GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2001

Fluid Mechanics
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

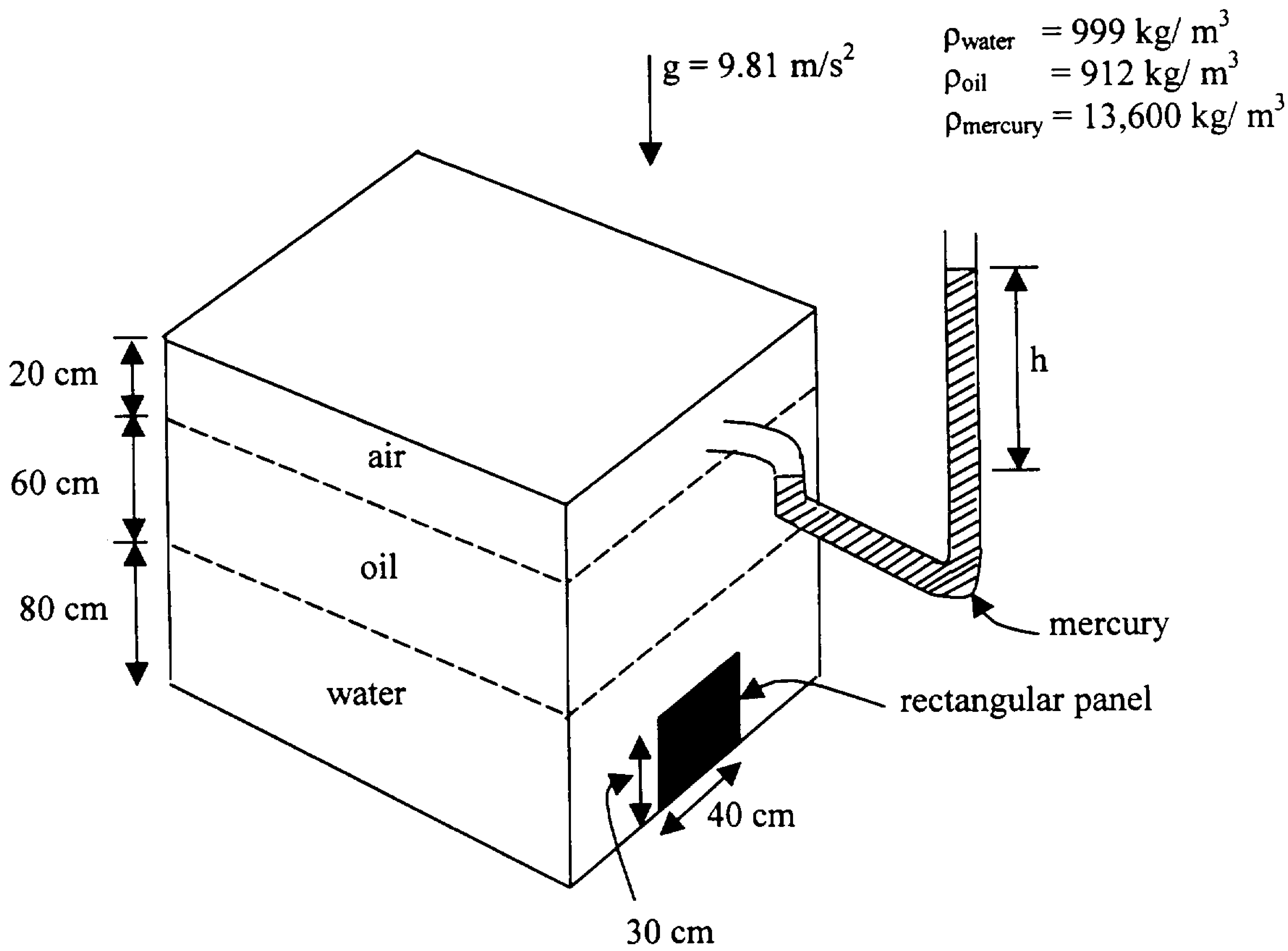
- **Please sign your name on the back of this page—**

1) Consider the closed tank shown below, in which the air space is pressurized. The net outward hydrostatic force on the rectangular panel (in the wall) is 4200 N.

a) Find the pressure in the air space at the top of the tank.

b) Find h, the reading on the mercury manometer.

c) A ball of radius 1.0 cm and mass 0.004 kg is placed in the air space. Will it sink to the bottom? If not, what will happen? Explain your reasoning.



