GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - FALL Semester 2001

Fluid Mechanics
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

1. In the troposphere (the region of Earth's atmosphere at altitudes $z < 11$ km) the temperature decreases linearly with altitude:

$$dT = -\lambda\, dz\,,$$

where λ is a constant known as the lapse rate ($\lambda > 0$). The temperature at sea level ($z = 0$) is $T(0) = T_o$. You may assume that the air in the troposphere has ideal-gas behavior with an ideal gas constant R.

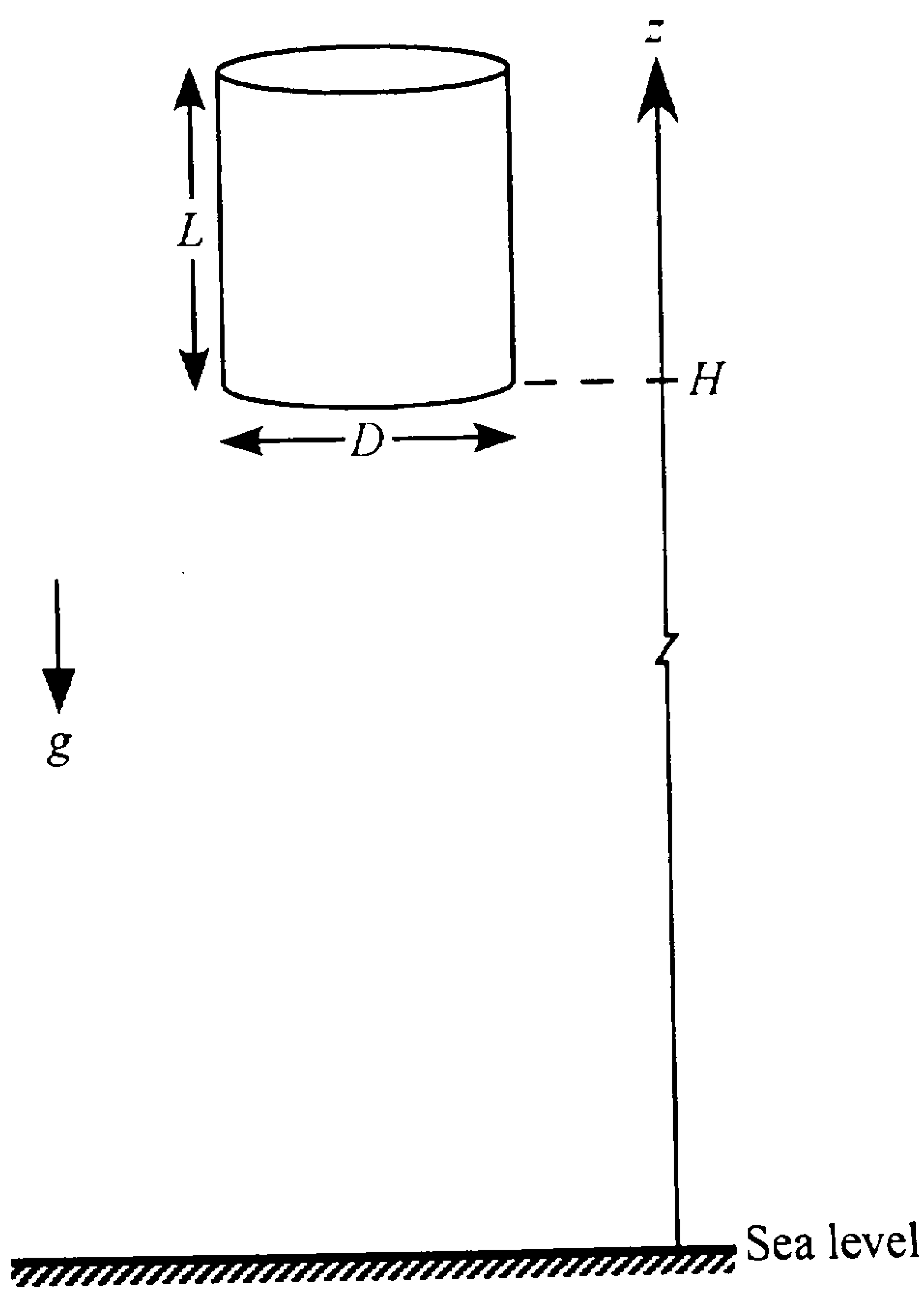
a) Find $p(z)$, the pressure distribution in the troposphere as a function of the altitude z, given that $p(0) = p_o$.

