GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2003

Fluid Mechanics
EXAM AREA

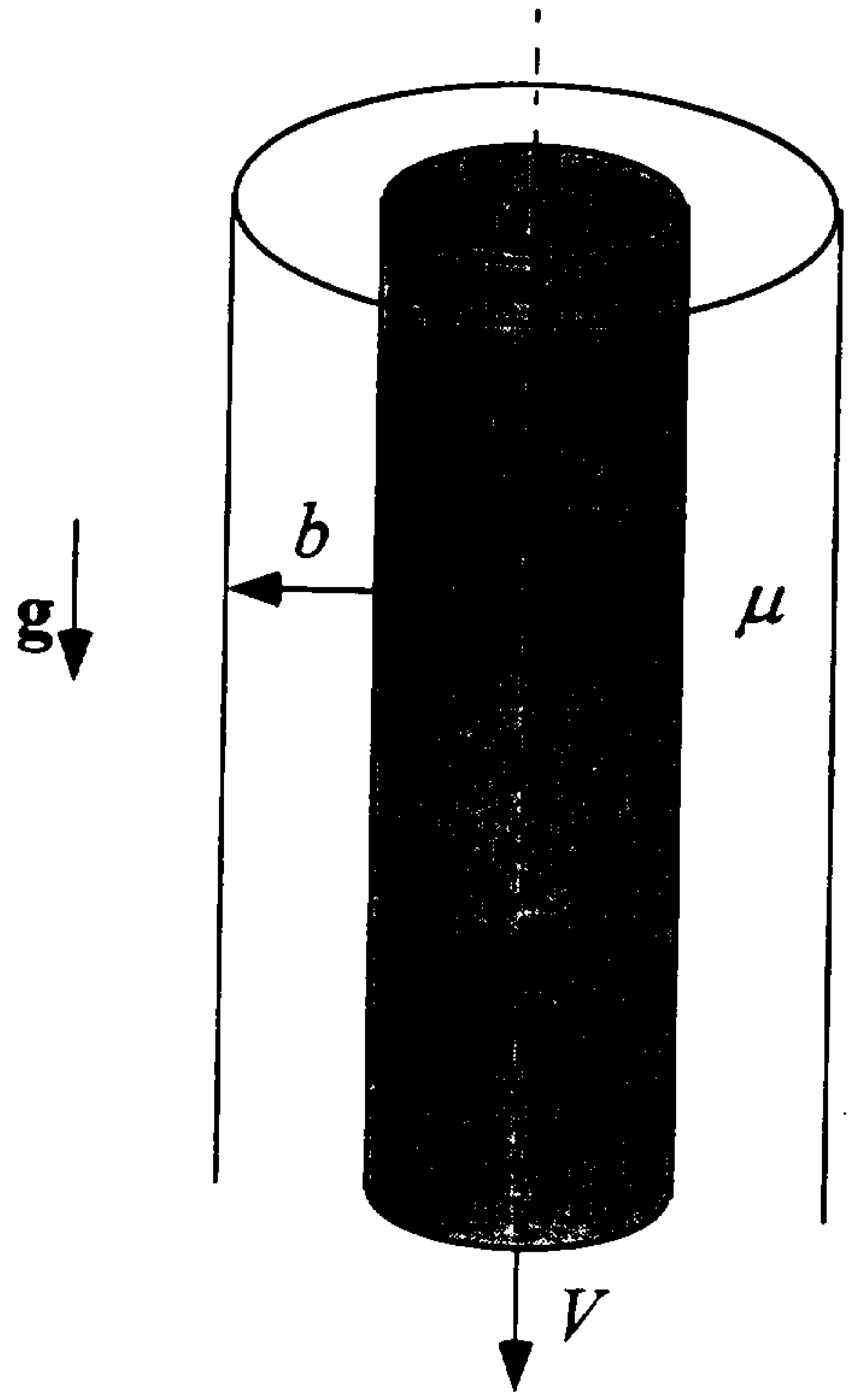
Assigned Number **(DO NOT SIGN YOUR NAME)**

* Please sign your **name** on the back of this page —

ME Fluid Mechanics Qualifying Examination

Fall, 2003

All problems are of equal weight—WORK ALL PROBLEMS!



1. A rod of density ρ and radius a falls along the center of a very long, vertical tube of radius b that is filled with oil of viscosity μ.

a) What is the terminal speed V of the rod?

b) A pressure gradient is established in the oil to drive flow in an *upward* direction. What is the magnitude of the pressure gradient required to keep the rod motionless?