2) It is proposed to propel a cart impulsively on frictionless tracks by releasing it after the attached blower system shown in Figure 1a below is turned on. Each section of the blower system is axisymmetric, and the volume flow rate is Q. The blower is located halfway between the front and rear wheels A and B, respectively, as shown in Figure 2a below. The cross sectional areas of the inlet and outlet (sections "1" and "2") are A_1 and A_2, respectively ($A_1 < A_2$). It is assumed that the inlet and outlet flows are uniform (the wall boundary layers are very thin), and that the static pressure in sections "2" and "1" is nearly atmospheric.

a. Neglecting contact friction on the tracks, determine the magnitude and direction of the cart acceleration immediately after it is released if the mass of the entire system is M.

b. The upward bend in the blower system is designed to reduce the load on the wheels. Assuming that the flow upstream and downstream of each bend is uniform and the overall dimensions of the duct system are given, determine the direction of the moment on the cart due to the blower system for a given flow rate. Explain how this will qualitatively affect the load on wheels A and B.

c. The designer is concerned that the flow might separate upstream of section "2" and that the performance of the blower system may degrade. It is therefore proposed to install internal thin conical partitions in the expansion duct that leads to section 2 as shown in Figure 2b below. The partitions are designed such that the areas of the axisymmetric ducts that are formed between them are equal. Furthermore, it is assumed that the boundary layers on these surfaces are thin enough so that the flow between the partitions at section"2" is horizontal and uniform. Determine whether the initial acceleration in part (a) will increase. *Explain your answer.*

