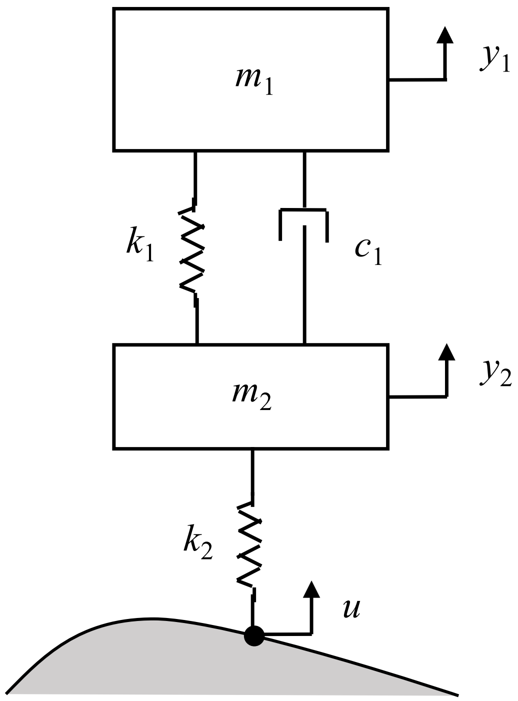
Dynamics Systems & Control Ph.D. Qualifying Exam
Fall 2017

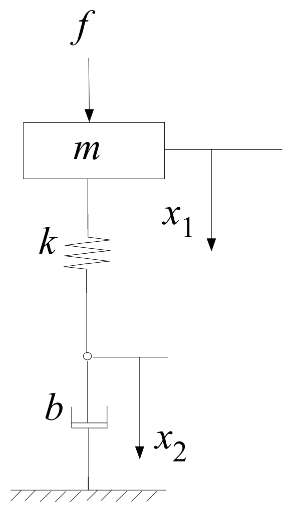
Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem: The system pictured below is a simplified model of a passive quarter-car suspension system. The mass m_1 represents the mass of the car body, k_1 and c_1 represent the suspension mechanism, and m_2 and k_2 represent the wheel mass and tire stiffness, respectively. The wheel contacts the ground at point P, and the vertical motion of this point forms the input u. Note that the displacements y_1 and y_2 are measured from their respective equilibrium positions with no input applied ($u = 0$).

Find the transfer function $Y_1(s)/U(s)$ for the system shown below.



Problem: Consider the mechanical system shown below and answer the questions.



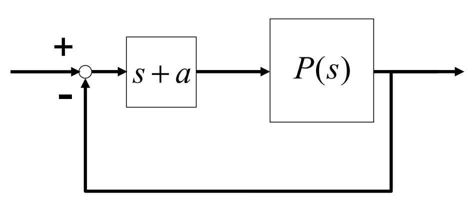
1) Obtain the transfer function from input force *f* to output displacement x_2, or $P(s) = \dfrac{X_2(s)}{F(s)}$.

2) Briefly discuss the stability of $P(s)$. (1 sentence OK)

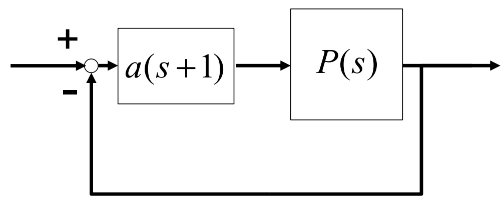
3) Assume $m = b = k = 1$. Use the result in 1) and sketch the unit impulse response of the system output x_2 versus time. Note: A general sketch is acceptable where complete mathematical derivations using the inverse Laplace transformation are NOT required. However you should a) determine *the final value of the output*, and b) provide sufficient discussion or justification about *whether the response exhibits an overshoot or not*.

4) Determine the condition between parameters m, b, k so that the system is critically damped.

5) Consider the feedback system shown below. Determine the range of parameter *a* such that the closed system is stable. Assume $m = b = k = 1$.

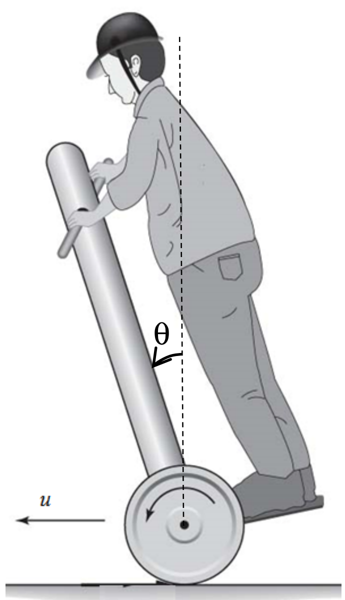


6) Consider the feedback system and assume $m = b = k = 1$. This closed-loop system is stable for any $a > 0$. *TRUE* or *FALSE*? Briefly explain why.

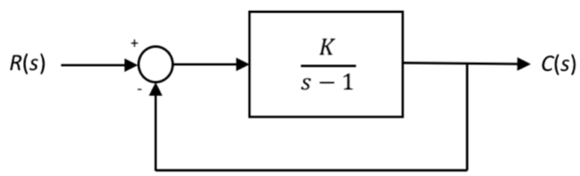


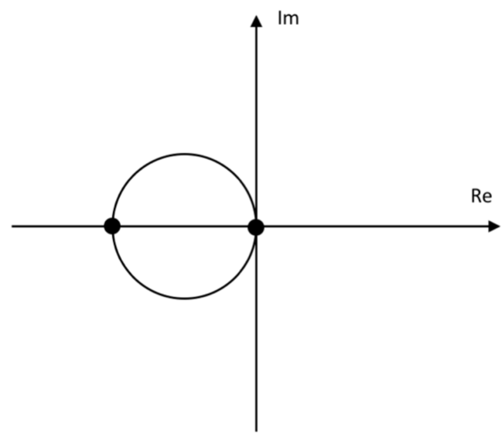
Problem: The linearized equation of motion of the Segway transportation device shown below is given by $\ddot{\theta} - \theta = bu$ and $\dot{v} = u$ where b is a positive constant, u is the linear acceleration input, θ is the angular displacement output, and v is the linear velocity output. To simultaneously regulate velocity and to keep the rider upright, a controller of the form $u = -K_p\theta - K_d\dot{\theta} + K_v(r - v)$ where r is the desired velocity.

a) Draw a block diagram showing the interaction between the plant (Segway) and its controller. Find the resulting closed-loop transfer function V(s)/R(s).
b) Find the control gains K_p, K_d, and K_v for b=1 such that all the closed loop poles of the system are placed at –1.
c) Fixing the control gains you found in (b), plot the closed-loop poles (root locus) as b varies from 0 to ∞. For what values of b is the closed-loop system stable? [You may use K_p=K_d=4 and K_v=-1 if you are not sure about your answers in (b)]



Problem: Consider the unity feedback system shown below. The gain K is to be set by the control designer. The Nyquist plot for this system is shown in the figure to the right.





a) Label points on the $G(j\omega)$ locus above corresponding to $\omega = 0$, $\omega = +\infty$, and $\omega = -\infty$. The value at one or more of these points can be written in terms of the gain K. Also, label the direction of the $G(j\omega)$ locus in terms of increasing frequency.

b) By analyzing the Nyquist plot, determine the value(s) of the gain K for which the system stable. Discuss your reasoning using the Nyquist stability criterion.

c) Let $K = 2$. Using the Nyquist plot, determine the phase margin for this system at this gain value.