Dynamics Systems & Control Ph.D. Qualifying Exam
Spring 2017

Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1

An electric motor drive system is used to turn a wheel through a set of gears with ratio N_2/N_1 (where N_1 is the number of teeth on the gear on the input shaft, and N_2 is the number of teeth on the gear on the output shaft). The viscous friction on the input shaft is given by b_1, and the viscous friction on the output shaft is given by b_2. Let I_1 represent the combined inertia of the motor shaft, input shaft, coupling and first gear. Let I_2 represent the combined inertia of the second gear, output shaft, and load.

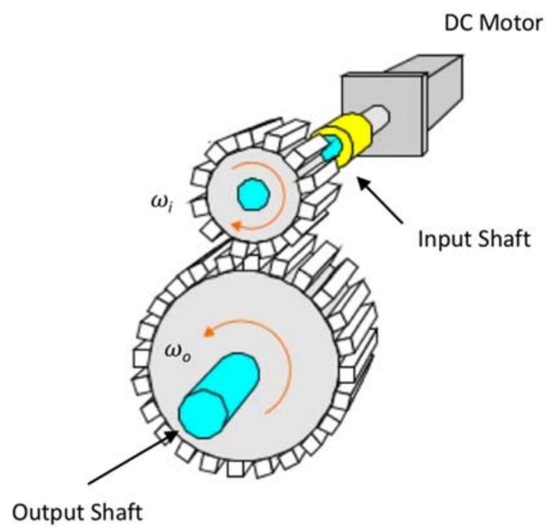
Develop a dynamic model that relates the voltage input to the motor (V) to the angular velocity of the output shaft (ω_o). **Provide the Laplace domain (transfer function) representation of this model.** The equations that govern the torque, current, and voltage relationships for a DC motor are given below:

$$I = \left(V - V_{bemf}\right)/R$$

$$V_{bemf} = K_E \omega_i$$

$$T = K_T I$$

R = terminal resistance of the motor
I = current through the motor
K_E = back-EMF constant of the motor
K_T = motor torque constant
V_{bemf} = back-EMF voltage
T = torque exerted on motor (input) shaft



Problem 2

Consider a dynamic system whose input-output transfer function G(s) is given by

$$\frac{Y(s)}{R(s)} = G(s) = \frac{2s+1}{s^2-3s+2}$$

a) What is the definition for a stable system and unstable system?

b) Based on your definition, is the system stable?

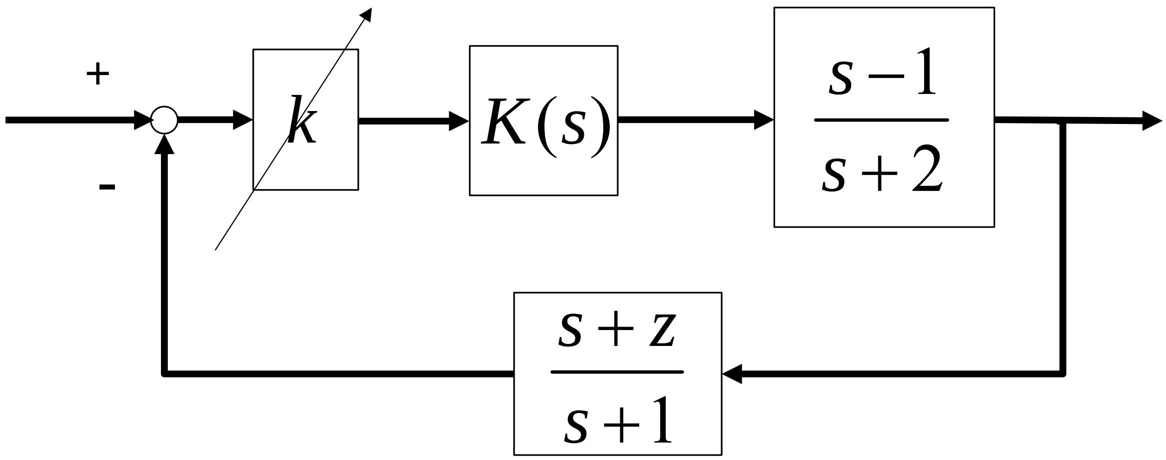
c) Suppose $u(t) = \delta(t)$ (Dirac delta function), $y(0) = 1$, $\dot{y}(0) = 2$. Find $y(t)$, $t \geq 0$.

d) Using the same input, do there exist other initial conditions $y(0)$ and $\dot{y}(0)$ which render $y(t) = 0$, $t \geq 0$?

e) Following (4), if your answer is yes, find all these initial conditions. Furthermore, does your answer contradict your answer in (a)? If your answer is no, stop.

Problem 3

Consider the following feedback system with a loop gain *k* (>0). z is a real number.



a) Assume the unity proportional controller or $K(s) = 1$. Also assume $z = -1$.

(a-1) Sketch the root-locus plot of the closed-loop system. Determine the angles of departure from complex poles, intersection of asymptotes, break-in/away points, if they exist.

(a-2) Determine the range of *k* such that the closed-loop system is *stable*.

(a-3) Determine *k* such that the closed-loop system is *stable* and *critically damped*.

(a-4) Determine the range of *k* such that the closed-loop system is *stable* and *underdamped*.

(a-5) You want to determine *k* such that a typical response of the closed-loop system is oscillatory and its frequency of oscillation is the *greatest*. Specify such closed-loop pole(s) on the root-locus diagram you gave in (a-1). Briefly explain why such pole(s) satisfy the requirement. No calculation is necessary.

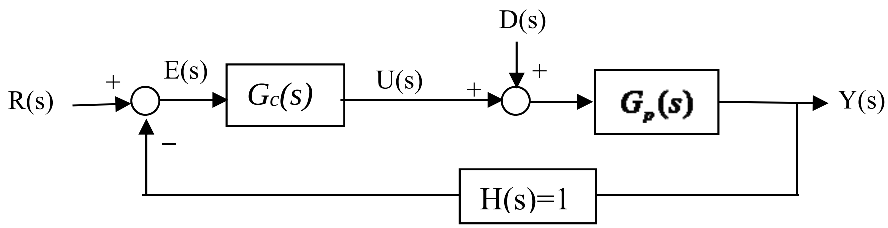
b) Assume the unity proportional controller or $K(s) = 1$.

Find z so that the closed-loop system is always overdamped when the system is stable. The answer is not unique. Justify your answer using the root-locus method. No calculation is necessary.

c) Assume $z = 3$. A control engineer proposes to use $K(s) = \frac{1}{s-1}$ saying that the controller cancels the unstable zero at 1 and stabilizes the closed-loop system for any *k* (>0). Explain why such a controller should NOT be used practically.

Problem 4

Consider a feedback system shown below where $G_p(s) = \dfrac{1}{s(s+1)(0.5s+1)}$.



a) Sketch the bode diagram of $G_p(s)$. Determine the gain and phase margins.

b) Design a lag compensator of the form $G_c(s) = K\dfrac{(s/a)+1}{(s/b)+1}$ so that the static velocity error constant (to a unit ramp reference input) is 5 sec^{-1} without significantly affecting the phase and gain margins.