GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1996

SYSTEM DYNAMICS & CONTROLS
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

George W. Woodruff School of Mechanical Engineering

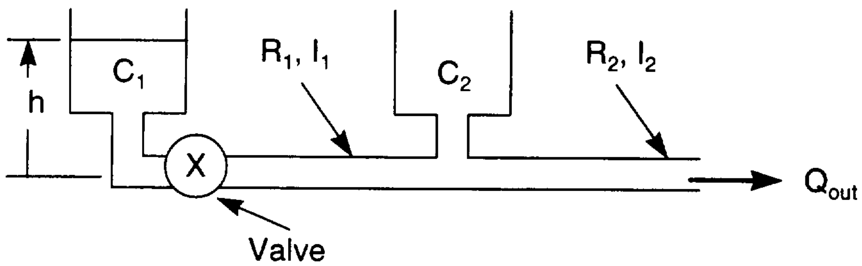
Fall 1996 Doctoral Qualifying Examination

INSTRUCTIONS

There are 4 questions attached, please solve all questions as completely as possible. State all assumptions, and make sure that you clearly indicate the thought processes that you employed to arrive at your answer.

MODELING QUESTION

Shown in the figure below is a tanker transfer system. It is used to transfer crude oil (density, ρ) from the tanker modeled as a fluid capacitor, C_1. The system also includes an on/off valve to connect the tanker to the transfer system, a long pipe that has resistance and inertance, R_1, and I_1, respectively, a intermediate tank with capacitance, C_2 and finally a second pipe with resistance and inertance, R_2, and I_2. The height of the crude oil in the tanker is given by h, and the out flow of the system into the refinery is given by Q_{out}. For this system, please model the first pipe with R_1 and I_1 in parallel and the second pipe with R_2 and I_2 in series. For this question, please state all assumptions and identify any parameters that you will need to generate the appropriate answers.



Model of a Tanker Crude Oil Transfer System.

Part A

Determine the transfer function between h and Q_{out}. Please state all assumptions that you make when determining the transfer function.

Part B

Discuss the differences in the modeling of the two pipes. In particular, please compare and contrast that use of the resistances and inertances in parallel and series. Which one is correct?

Part C

How big would C_1 have to be before you would change your model, and how would your model change?

Part D

What is the purpose of the intermediate tank, C_2, and how does it affect the dynamics between h and Q_{out}?

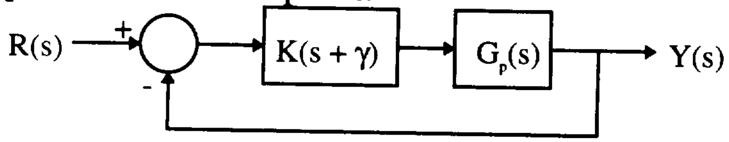
Part E

Sketch the equivalent mechanical rotational system. Make sure to label all parameters and variables and to link them to parameters and variables represented in the crude oil transfer system.

Your company has a very old plant that has the following transfer function:

$$G_p(s) = \frac{1}{(s+5)\left[(s+1)^2+1\right]}$$

Your boss would like to add an external PD controller of the following form to increase the response time of the plant:



Part a)

Sketch the root locus diagram of the system as a function of the gain, K, with $\gamma = 2$. Will this controller increase the response time of the plant to a step input in reference signal, r? Briefly explain how you arrived at your answer. What effect might the PD controller have on the stability of the system?

Part b)

What effect would increasing the parameter γ have on the root locus plot you sketched in Part a? What would the effect be on the stability of the closed loop system? Hint: look at the effect of very large γ.

Part c)

Your expense-conscious boss has decided that it might be easier to alter the damping in part of the original plant than add an external control system. With the adjustable parameter δ, the plant transfer function becomes:

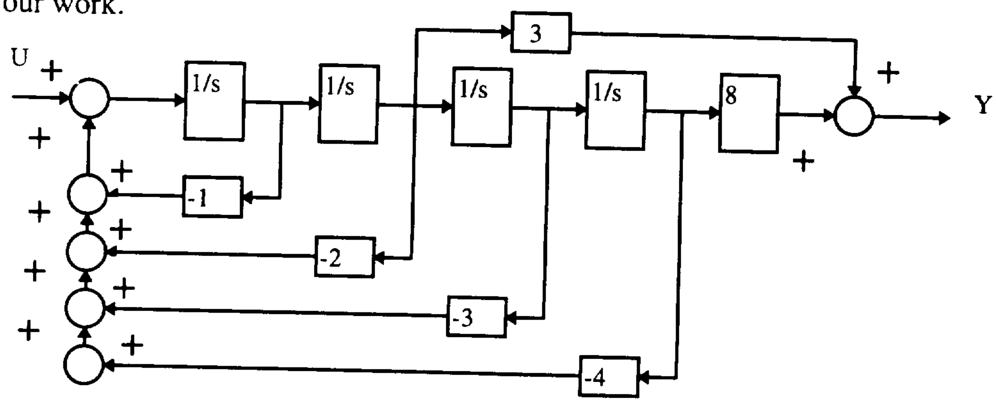
$$G_p(s) = \frac{1}{(s+5)(s^2+\delta s+2)}$$

Draw the root locus illustrating the effects of variations in δ on the poles of the <u>plant transfer function</u>, $G_p(s)$. What benefits would the system in part a give us that this open loop system would not? Are all of the poles of $G_p(s)$ illustrated on your root locus diagram? Why or why not?

Part d)

Your boss seems intrigued by the possibility of simultaneously altering δ and K to design the closed loop system. Without drawing a root locus diagram, what would be the effect of altering δ on the root locus you drew in Part a ($\gamma = 2$)? Is it possible to alter δ in such a way that the gain, K, can cause the system to become unstable? Why or why not?

(a) The figure shows the block diagram of a linear system to be represented in state space form. You may use any appropriate set of states to represent the overall behavior. Show your work.



(b) The state equations of a different linear system are represented by the usual equation as shown below. Determine the overall transfer function from input u_1 to output y_2.

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 7 \\ 7 & 6 \\ 6 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 & 1 \\ 9 & 5 & 8 \end{bmatrix}$$

(c) What will be the response of the system in (b) above to an impulse input into u_1 when u_2 remains zero?

Consider a unity-feedback system whose open-loop transfer function is

$$G(s) = \frac{Ke^{-\tau s}}{s+1}$$

Sketch the Nyquist or polar plot for $\tau=0$ and $\tau=0.8$ second. For both cases, determine the critical value of K for stability.