(You assume the specific weight of the rod is much greater than that of the oil and consequently may ignore gravitational forces *in the oil* for both parts of this problem.)

The Navier-Stokes and continuity equations, expressed in cylindrical coordinates, are given below.

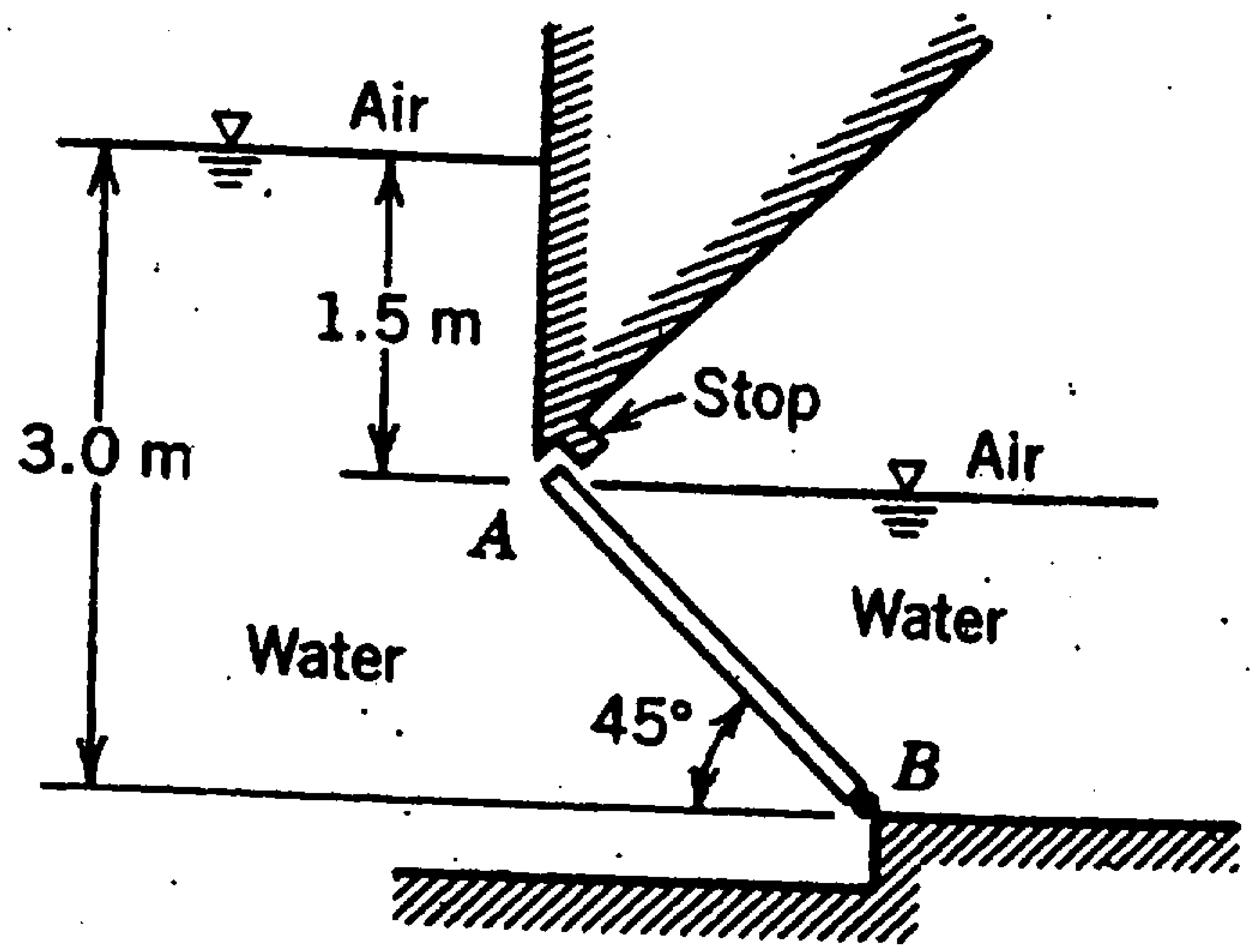
$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]-\frac{\partial p}{\partial r}+\rho g_r,$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta,$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z,$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0.$$

2. The rectangular gate AB shown in the diagram below has a width w. Find the force per unit width exerted against the stop at A. Assume the mass of the gate is negligible and that the fluid is water with a density of $1000\ kg/m^3$.

