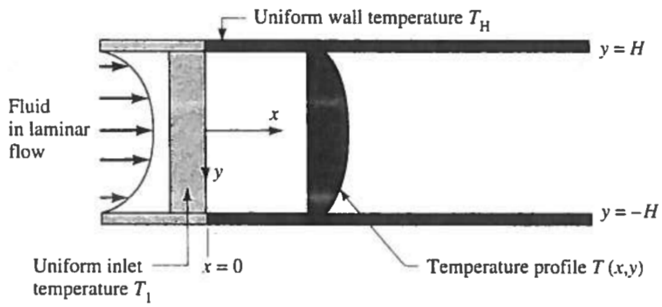
a. Develop an expression for the temperature profile in the quartz window in terms of the above parameters.
b. Find the location of the maximum temperature in the window
c. Sketch qualitatively the variation in temperature across the domain of interest in the problem, including the gases inside the chamber and the air outside
d. Repeat Part c for a case where the same total heat generation occurs uniformly in the window
e. Repeat Part c for a case where the combustion gas and outer air temperatures are the same, but there is no absorption of radiation and the consequent heat generation within the window.

## Problem 2

Consider laminar flow of a fluid between two parallel flat plates. At the inlet, the velocity profile is known, since it is fully developed, and is given by,

$$u(y)=\frac{3}{2}U\left[1-\left(\frac{y}{H}\right)^2\right]$$

where $U$ is the average velocity and the separation of the plates is $2H$. The temperature distribution at $x=0$ is uniformly at $T_1$.

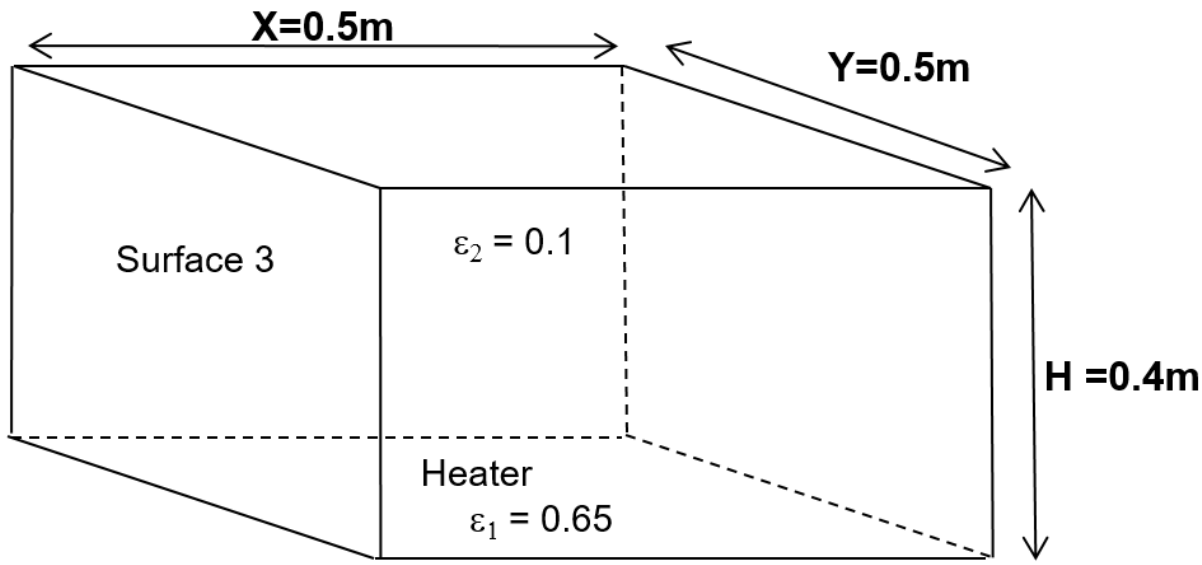



1. **[3 points]** Assuming that axial conduction through the fluid is negligible, write down the governing differential equations and boundary conditions that would be needed to solve for the temperature distribution as a function of $x$ and $y$. Assume the two solid walls are at a constant temperature $T_H$ for all values of $x$.
2. **[1 point]** Rewrite the energy balance and boundary conditions in non-dimensional form in terms of $x^*$ $y^*$ and $\Theta$
3. **[5 points]** Assuming the solution to the temperature distribution can be written as,

   $$\Theta=\sum_{m=0}^{\infty}\left[A_m \exp\left(\frac{-2\lambda_m^2 x^*}{3}\right)\sum_{n=0}^{\infty} a_{mn}(y^*)^n\right]$$

   and assuming the convective heat transfer is defined based on the average fluid temperature at a given location $x^*$, $q=h(T_H-\overline{T})$, determine the convective heat transfer coefficient as a function of $x^*$
4. **[1 point]** Sketch how the heat transfer coefficient will vary as a function of $x^*$

**Problem 3**




A rectangular furnace has diffuse gray walls and top of emissivity 0.1 and reaches a power input of 900W. It reaches a steady-state conditions where the heater temperature is 900 $^{o}$C. The furnace is insulated and inside a room at 27 deg C.

