

PLEASE NOTE:

You need to correctly answer only 3 out of the 4 problems to receive full credit. In case you attempt all 4 problems, clearly state which 3 problems you want to be graded. If you do not explicitly so indicate, the 3 lowest scores will be used.

Problem #1

A prismatic bar with two sections is mounted between two rigid walls, as shown in Figure 1. The structure is initially stress free, and is subsequently subjected to temperature change and a concentrated force F applied at D . The modulus, cross-sectional area, thermal expansion coefficient, and temperature change in each section are marked in the figure, respectively. The length of each section is also marked in the figure. Assume the structure remains elastic. Please find the axial stress in section AD .

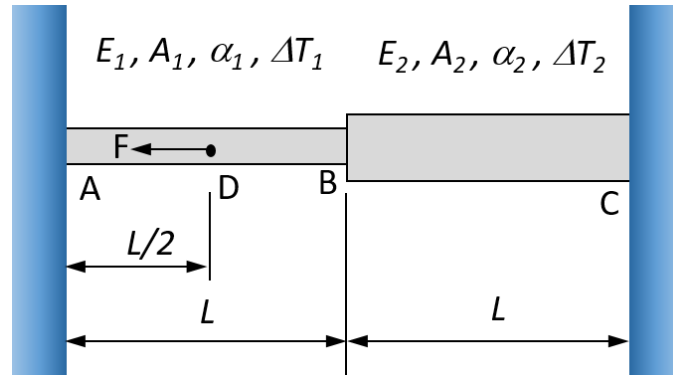
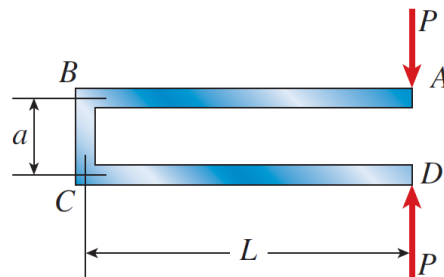


Figure 1

Problem #2

The frame $ABCD$ shown in Figure 2 is squeezed by two collinear forces P acting at points A and D . The flexural rigidity EI is **constant throughout the frame**. *Note: Draw necessary free-body diagrams.* Disregard the effects of axial deformations and consider only the effects of bending due to the loads P .

- 1) What is the decrease in the distance between points A and D when the loads P are applied?
- 2) If C is perfectly welded on a support against all degrees of freedom, what is the decrease in the distance between points A and D when the loads P are applied?
- 3) If member BC is firmly fixed on a rigid wall, what is the decrease in the distance between points A and D when the loads P are applied?

**Figure 2**

Problem #3

A cylindrical oxygen tank made of an E-glass fiber-reinforced epoxy contains oxygen at a pressure of 10 MPa. The tank has a mean diameter of 300 mm and a wall thickness of 8.9 mm. The fiber orientation angles in various layers of the tank wall are $\pm 55^\circ$ with its longitudinal axis.


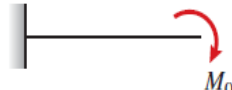
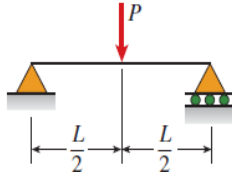
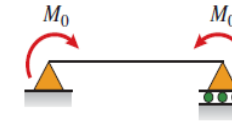
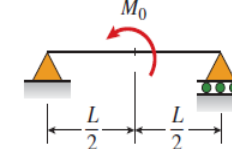
- (a) Neglecting the interaction between the layers, calculate the stresses in the principal material directions for both fiber orientation angles.
- (b) Will any of the layers fail under the applied load if the tensile strengths in the longitudinal and transverse directions of E-glass fiber-epoxy are 140 MPa and 120 MPa respectively and the shear strength is 100 MPa?

Problem #4

- (a) You are to estimate the fatigue life (consisting of initiation fatigue life and propagation fatigue life) of a notched component tested under uniaxial loading with non-negligible plastic deformation occurring at the notch. Assume a constant amplitude loading (far-field stress S is nominally elastic) with a mean stress S_m . Describe the various steps required to predict the fatigue life using relevant equations, relevant material properties (that you need to define), and schematics to illustrate your reasoning.
- (b) You are to measure the fatigue endurance limit, σ_e , for 4 types of specimens made of the same metal and being tested under identical conditions (fully-reversed, constant amplitude loading), except for the following:
- Specimen 1 is large (i.e., width more than 250 mm) and tested under uniaxial load;
 - Specimen 2 is small (i.e., width less than 8 mm) and tested under uniaxial load;
 - Specimen 3 has the same size as Specimen 1 but is tested under bending mode;
 - Specimen 4 has the same size as Specimen 2 but tested under bending mode.
- Rank the expected σ_e values for the 4 specimens (σ_{e1} , σ_{e2} , σ_{e3} , σ_{e4}) from largest to smallest value, and explain your reasoning.

Supplemental Information

A table of beam deflections is given below for your reference. It may or may not be useful.

<p>a</p> 	$v = -\frac{Px^2}{6EI}(3L - x) \quad v' = -\frac{Px}{2EI}(2L - x)$
<p>b</p> 	$v = -\frac{M_0x^2}{2EI} \quad v' = -\frac{M_0x}{EI}$ $\delta_B = \frac{M_0L^2}{2EI} \quad \theta_B = \frac{M_0L}{EI}$
<p>c</p> 	$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \quad v' = -\frac{P}{16EI}(L^2 - 4x^2) \quad \left(0 \leq x \leq \frac{L}{2}\right)$ $\delta_C = \delta_{\max} = \frac{PL^3}{48EI} \quad \theta_A = \theta_B = \frac{PL^2}{16EI}$
<p>d</p> 	$v = -\frac{M_0x}{2EI}(L - x) \quad v' = -\frac{M_0}{2EI}(L - 2x)$ $\delta_C = \delta_{\max} = \frac{M_0L^2}{8EI} \quad \theta_A = \theta_B = \frac{M_0L}{2EI}$
<p>e</p> 	$v = -\frac{M_0x}{24LEI}(L^2 - 4x^2) \quad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \quad \left(0 \leq x \leq \frac{L}{2}\right)$ $\delta_C = 0 \quad \theta_A = \frac{M_0L}{24EI} \quad \theta_B = -\frac{M_0L}{24EI}$