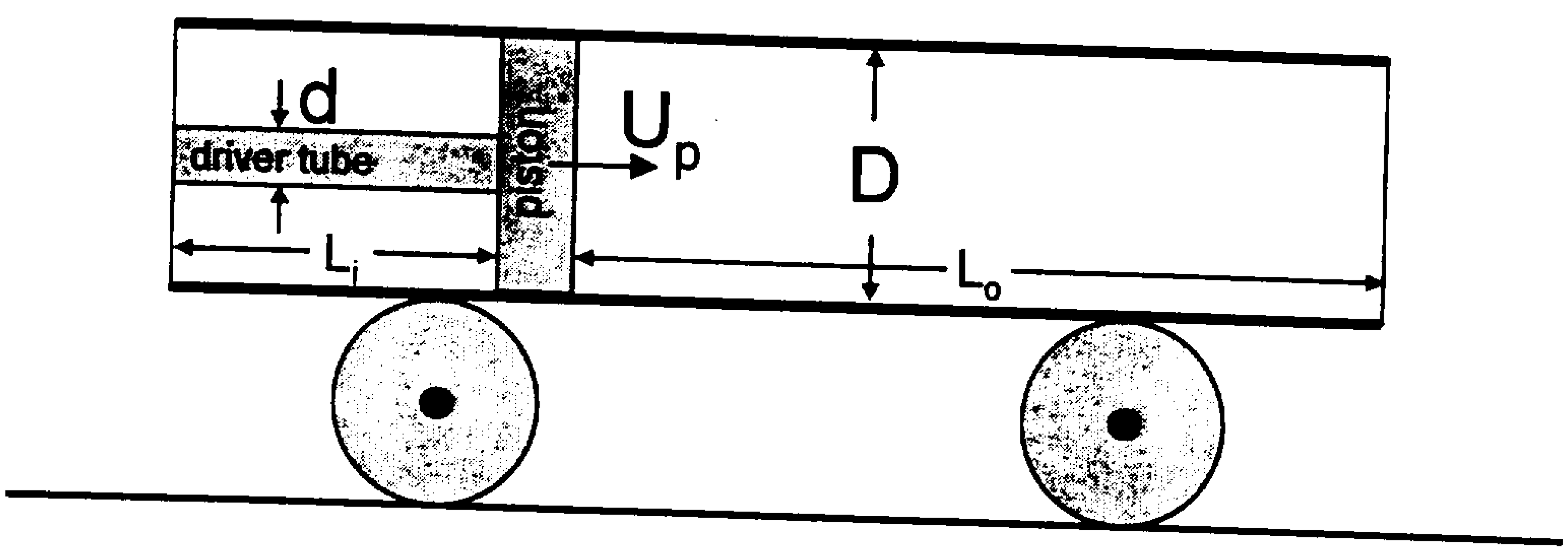


3. It is proposed to propel a cart of mass M_c on a horizontal, frictionless surface by ejecting a momentary air jet (density ρ_a) out of a cylindrical tube having an inner diameter D, as shown in the sketch below. The jet is ejected by the motion of a massless piston that is driven to the right at a constant speed U_p using a massless pneumatic mechanisms housed in a coaxial telescopic cylinder of diameter d as shown below. Before the motion begins, the piston is located distances L_i and L_o from the left and right edges of the tube, respectively. It may be assumed that all transients that are associated with the onset of the motion as well as compressibility and friction effects within the tube are negligible and that the air speed through the tube's left and right end cross sections is uniform and the pressure there is atmospheric.

Using a control volume analysis and the information that is given above, determine:

a) The force on the cart during the piston's motion. (Please list and explain your assumptions.)

b) How would you maximize the force?



4. In the transport of sand particles by ocean waves, the shear stress τ on the ocean bottom is a function of the particle length scale d, the acceleration g due to gravity, the difference $\Delta\rho = \rho_s - \rho_f$ between the sand and seawater densities (subscripts s and f indicate sand and fluid, respectively), and the viscosity μ and density ρ_f of the seawater.

a) Use dimensional analysis to find the appropriate Π groups for this problem, and write the shear stress as a function of all the other parameters.

b) For a given type of sand, suggest a way to plot these data in terms of Π groups analogous to the drag coefficient plot for a sphere. In other words: how would you define a sand Reynolds number for this problem? How would you define a normalized shear stress (also known as the Shields number)?