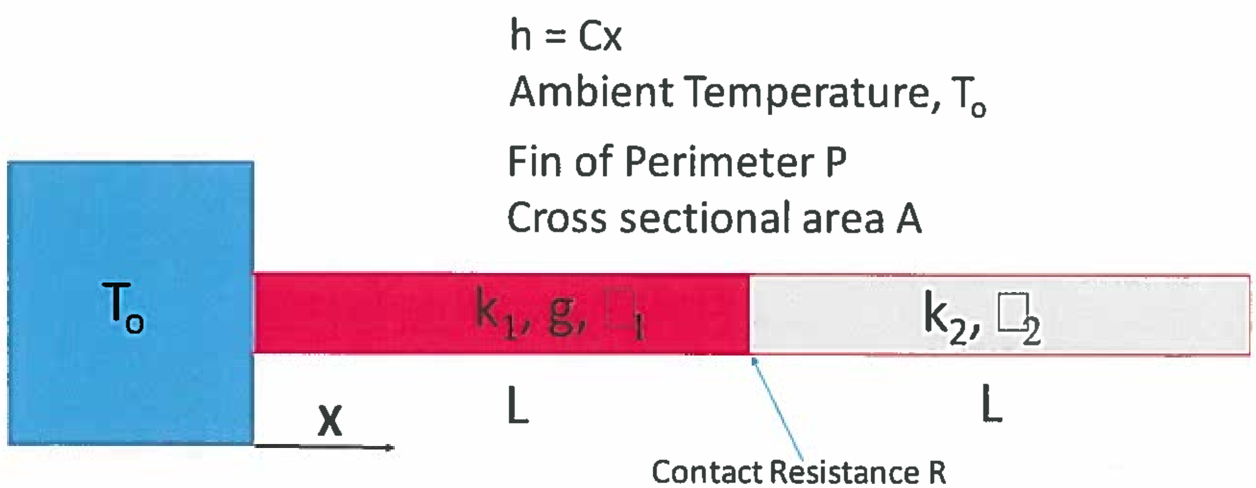
1. A cylindrical fin is composed of two materials each of length L are joined at x=L. Material 1 has a thermal conductivity k_1 and emissivity ε_1 and a heat generation rate g per unit volume. Material 2 has a thermal conductivity k_2 and emissivity ε_2 and does not generate any heat. There is a contact resistance of R at the junction between materials 1 and 2. The base of the fin and the large surroundings are held at a fixed temperature of T_o. There is a convection heat transfer coefficient that increases linearly with x applied to the fin (h = Cx).

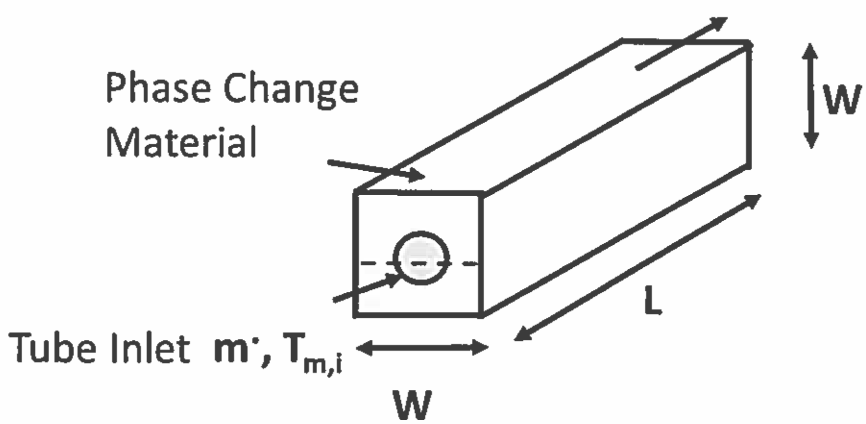
1. Please develop the governing differential equation(s) and appropriate boundary conditions needed to solve for the steady state temperature distribution in the fin. DO NOT try to solve for the temperature solution. (5 pts)
2. Sketch the qualitative temperature distribution in the fin. (3 pts)
3. What conditions (boundary conditions and material properties) should be met for the maximum temperature to occur as close as possible at x = L/2? (2 pts)