b) Show that the behavior of the air in the troposphere can be described by a polytropic process:

$$\frac{p}{\rho^n} = \text{constant}\,,$$

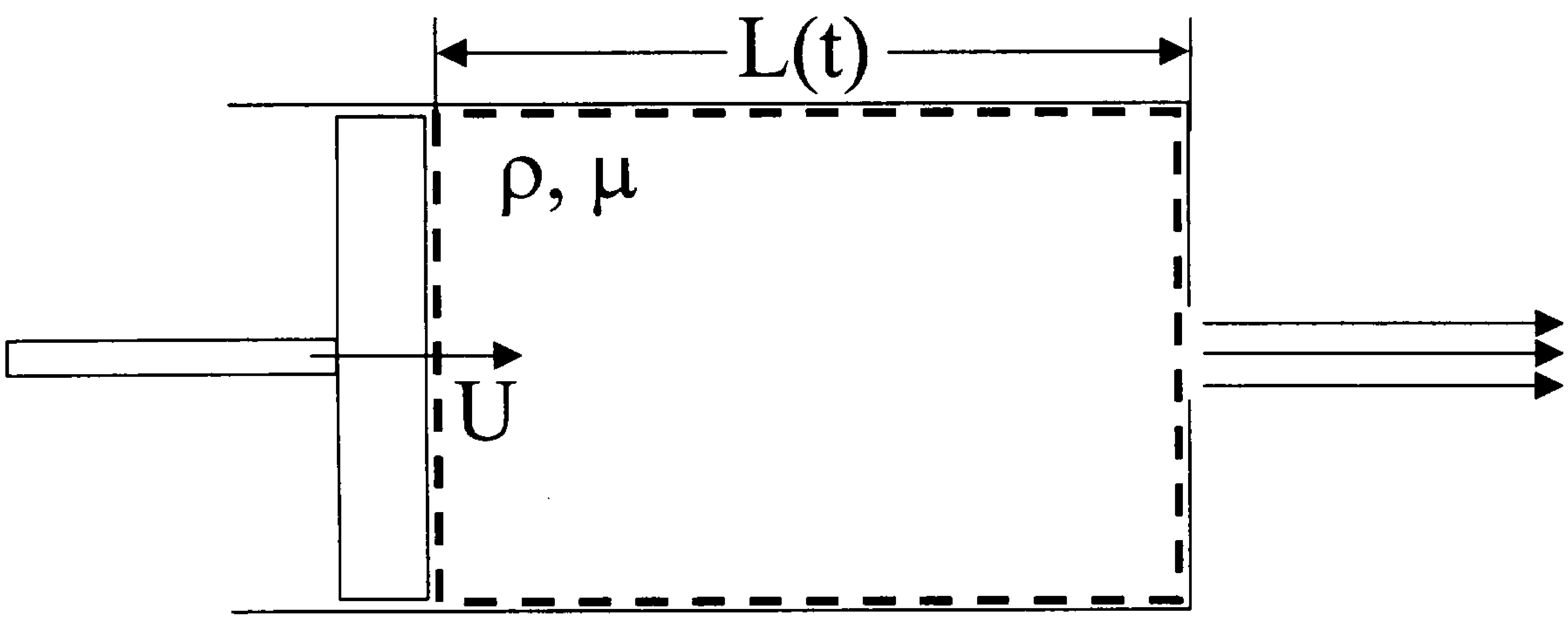
where ρ is the density of the air. Determine the polytropic exponent n.

c) Find the buoyancy force acting upon a cylindrical body of diameter D and height L if the bottom of the cylinder is at an altitude $z = H$.



