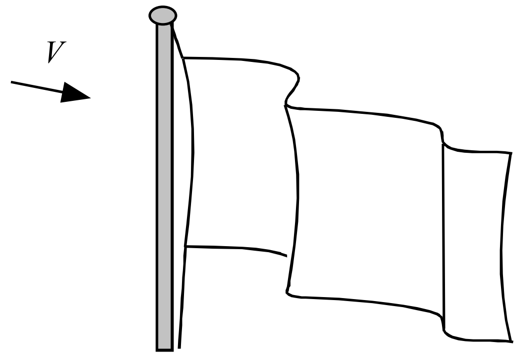
For your reference, the basic equations in cylindrical polar coordinates are given after Problem #4.

1)



Wind blowing past a flag causes it to "flap" in the breeze. The flapping frequency ω is assumed to be a function of the wind speed V, air density ρ, gravitational acceleration g, length of the flag L, and the "area density" ρ_A (which has dimensions M/L^2) of the flag material.

a) Using the Buckingham Pi Theorem, determine the relevant dimensionless groups relating the flapping frequency and the other variables.

For parts (b) – (d): We wish to test a model of a large flag in a wind tunnel to determine the flapping frequency by using a 1/10 scale model.

b) What is the required area density of the model flag material in terms of that of the actual flag?

c) At what wind speed should the model flag be tested in the wind tunnel?

d) If the model flag flaps at a frequency of ω_M , given the parameters above, what will be the flapping frequency of the actual flag?

2) Consider a cylindrical container of inner radius R = 25 cm containing liquid water (density ρ = 1000 kg/m^3), initially at rest, of depth h_o = 10 cm.

a) The container is initially sealed, with an absolute pressure at the free surface of p_o = 30 kPa. What is the absolute pressure at the bottom of the water inside the container?

For parts (b) – (d): The container is then opened to air at an absolute pressure of 100 kPa, and rotated about its centerline at a constant angular speed ω. The water inside the container rotates with the container as a rigid body.

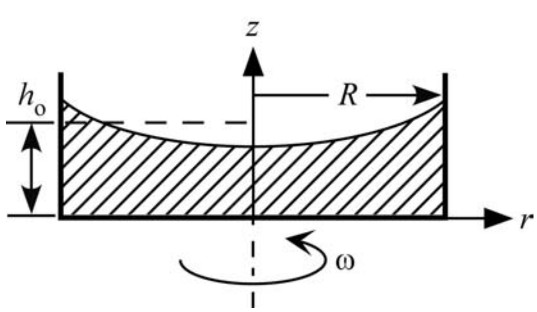
b) Consider Euler's equations (*i.e.*, the equations of motion for an inviscid fluid in the presence of gravity). Use these equations to derive the radial pressure gradient $\partial p / \partial r$ in the water.

c) Euler's equations can be used to show that the equation for the free surface of the water in the rotating container is:

$$z = h_o - \frac{(\omega R)^2}{2g}\left[\frac{1}{2} - \left(\frac{r}{R}\right)^2\right]$$

What is the maximum angular speed ω_w where water completely covers the bottom of the container?

d) If the water is replaced with oil (density ρ_o = 900 kg/m^3), what is the maximum angular speed ω_o where oil completely covers the bottom of the container?



3) A two-dimensional body moves in still air to the left along the x-direction at a speed of 100 ft/s. For a fixed coordinate system, then the velocity profile $u(y)$ of the air past at Section 2 (which was previously disturbed by the body) can be approximated as:

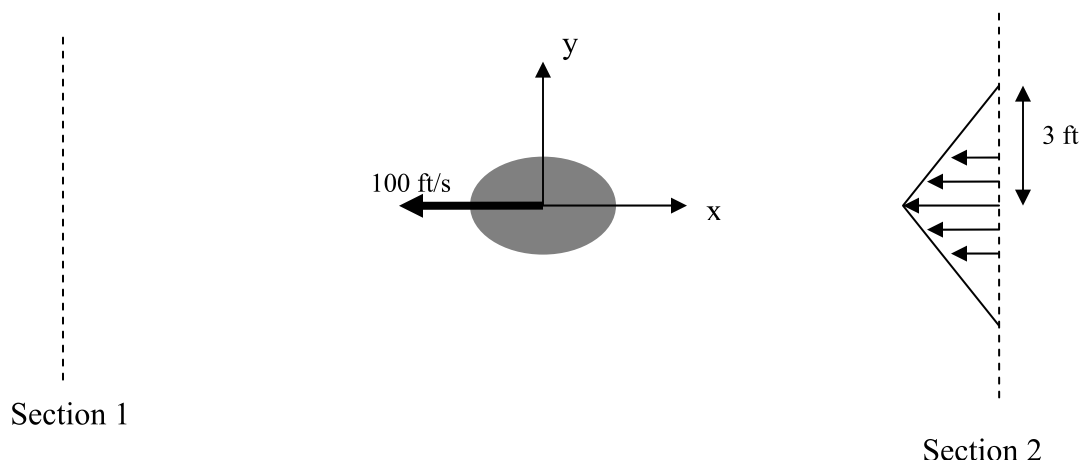
$$u = -30\left(1 - \frac{|y|}{3}\right) \text{ ft/s} \qquad \text{when } |y| \le 3 \text{ ft}$$

