GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1995

Fluid Mechanics
EXAM AREA

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Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

Work all problems.

All problems are of equal weight.

1. Consider an infinitely long horizontal rigid rod having a circular cross section with diameter D in an unbounded fluid (air) with density ρ and kinematic viscosity ν. The rod is oscillating time-harmonically in the vertical direction with frequency ω and amplitude A.

   (a) Use appropriate scales to write the momentum equation for this problem in dimensionless form and define the relevant dimensionless parameter(s).

   (b) Simplify the equations for the case of small amplitude oscillation when $A \ll \nu/\omega D$.

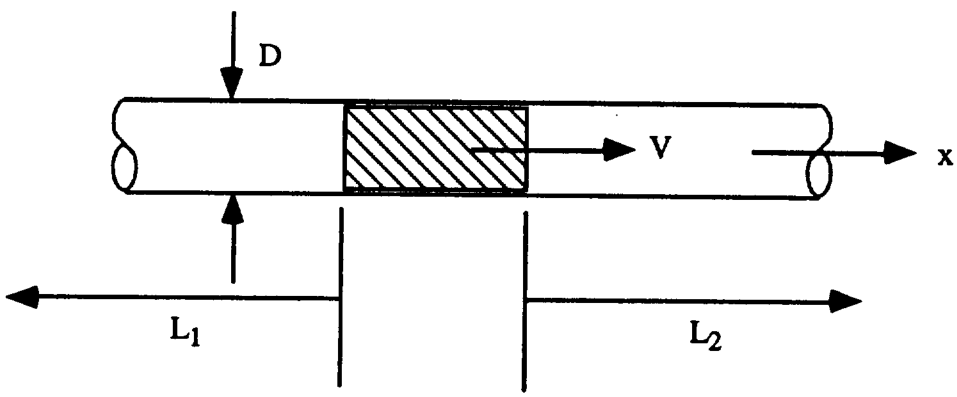
   (c) If the diameter of the rod is decreased by a factor of 2, how should frequency and amplitude of oscillation be changed in order for the flow field to remain dynamically similar?

2. A cylindrical body is moving with constant velocity V through a pipe of length L and diameter D, as shown below. The pipe is open to the atmosphere at both ends and the distances between the rear and front surfaces of the cylindrical body and the inlet and outlet of the pipe are L_1 and L_2, respectively. The clearance between the body and the pipe is very small, so that all the air in front of the body is pushed forward and discharged from the pipe. Assume that the air flow in the pipe is fully-developed, incompressible and laminar.

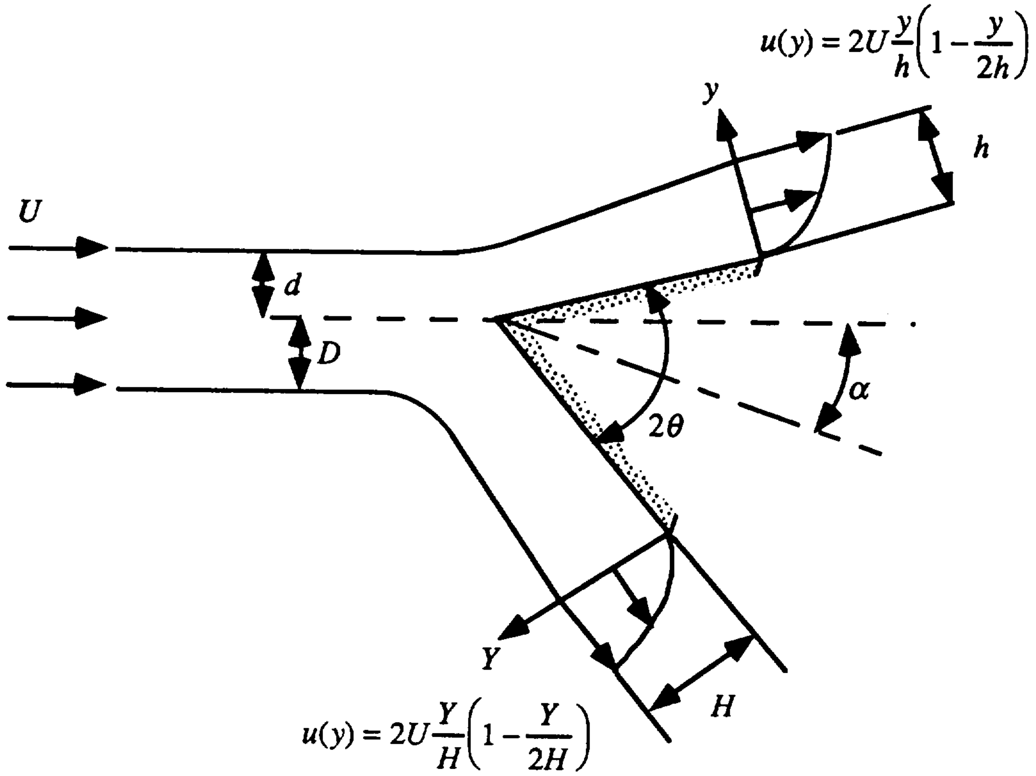
   (a) Determine the change in pressure between the front and rear of the body $\Delta p = p_f - p_r$ as a function of L_1, L_2, D, μ, V, and ρ.