2. A jet of liquid of density ρ and viscosity μ is ejected out of an orifice (diameter *d*) by a piston that is moving with constant velocity *U* within a circular cylinder (diameter *D*) as shown in the sketch below. During the portion of the piston motion that is of interest here, $L(t) \gg D$ and the jet velocity is uniform across the orifice. The distribution of pressure along the internal surfaces of the cylinder p_i may be approximated as uniform (except across the jet orifice where it may be taken to be atmospheric). It is assumed that there are no secondary motions within the cylinder, and that the distribution of shear stress τ on the inner surface of the cylindrical shell is uniform. The friction force on the piston (as a result of its motion within the cylinder) is $F_f = kU$.

Using a control volume analysis ***and the control volume that is marked (by a dashed line) in the sketch*** (i.e., bounding the internal cavity of the cylinder), determine the force F_p that is necessary to maintain the piston velocity in terms of some (or all) of the given variables.

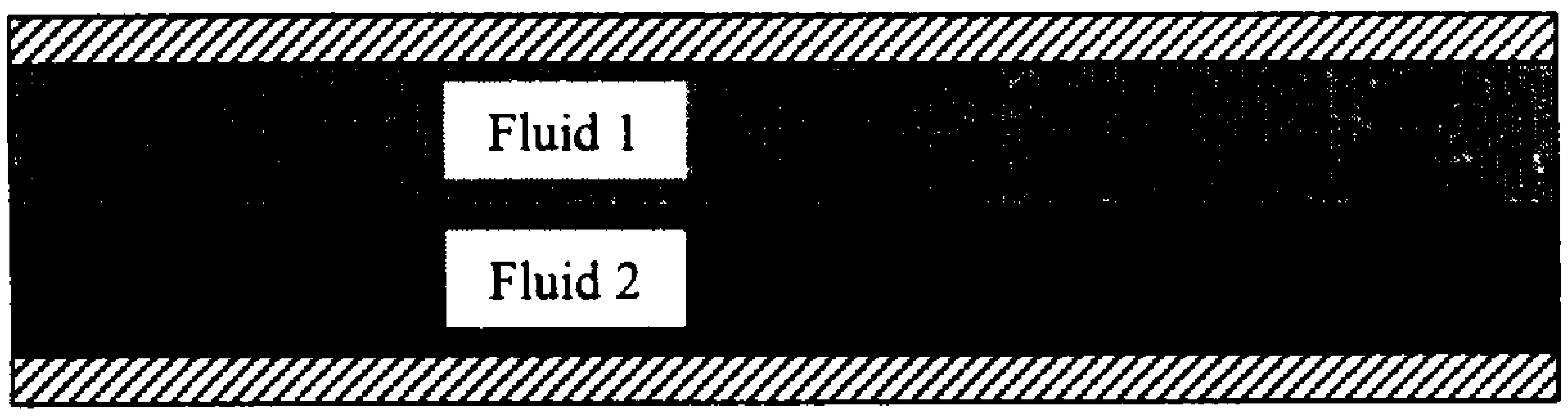


3. Consider a steady, planar, laminar Couette flow of two immiscible liquid layers of equal thickness confined within a narrow gap between two long horizontal plates.

a) Find the velocity and shear-stress distributions across the gap when the upper plate moves to the right at a constant velocity and the lower one remains stationary.

b) Repeat part (a) when the lower plate moves to the left at the same speed as the upper plate moves to the right.

c) Develop a criterion for no net flow across the gap for the case described in part (b).



4. A liquid droplet suspended in space is given a slight deformation and then released so that it oscillates with a frequency ω, which appears to be a function of the density ρ, the viscosity μ, and the droplet radius R. Using dimensional analysis, find the proper nondimensional parameter for the frequency of oscillation and the time τ it takes for the oscillations to die-down as a function of the flow properties.