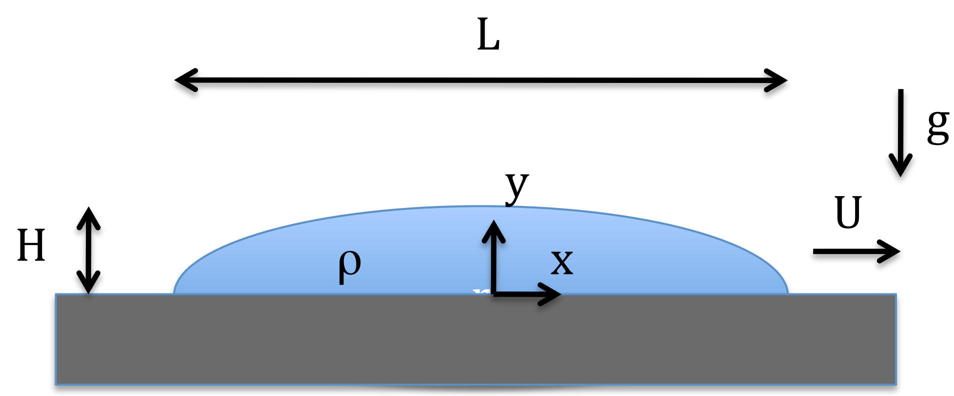
Problem 1

2-D Viscous Gravity Current



Consider a large 2-D drop, composed of a constant volume/unit length V of lava released from a crack. Over time, the drop will spread, increasing in length L and decreasing in height H.

a. What force is driving the flow?

a. In the lubrication limit, where viscous terms dominate inertia, the Navier Stokes equations simplify to:

$$\hat{y}: \quad \frac{\partial p}{\partial y} = -\rho g$$

$$\hat{x}: \quad \frac{\partial p}{\partial x} = -\rho \nu \nabla^2 \vec{u}$$

Scaling y by H, x by L, and p by p_0, first non-dimensionalize the y-momentum equation. Now determine the pressure p_0 in terms if ρ, g and H.

b. Now non-dimensionalize the x-momentum equation.

c. Determine how the length L and height H of the drop changes with time T. Use the fact that the speed U ~ L/T and the volume/unit length $V \sim HL$.

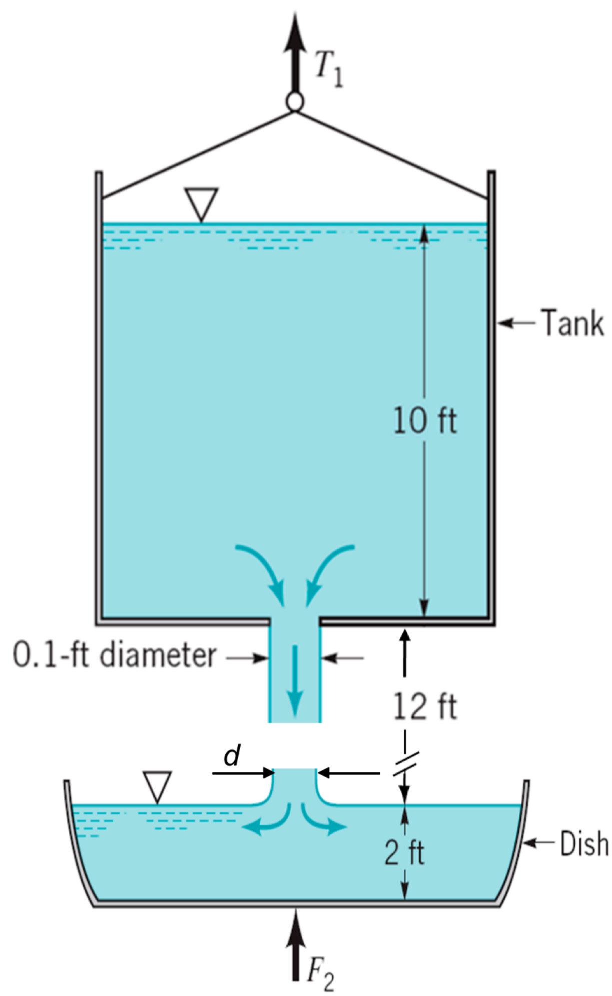
d. For syrup, what is the time-scale T of spreading? Assume $V \sim 10$ cm^2, $g \sim 10^3$ cm/s^2, $\nu \sim 10$ cm^2/s.

