GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1998

Thermodynamics
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

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PhD Qualifying Examination in Thermodynamics

Fall 1998

The compressor in a vapor compression refrigeration system that uses R-12 has an adiabatic efficiency of 90%. The system is designed to keep a food freezer at -20°C, when the ambient air temperature is 25°C. The evaporator operates at -30°C and the condenser at +30°C

a) Calculate and compare the decrease of availability in each process of the cycle.

b) Ascertain where (which process) you might realistically focus your efforts to minimize the loss of availability.

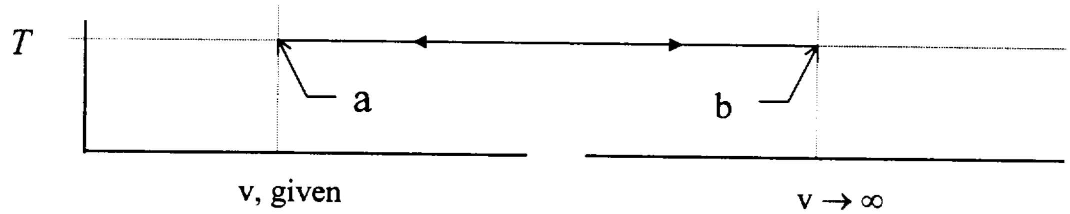
Please quickly **review** the background information in the next three paragraphs and then address the **questions** and **problems** in the next section.

BACKGROUND

Definition of a Residual Property. A residual property is defined for u or h or any other so-called "specific property". It is the difference between the value of the property for a realistic fluid at some specified state (i.e., specified T and v) and the value of the property as if the fluid behaved as an ideal gas at the same T and v, or

$$u_{res} = u(v,T) - u_{ig}(v,T)$$

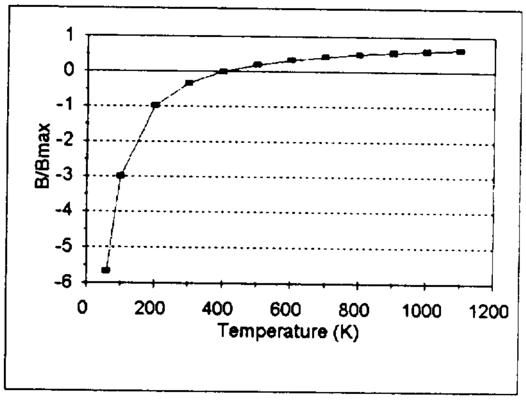
Typically the residual property is evaluated by integrating a property differential, such as du, with respect to the volume over an especially convenient path. This path is illustrated by the line a to b in the following sketch. The first part of the path is a to b, integrating from the real state at the given volume to a state where the volume is very large. The next part is the reverse, from b to a. After assuming ideal gas properties, the property is integrated from the large volume state to an ideal gas state at the given volume. Note that the path return to the same properties T and v but not the same behavior. The complete path is illustrated below



Virial Equation of State. In many applications, a real vapor is commonly considered to be governed by the truncated virial equation of state,

$$\frac{Pv}{RT} = 1 + \frac{B}{T}$$

A typical variation in the so-called second virial coefficient B is shown below. In further calculations, do not attempt to evaluate the temperature derivative of B but rather use the symbol B', i.e. "B-prime", to represent this derivative.



General Property Relations. Two important sets of general property relations useful in developing general property formulations are tabulated below.

Table 1. The Gibbs Equations

$$dU = TdS - PdV$$
$$dH = TdS + VdP$$
$$dA = -SdT - PdV$$
$$dG = -SdT + VdP$$

Table 2. The Maxwell Relations

$$\left(\frac{\partial T}{\partial V}\right)_{S,N_k} = -\left(\frac{\partial P}{\partial S}\right)_{V,N_k}$$
$$\left(\frac{\partial T}{\partial P}\right)_{S,N_k} = \left(\frac{\partial V}{\partial S}\right)_{P,N_k}$$
$$\left(\frac{\partial S}{\partial V}\right)_{T,N_k} = \left(\frac{\partial P}{\partial T}\right)_{V,N_k}$$
$$-\left(\frac{\partial S}{\partial P}\right)_{T,N_k} = \left(\frac{\partial V}{\partial T}\right)_{P,N_k}$$

QUESTIONS AND PROBLEMS

1. Using the Gibbs Equations and the Maxwell Relations derive the following differential formulation for the internal energy. In your derivation, please identify and explain each step.

$$dU = \left\{T\left(\frac{\partial S}{\partial T}\right)_V\right\}dT + \left\{T\left(\frac{\partial P}{\partial T}\right)_V - P\right\}dV$$

2. The coefficient of dT in the preceding equation is better known by what name and symbol?

3. Using the path of integration suggested above and the result of Part 1 above, calculate the residual internal energy of a vapor governed by the truncated virial equation of state.

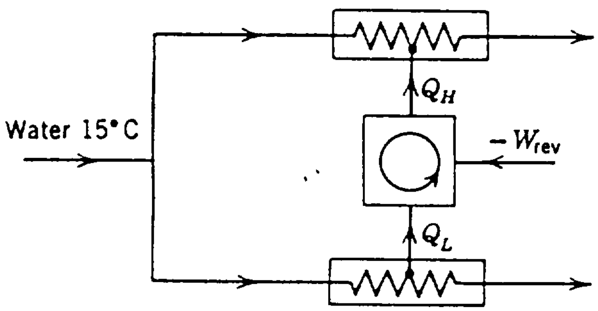
4. Explain why the suggested path of integration is so convenient.

5. How much does the internal energy change during the part of the path from b to a.

6. Is the residual internal energy positive or negative? Please explain the reason for this behavior of the residual internal energy on the basis of simple physical principles, not mere mathematics.

A water supply is at 15C and one atmosphere. Chilled water is to be made from this water supply by the process shown below. The exit temperature of the chilled water is 5C, and the exit temperature of the water acting as a heat sink is 30C. There is no pressure drop in the water flow circuit, so both the hot and chilled water streams exit at 1 atmosphere.

What is the minimum possible power required to produce a chilled water flow of 1 kg/sec?



A steady-flow, water-cooled, air compressor is used to compress air from 15 psia and 80 F to 150 psia and 260 F. The air enters the compressor at a rate of 350 ft^3/min.

Cooling water enters the cooling passages of the compressor at a rate of 20 lbm/min. at a temperature of 60F and a pressure of 30 psia. The water experiences an increase of temperature of 20 F and a negligible pressure drop. Determine the power input, in horsepower, required to drive the compressor