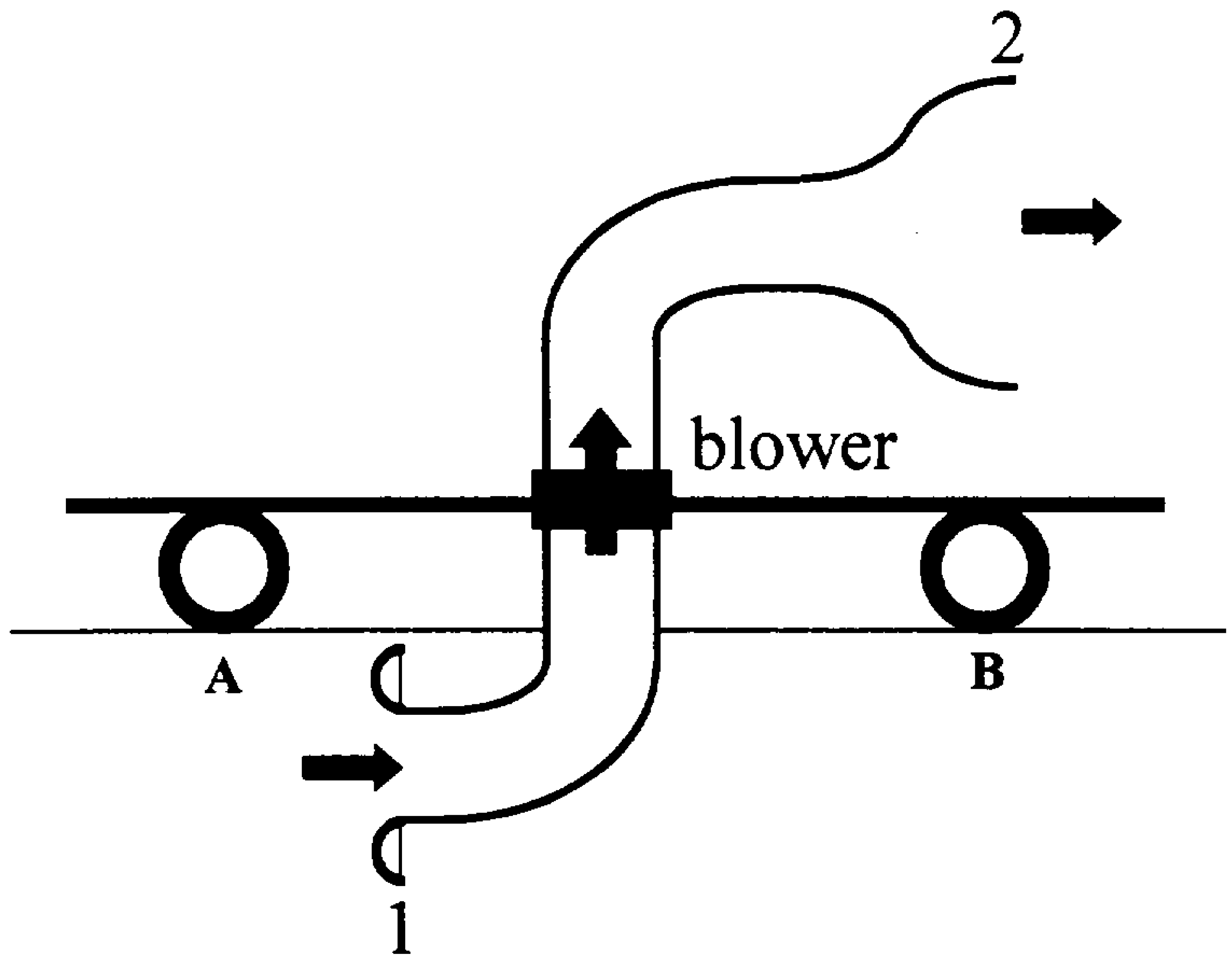


Figure 2a

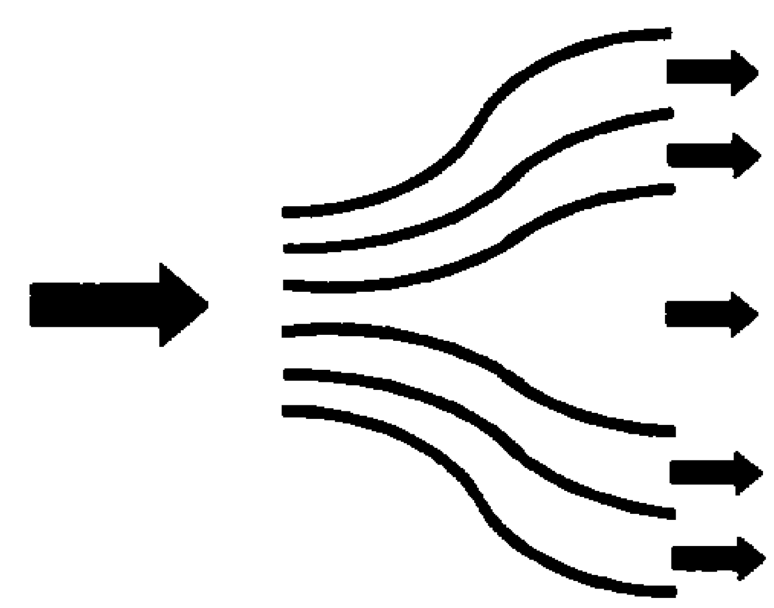


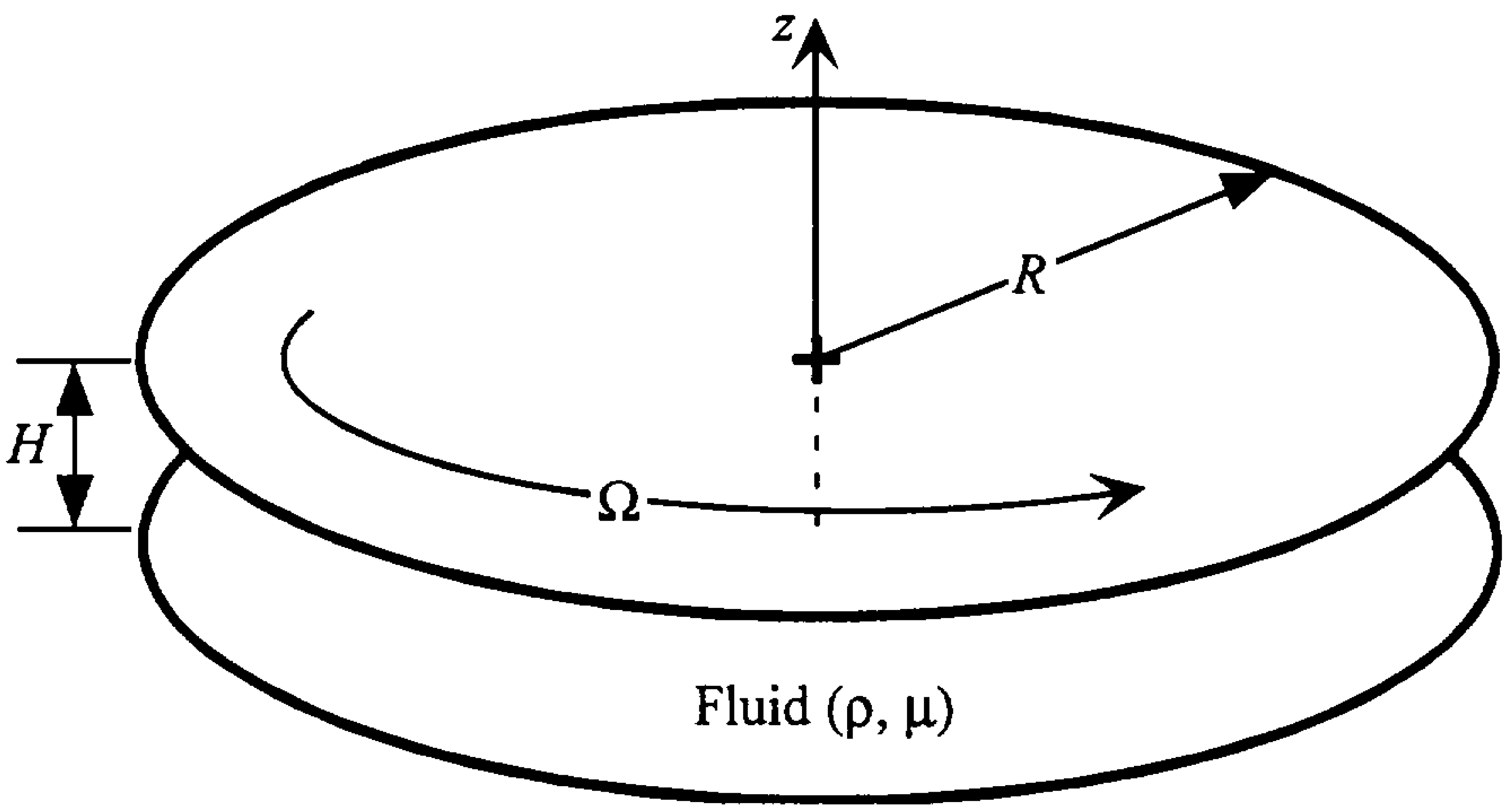
Figure 2b

3) Consider the flow of a Newtonian fluid of density ρ and viscosity μ between two parallel disks of radius R separated by a vertical distance H ($R \gg H$). The top disk rotates along the +z direction with an angular speed Ω, while the bottom disk is stationary as shown below.

a) For *creeping* flow (Reynolds numbers $Re < 1$), what is the velocity field $\vec{V}(r,\theta,z)$ in this geometry? Please clearly state the assumptions and boundary conditions used to solve this problem. [Hint: just as in solid-body rotation, the azimuthal velocity is proportional to r.]

b) If edge effects are negligible, what is the net viscous force exerted by the flow on the bottom disk? What is the viscous force on the top disk?

c) *Briefly* explain what happens in this flow for $Re > 1$. Neglect edge effects.



Continuity Equation (cylindrical polar coordinates):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

Navier-Stokes Equations (cylindrical polar coordinates):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

4) Consider three-dimensional flow around a small sphere of radius a. In general, the drag D on the sphere is a function of the fluid viscosity μ, the fluid density ρ, the sphere velocity U, and a.

a) Use dimensional analysis to determine the important dimensionless groups for this problem.

b) If the Reynolds number is less than unity (Stokes flow), the Stokes drag D_S on the sphere turns out to be independent of ρ. Find the relevant dimensionless groups and explain why D_S no longer depends upon ρ for this case.

c) Now consider *two-dimensional* flow around a cylinder of radius a. Following the previous analysis, determine the Stokes drag *per unit length* D_{2D} assuming that D_{2D} is independent of ρ.

d) Does your answer to part (c) make physical sense? Why or why not? What conclusions can be drawn from this dimensional analysis regarding Stokes drag for two-dimensional, *vs.* three-dimensional, flows?