Problem 2

Water flows from a large tank into a dish by a free surface falling liquid jet, as shown in the figure.

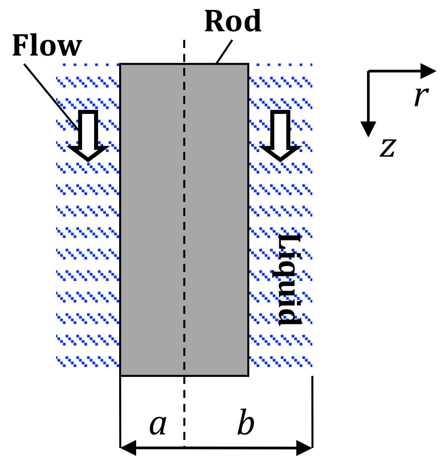
a) If at the instant shown, the tank and the water inside it weight W_1 (N), what is the tension T_1 in the cable supporting the tank?
b) At the instant shown, what is the diameter d of the water jet as it reaches the surface of the water in the dish assuming that the axial velocity of the water jet is only a function of axial location. (You can neglect the liquid spreading right at the liquid surface in the dish)
c) If at the instant shown, the dish and the water in it weight W_2 (N), what is the force F_2 needed to support the dish?

(Specific weight of water is 9800 N/m^3)



Problem 3

A viscous, incompressible, Newtonian fluid flows down along a vertical rod of radius *a*. The fluid forms a uniform film with outer radius *b*. The flow is fully developed and laminar. Assume that the resistance of ambient air does not affect the flow.



Formulate relevant assumptions and state boundary conditions for this problem.

Find velocity distribution in the film.

Continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial(r\rho v_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta}+\frac{\partial(\rho v_z)}{\partial z}=0$$

Linear momentum equations

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right)$$

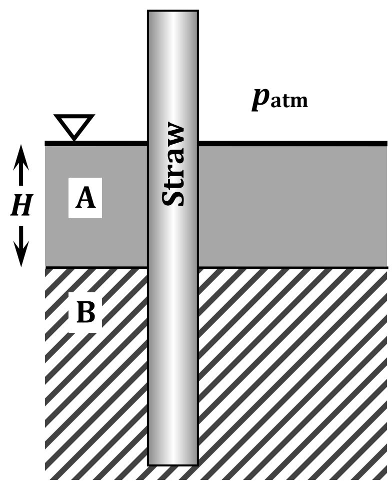
$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$$

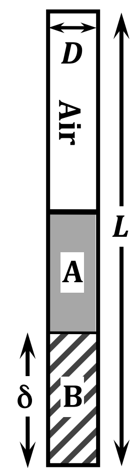
Problem 4

A hollow cylindrical straw with an inner diameter of D and a length of L is partially submerged in two layers of immiscible liquids, A and B, with densities ρ_A and ρ_B, respectively, where $\rho_B > \rho_A$. As shown below on the left, the depth of the layer of liquid A is H. The pressure at the free surface above liquid A is p_{atm}.

Partially submerged straw



Straw in air



You then seal the top of the straw with your thumb, and slowly lift the straw so that it ends up out of both liquids and the straw is surrounded by air on all sides. The depth of liquid B in the straw is now δ, as shown above on the right. Neglect any surface tension effects, and *list all additional assumptions*.

a) What is the gage pressure in the air under your thumb p_g?

b) What was the vertical extent of the straw originally submerged in liquid B Δ_B?

c) Finally, consider the effect of surface tension only at the interface between liquids A and B. The surface tension between liquids A and B is σ, and the contact angle of the meniscus between the two liquids measured with respect to the vertical is θ, as shown below. What is the height difference Δh in the interface between liquids A and B inside the straw and outside the straw? Note that the force due to surface tension pulling the meniscus "up" along the walls of the straw acts only where the meniscus touches the walls.

Closeup of partially submerged straw