$$u = 0 \text{ ft/s} \qquad \text{when } |y| > 3 \text{ ft}$$

as shown in the sketch below.

a) Draw the velocity profile and the control volume for a coordinate system fixed on the body.

b) Calculate the drag force exerted by air on the body per unit length normal to the page.



4) A massless piston moving at a constant velocity U pushes fluid (of viscosity μ and density ρ) through a long pipe of internal diameter D as shown schematically in the sketch below (pipe length >> D).

Assume that:

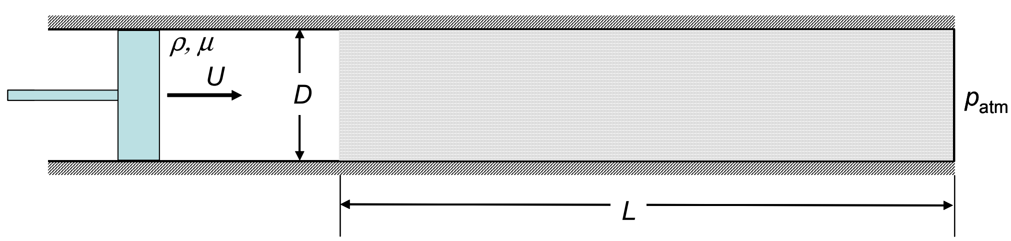
i) the transients associated with the onset of the motion have died out
ii) the motion of the fluid becomes fully-developed and laminar within a few pipe diameters downstream of the piston
iii) the motion of the piston is frictionless and the effects of the fluid on its left side are negligible
iv) the pressure at the downstream end of the pipe is atmospheric.

Determine:

a) The velocity distribution in the fully-developed section of the pipe.

b) The pressure gradient in the fully-developed section of the pipe in terms of the piston velocity and diameter (U and D, respectively) and the other relevant parameters.

c) The force that is necessary to push the *fluid* through the segment L of the pipe in which the fluid's motion is fully-developed and laminar (marked by the shaded domain in the sketch). Is that force the same as the force that is needed to push the piston? ***Explain why***.



Continuity

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r \upsilon_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho \upsilon_\theta)+\frac{\partial}{\partial z}(\rho \upsilon_z)=0$$

Navier-Stokes Equations

$$\rho\left(\frac{\partial \upsilon_r}{\partial t}+\upsilon_r\frac{\partial \upsilon_r}{\partial r}+\frac{\upsilon_\theta}{r}\frac{\partial \upsilon_r}{\partial \theta}-\frac{\upsilon_\theta^{\,2}}{r}+\upsilon_z\frac{\partial \upsilon_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_r)\right)+\frac{1}{r^2}\frac{\partial^2 \upsilon_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial \upsilon_\theta}{\partial \theta}+\frac{\partial^2 \upsilon_r}{\partial z^2}\right]$$

$$\rho\left(\frac{\partial \upsilon_\theta}{\partial t}+\upsilon_r\frac{\partial \upsilon_\theta}{\partial r}+\frac{\upsilon_\theta}{r}\frac{\partial \upsilon_\theta}{\partial \theta}+\frac{\upsilon_r\upsilon_\theta}{r}+\upsilon_z\frac{\partial \upsilon_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 \upsilon_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial \upsilon_r}{\partial \theta}+\frac{\partial^2 \upsilon_\theta}{\partial z^2}\right]$$

$$\rho\left(\frac{\partial \upsilon_z}{\partial t}+\upsilon_r\frac{\partial \upsilon_z}{\partial r}+\frac{\upsilon_\theta}{r}\frac{\partial \upsilon_z}{\partial \theta}+\upsilon_z\frac{\partial \upsilon_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z$$
$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \upsilon_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \upsilon_z}{\partial \theta^2}+\frac{\partial^2 \upsilon_z}{\partial z^2}\right]$$