(a) State assumptions for solving this problem
(b) Develop an equivalent thermal circuit for the radiation exchange
(c) Calculate the temperature of the oven walls.
(d) A window which is transparent to a portion of the IR and visible spectrum replaces "Surface 3" of the furnace. The spectral transmissivity, emissivity and reflectivity of this window is given in table below. Indicate how you would modify the thermal equivalent circuit to estimate the temperature of the window under steady-state conditions.
(e) Approximately how much higher would the input power be to maintain the oven at the same operating temperature if the glass pane is thin.

Spectral properties of window are:

| | | | |
|---|---|---|---|
| $0 < \lambda < 2.75$ um | $\tau = 1$ | $\varepsilon = 0$ | $\rho = 0$ |
| $2.75 < \lambda <$ infinity | $\tau = 0$ | $\varepsilon = 0.8$ | $\rho = 0.2$ |

**TABLE 12.1 Blackbody Radiation Functions**

| $\lambda T$ ($\mu$m · K) | $F_{(0 \rightarrow \lambda)}$ | $I_{\lambda,b}(\lambda, T)/\sigma T^5$ ($\mu$m · K · sr)$^{-1}$ | $\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$ |
|---|---|---|---|
| 200 | 0.000000 | $0.375034 \times 10^{-27}$ | 0.000000 |
| 400 | 0.000000 | $0.490335 \times 10^{-13}$ | 0.000000 |
| 600 | 0.000000 | $0.104046 \times 10^{-8}$ | 0.000014 |
| 800 | 0.000016 | $0.991126 \times 10^{-7}$ | 0.001372 |
| 1,000 | 0.000321 | $0.118505 \times 10^{-5}$ | 0.016406 |
| 1,200 | 0.002134 | $0.523927 \times 10^{-5}$ | 0.072534 |
| 1,400 | 0.007790 | $0.134411 \times 10^{-4}$ | 0.186082 |
| 1,600 | 0.019718 | 0.249130 | 0.344904 |
| 1,800 | 0.039341 | 0.375568 | 0.519949 |
| 2,000 | 0.066728 | 0.493432 | 0.683123 |
| 2,200 | 0.100888 | $0.589649 \times 10^{-4}$ | 0.816329 |
| 2,400 | 0.140256 | 0.658866 | 0.912155 |
| 2,600 | 0.183120 | 0.701292 | 0.970891 |
| 2,800 | 0.227897 | 0.720239 | 0.997123 |
| 2,898 | 0.250108 | $0.722318 \times 10^{-4}$ | 1.000000 |
| 3,000 | 0.273232 | $0.720254 \times 10^{-4}$ | 0.997143 |
| 3,200 | 0.318102 | 0.705974 | 0.977373 |
| 3,400 | 0.361735 | 0.681544 | 0.943551 |
| 3,600 | 0.403607 | 0.650396 | 0.900429 |
| 3,800 | 0.443382 | $0.615225 \times 10^{-4}$ | 0.851737 |
| 4,000 | 0.480877 | 0.578064 | 0.800291 |
| 4,200 | 0.516014 | 0.540394 | 0.748139 |
| 4,400 | 0.548796 | 0.503253 | 0.696720 |
| 4,600 | 0.579280 | 0.467343 | 0.647004 |
| 4,800 | 0.607559 | 0.433109 | 0.599610 |
| 5,000 | 0.633747 | 0.400813 | 0.554898 |
| 5,200 | 0.658970 | $0.370580 \times 10^{-4}$ | 0.513043 |
| 5,400 | 0.680360 | 0.342445 | 0.474092 |
| 5,600 | 0.701046 | 0.316376 | 0.438002 |
| 5,800 | 0.720158 | 0.292301 | 0.404671 |
| 6,000 | 0.737818 | 0.270121 | 0.373965 |
| 6,200 | 0.754140 | $0.249723 \times 10^{-4}$ | 0.345724 |
| 6,400 | 0.769234 | 0.230985 | 0.319783 |
| 6,600 | 0.783199 | 0.213786 | 0.295973 |
| 6,800 | 0.796129 | 0.198008 | 0.274128 |
| 7,000 | 0.808109 | 0.183534 | 0.254090 |
| 7,200 | 0.819217 | $0.170256 \times 10^{-4}$ | 0.235708 |
| 7,400 | 0.829527 | 0.158073 | 0.218842 |
| 7,600 | 0.839102 | 0.146891 | 0.203360 |
| 7,800 | 0.848005 | 0.136621 | 0.189143 |
| 8,000 | 0.856288 | 0.127185 | 0.176079 |
| 8,500 | 0.874608 | $0.106772 \times 10^{-4}$ | 0.147819 |
| 9,000 | 0.890029 | $0.901463 \times 10^{-5}$ | 0.124801 |