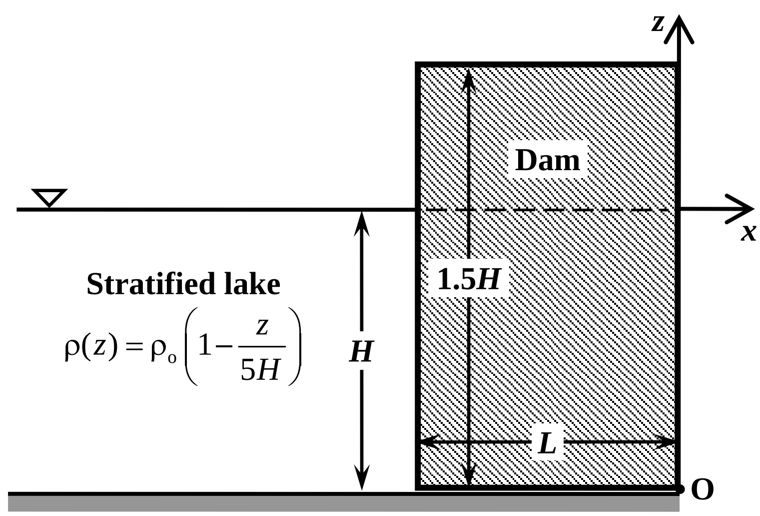
1) Consider a stratified lake with a depth of *H* and a linearly varying density

$$\rho(z) = \rho_o\left(1 - \frac{z}{5H}\right),$$

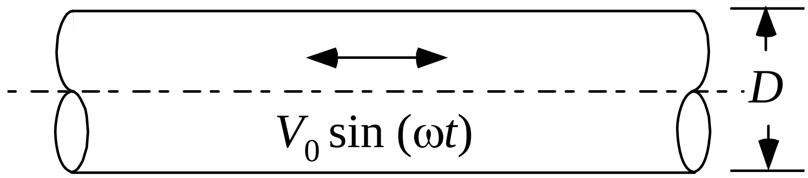
where ρ_o is a constant. A dam on this lake is modeled as a rectangular slab of concrete with a density $2\rho_o$ with a width (*x*-dimension) of *L*, a height (*z*-dimension) of 1.5*H*, and a unit (*y*) dimension normal to the page. The dam and lake are surrounded by air at atmospheric pressure.

a) Find $p_g(z)$, the gage pressure distribution in the lake.

b) If the bottom of the dam is sealed against leakage, find the ratio *L*/*H* at which the dam will tip over, assuming that it will tip around point O.



2)



An incompressible fluid oscillates harmonically ($V = V_0 \sin(\omega t)$, where V is the velocity) in a pipe with diameter D. A 1/4-scale model is to be used to determine the pressure difference per unit length Δp_l along the pipe at any instant.

a) Assuming that $\Delta p_l = f(D, V_0, \omega, t, \mu, \rho)$, where ω is the frequency, t the time, μ is the fluid viscosity and ρ the fluid density, determine the relevant dimensionless parameters for this problem.

b) Determine the similarity requirements for the model and the prediction equation for Δp_l.

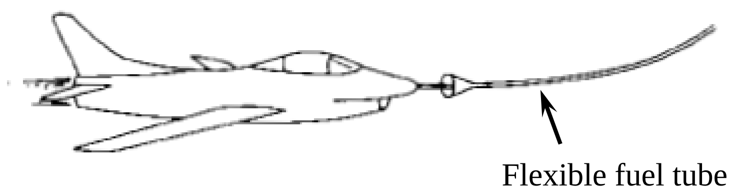
c) If the same fluid is used in the model and in the prototype studies, at what frequency ω_m should the model be operated to correspond to frequency ω_p in the prototype?

3) A fighter plane is being refueled in flight with a fuel of specific gravity SG, as shown below, at a volume flow rate Q. The inside diameter of the flexible tube is D. The fluid gage pressure in the pipe at entrance to the fuel tank in the fighter is p_0.

a) What additional thrust is required for the plane to maintain a constant velocity before and during refueling? You may make the following assumptions:

- the mechanical force exerted by the flexible tube on the plane is negligible;
- the flow at the entrance of the fuel tank is one-dimensional;
- the average velocity in the fuel tank is zero measured with respect to the fighter.

b) How would your result change if the velocity profile in the fuel tube leading to the tank was that for parabolic laminar pipe flow instead at the same flow rate Q?



4) For mixing purposes, it is necessary to regulate the flow rate of two incompressible fluids having the same density ρ but different viscosities μ_1 and μ_2. The fluids are transported within long horizontal channels separated by a partition having negligible mass and thickness, as shown below. The length and width of each channel are much greater than its height, and it can be assumed that the motion within each channel is two-dimensional and independent of *x*.

It is proposed to regulate the flow rate of the two fluids by moving the partition either to the left or to the right at a constant speed U while subjecting the fluids in each of the channels to opposing pressure gradients such that for $0 < y < h_1$, $P_1 \equiv dp/dx < 0$; for $-h_2 < y < 0$, $P_2 \equiv dp/dx > 0$.

a) Determine the force that is necessary to move the partition to the left at a constant speed U.

b) If $\mu_1 = 2\mu_2$ and $h_1 = h_2$, determine the relationship between the volume flow rate (per unit width) q_1 and q_2 through each of the channels and U.