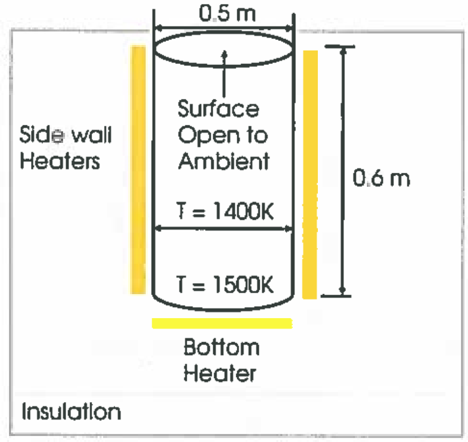
2. In order to harvest and store energy from hot industrial waste gases, they are passed through tubes of length L, radius r and thickness t, and thermal conductivity k_s. The tubes are embedded in a low temperature phase change material with a very high latent heat of fusion and high thermal conductivity that melts at temperature = T_p. The latent heat and density of the phase change material are h_{fg} and $\dot{\rho}_p$. A unit cell of this system can be considered as a tube embedded in a rectangular box of the phase change material with dimensions W x W x L as shown in the figure below. The specific heat of gas is C_p, conductivity is k, viscosity is μ, density is $\dot{\rho}$, and Prandtl Number is Pr. The mass flow rate of gas is m and its mean temperature at inlet is $T_{m,i}$. The flow in tube can be considered as laminar, thermally and hydraulically fully developed. Axial conduction in the pipe and viscous dissipation are negligible and fluid properties can be assumed constant. The gas inlet temperature is high enough such that the phase change material is starting to melt where it contacts the pipe. Convection effects in the phase change material when it turns liquid can be neglected.

1. Plot temperature along the dashed line (passing through tube center) shown in the figure at the inlet and outlet of the tube at a time instant when paraffin in the box has not been completely melted. List your assumptions. **(3 pt)**

2. Perform energy balance and derive an equation to predict temperature at the outlet of the tube, heat exchange between tube and phase change material, and the approximate time to melt all phase change material in the box. **(7 pt)**



3. A specialty metal component is to be heat treated at high temperature in an oven that is open to the surrounding air as illustrated in the figure. The furnace is an open cylinder where the side walls and bottom faces are made of a high thermal conductivity material that spreads heat effectively and provides a uniform temperature on each face. The furnace walls (the top is open to the environment) is then insulated from environment to minimize heat loss, but the top is left open so the part can be inserted and removed easily. The Heat treatment of the part requires that at steady state, when there is no part in the furnace, the side walls reach steady state temperatures of 1400K and the bottom reaches 1500K, while the surrounding ambient is maintained at 298K. The dimensions are given in the figure and all surfaces can be treated using the diffuse-gray approximation. There is an effective resistance due to conduction through the insulation along with natural convection and radiation to the environment. For the side walls this resistance is 2.91 K/W, and for the bottom it is 2.67 K/W. You can neglect direct convection from the heated surfaces to the environment, since the predominant heat loss will be through radiation. Write an appropriate energy balance for the bottom surface including its own heater and assuming no lateral conduction to the side walls. Assume all electrical work input to the heater goes to the hot surface as opposed to leaking through the insulation. Separately determine the amount of electrical power required for the bottom heater and side wall heater to reach the desired steady state condition whereby the side walls are held at 1400K and the bottom is at 1500K, when no part is inserted. The emissivity of the bottom and side walls is 0.5. The view factor between two parallel coaxial disks of radius r, separated by a distance L, is given by:



$$\text{For } r_i = r_j = r \text{ and } R = r/L: \qquad F_{i\to j} = F_{j\to i} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2}$$

Coaxial parallel disks