GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1999

Fluids
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

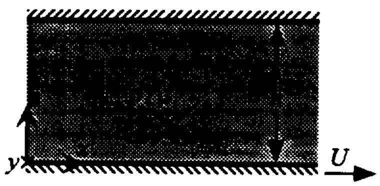
Problem 1

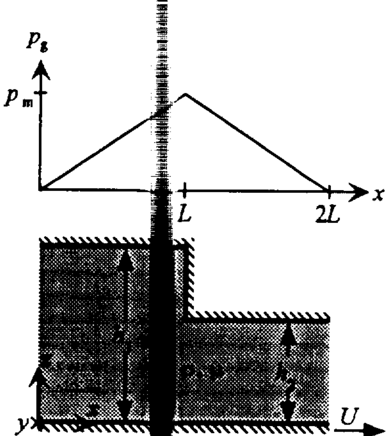
Consider two-dimensional steady Couette-Poiseuille flow of a viscous fluid (den[illegible] viscosity μ) between two parallel plates (below left). The lower plate (z=0) mov[illegible] constant speed U; the upper plate (z=h) is stationary.

a) Find the velocity distribution in this flow. Clearly describe your assumpti[illegible] boundary conditions.

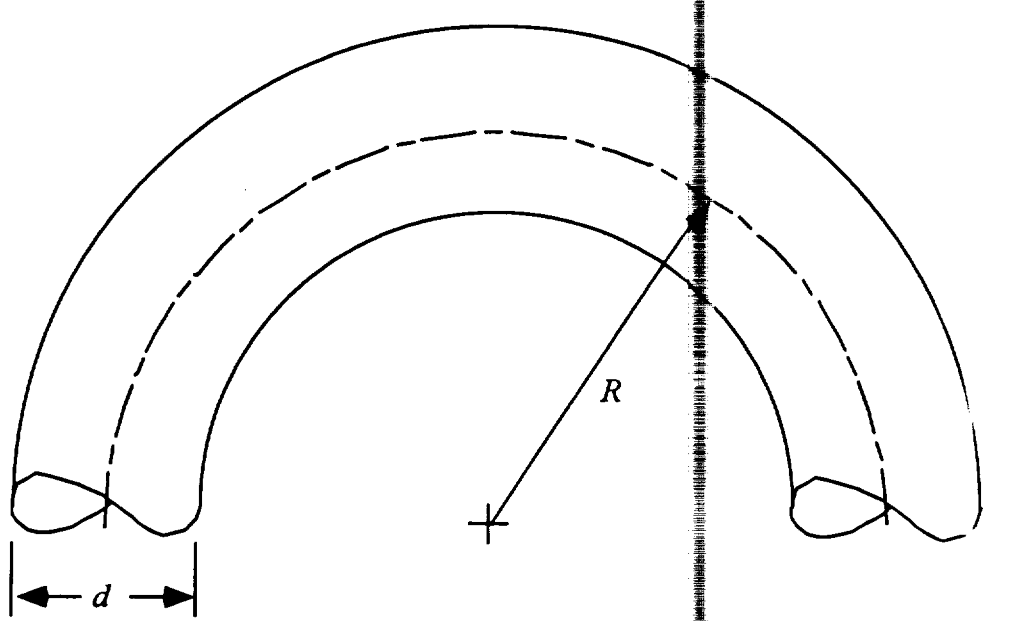
b) What is the volumetric flow rate q per unit width (y-dimension) between the two pl[illegible]

c) Now consider the two-dimensional steady flow in a *stepped bearing*, where the lowe[illegible] surface moves at a constant speed U, and the upper stationary surface contracts [illegible] gap width of h_1 to h_2 (below right). The gauge pressure in this stepped bearing [illegible] (below right) increases linearly along x to a maximum value p_m at the step, an[illegible] decreases linearly along x back to 0. Analyze this geometry as two separate Co[illegible] Poiseuille flows with the same q. Using your expressions for q in the two halves [illegible] stepped bearing and your answer from part (b), find p_m in terms of U, h_1, h_2, L and [illegible]





Problem 2



The aorta is the large artery which carries oxygenated blood from the heart to the rest of the body. Blood must flow through the "aortic arch" just after leaving the heart, and the pulsatile flow through this curved passage (sketched above) is of medical interest.

A bioengineer wishing to construct a model of the aortic arch has identified the following dimensional quantities as being important for study:

$\overline{\nabla p}$ - mean pressure *gradient* U - mean velocity ω - pulsation frequency
d - aorta diameter R - arch radius of curvature ρ - fluid density
μ - fluid viscosity

a. Determine the *dimensionless groups* which the bioengineer will have to match in performing her model studies.

b. The scale model is to be constructed to *twice* the size of the actual aorta and the transparent liquid to be used for flow visualization studies in place of blood has the same density and viscosity as blood. Determine the corresponding mean velocity, pulsation frequency and pressure gradient *relative to the actual heart* at which the model should be operated.

Problem 3

Water is pumped from an open reservoir at sea level through a straight horizontal rou[illegible] length L, and is discharged to the atmosphere (also at sea level) through a nozzle that i[illegible] to the end of the pipe without change in elevation. The friction factor for the pipe is [illegible] coefficient for the nozzle is k, and their respective cross sectional areas are A_1 and [illegible] operating characteristic of the pump (at a constant speed and for a given fluid) is

$$h_p = c - dQ$$

where h_p is the head increase (i.e., the increase in the mechanical energy of the flui[illegible] weight) across the pump, Q is the discharge rate, and c and d are constants.

a) Determine the discharge (volume flow rate) from the nozzle.

b) Discuss ***briefly*** (*no more than three sentences each*) the validity of each of th[illegible] statements:

   i. If the flow in the pipe is fully developed, steady, and laminar, Bernoulli's [illegible] can be used to compute the pressure drop between the inlet and the outlet.

   ii. Moody's diagram can be used to determine the diameter of the larges[illegible] circular pipe that delivers the volumetric flow rate Q (of the working fluid havi[illegible] density ρ and viscosity μ, respectively), a distance L for a given head loss.

   If the flow in the straight pipe is fully developed and turbulent:

   iii. the friction factor always increases with the Reynolds number,

   iv. the friction factor always increases with the length of the pipe.

   v. the mean static pressure decreases exponentially in the flow direction.

Problem 4

A source of strength m is located a distance ℓ from a vertical solid wall as shown in the figure below. The velocity potential for this incompressible, irrotational flow is given by

$$\phi = \frac{m}{4\pi}\{\ln[(x-\ell)^2 + y^2] + \ln[(x+\ell)^2 + y^2]\}$$

a) Show that there is no flow through the wall.

b) Determine the velocity distribution along the wall.

c) Determine the pressure distribution along the wall, assuming $p = p_o$ far from the source.

d) Show that the vorticity is zero everywhere.