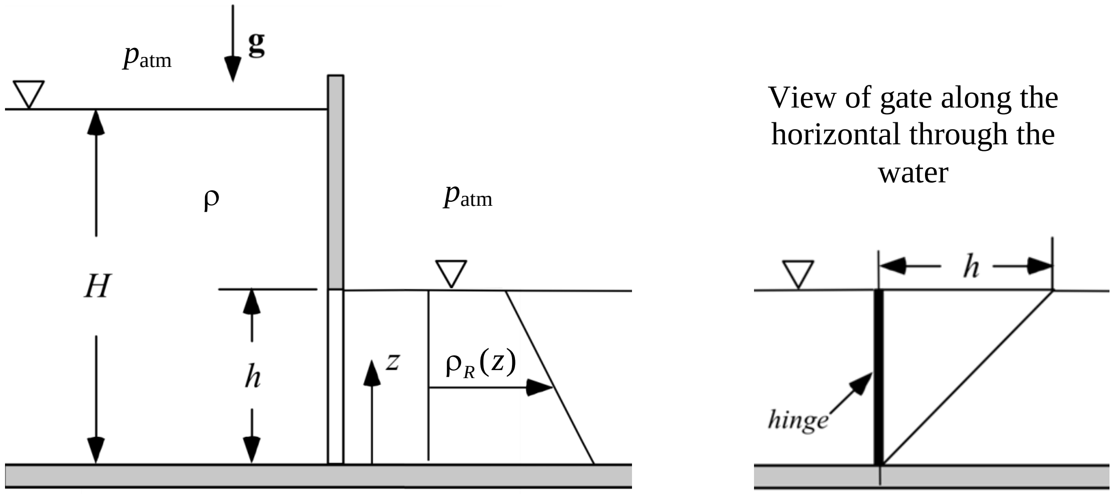
1) A hawk with a wing span five times larger than its prey, a sparrow, has a wing beat frequency $\omega = 1.2 \text{ s}^{-1}$. In order for the sparrow to survive, it must have a higher wing beat frequency.

a) Use dimensional analysis to find the relevant dimensionless groups if the wing beat frequency, ω, is a function of the wing span, S, the specific weight of the birds, γ (assume that the hawk and the sparrow both have the same specific weight), the acceleration due to gravity, g, and the density of air ρ. Then express the wing beat frequency as a function of the other relevant parameters.

b) What is the minimum wing beat frequency that the sparrow must have in order to avoid being captured by the hawk? Assume that there is a power-law relation between the wing beat frequency and wing span, so $\omega \propto S^n$.

2)

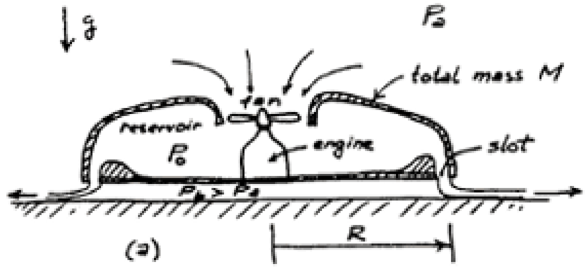


A triangular gate of width h and height h in a vertical wall divides two bodies of water, which are both bounded above by air at atmospheric pressure p_{atm}, as sketched above (note that the view of the gate and hinge on the right is what would be seen looking from left-to-right through the water). The water on the left side of the gate is of constant density ρ and depth H, while that on the right side of the gate extends only to the top of the gate and is linearly density-stratified through the use of salt, having a density profile given by $\rho_R(z) = \rho + \alpha(h - z)$, where $\alpha > 0$ is a constant (so that the density of the salty water at the top of the gate on the right-hand side matches the constant density of the fresh water on the left-hand side).

a) Determine the gage pressure distributions on both sides of the gate.

b) Determine the stratification parameter α that will keep the gate in a closed position, if no other forces are present.

3)



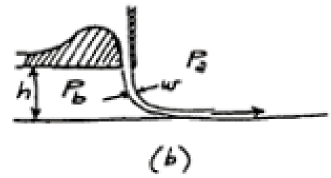


Figure (a) on the left shows the cross-section of an air cushion vehicle of the "peripheral jet" type, first developed by Christopher Cockerell in the mid-1950s. A fan draws air from the ambient atmosphere at pressure p_a through an intake of area A and compresses it to a stagnation pressure p_o in the reservoir inside the vehicle. The air then exhausts downward as a steady jet through a narrow slot of width w at the periphery of the vehicle, creating a positive gage pressure (*i.e.*, the "air cushion") under the vehicle, that allows the vehicle to float at a height h above the ground.

The circular vehicle has a total mass M and outer radius R. Let the pressure difference $\Delta p \equiv p_o - p_a$. Assume that $w << R$ and $h << R$, and that the flow from the reservoir to the slot is steady, incompressible and inviscid. Also assume that the pressure under the vehicle p_b is just slightly greater than p_a, so $p_b - p_a << \Delta p$.

The known parameters are M, g, R, w and Δp.

a) Derive an expression for the velocity of the air across the slot jet V_j.

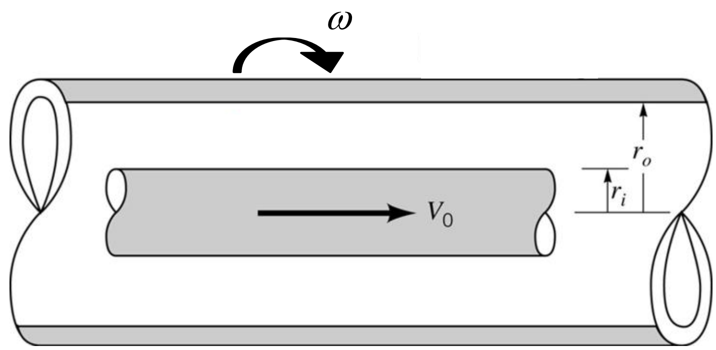
b) Show that the weight of the vehicle $Mg = (p_b - p_a)\pi R^2$. Hint: the pressure across the intake at the top of the vehicle is less than p_a when the fan is on, so you will have to show that the momentum flux is negligible compared with the net pressure force on a hemispherical control volume that encloses the vehicle.

Now consider the the operating condition for this vehicle when $h >> w$, as shown in the expanded view of Figure (b) on the right. Here, the jet issues from the slot as a thin, sheet of constant width w which becomes parallel to the ground, and flows radially outwards along the horizontal.

c) Obtain an expression for $p_b - p_a$ in terms of the known parameters and h, and show that $p_b - p_a << \Delta p$ for $h >> w$. Then determine h in terms of the known parameters.

d) The power delivered by the fan to the fluid $\dot{W} = (\Delta p)Q$, where Q is the volume flowrate through the vehicle. Show that for a given R and h, choosing a fan that delivers the power required at the lowest pressure Δp minimizes the specific power $\dot{W}/(Mg)$.

4)



A viscous, incompressible, and Newtonian fluid fills the gap between two infinitely long, concentric cylinders. The outer cylinder has inner radius r_o and the inner cylinder has outer radius r_i. The outer cylinder rotates with a constant angular velocity ω, whereas the inner cylinder moves with a constant longitudinal velocity V_0. Assume the flow is laminar, fully developed and axisymmetric.

a) Formulate the relevant assumptions and state the appropriate boundary conditions for this problem.

b) Find the velocity field in the fluid $\vec{\mathbf{V}}$.

c) Is the pressure field p uniform in the fluid? Explain your answer.

Continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial(r\rho v_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta}+\frac{\partial(\rho v_z)}{\partial z}=0$$

Navier-Stokes equations

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$