   (b) Neglecting friction between the cylindrical body and the pipe, what is the power required to produce this air flow?

   (c) Explain how the head $h(x) = p(x)/\rho g + z(x) + v^2(x)/2g$ varies with x. Sketch h(x) between the inlet and outlet of the pipe at the instant of time when the piston is in the position shown in the figure.



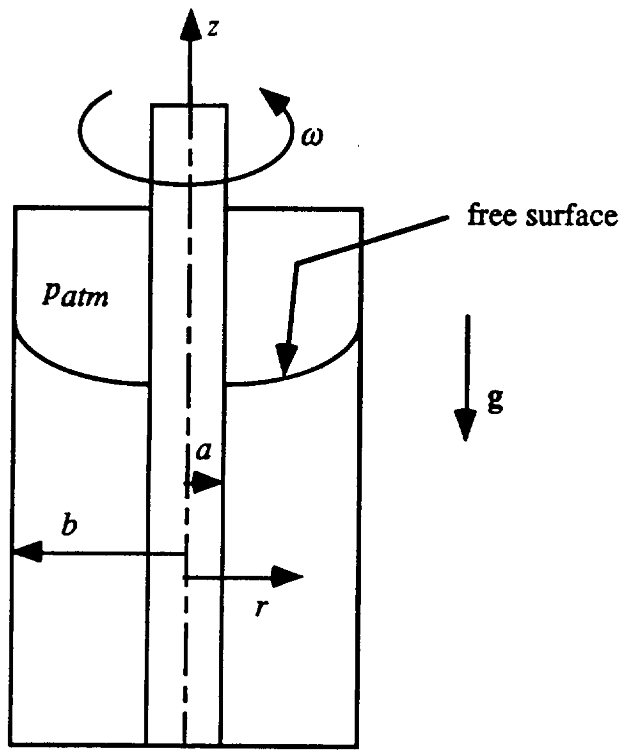
3.



A liquid sheet of thickness $d + D$ strikes a two-dimensional wedge of half-angle θ (inclined at an angle α to the oncoming liquid) with uniform speed U, as shown in the figure above. The nose of the wedge is *not* necessarily in the center of the sheet, i.e., $d \neq D$. The sides of the wedge are long enough that the velocity profiles on each side develop a parabolic shape, as shown in the figure.

Determine the horizontal (drag) and vertical (lift) components of force on the wedge. You may ignore the effect of gravity.

4.



Consider the purely circumferential, steady motion of a Newtonian fluid in the partially filled annular region $a \le r \le b$ between a rotating inner cylinder and a stationary outer one, as shown above.

a) Simplify the Navier-Stokes equations for thie flow, pose the necessary boundary conditions and determine the velocity and pressure fields.

b) Obtain the equation of the free-surface of the liquid.

The Navier-Stokes and continuity equations, in cylindrical coordinates, are

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]-\frac{\partial p}{\partial r}+\rho g_r,$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta,$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_z)\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z,$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial v_z}{\partial z}=0.$$