GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1997

System Dynamics & Controls
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

George W. Woodruff School of Mechanical Engineering

Fall 1997 Doctoral Qualifying Examination

INSTRUCTIONS

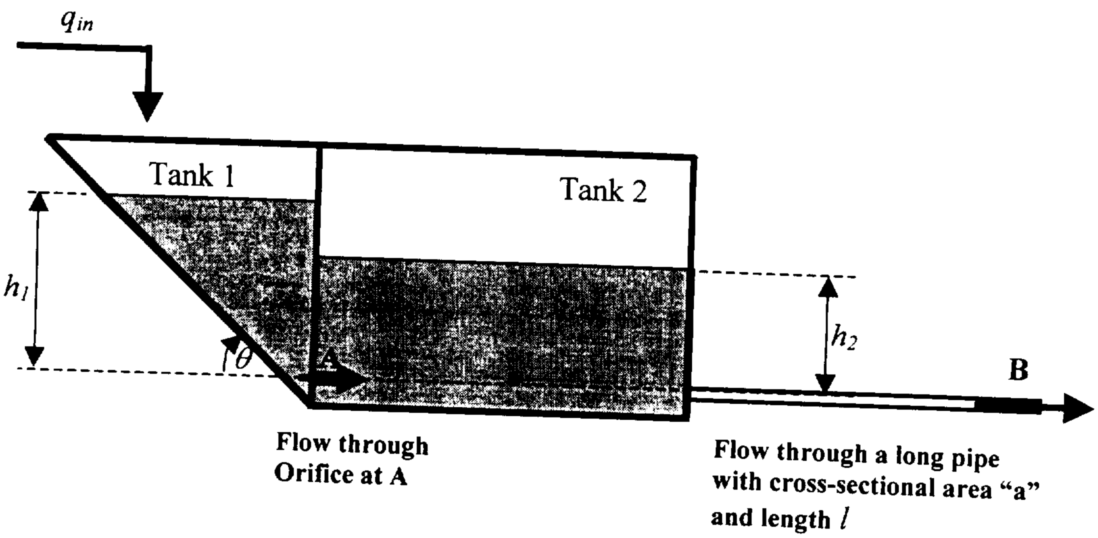
There are 3 questions attached, please solve all questions as completely as possible. State all assumptions, and make sure that you clearly indicate the thought processes that you employed to arrive at your answer.

1. The following figure shows a water supply system. Water is pumped into Tank 1 that has a semi-circular cross-section, and flows through an orifice to the rectangular Tank 2. It is of interest to control the flow-rate of the water at point B, q_B, where water from Tank 2 is drawn through a long pipe. You may assume that the flow through the orifice is proportional to the square root of the pressure drop across the orifice and the flow through the long pipe is linearly proportional to the pressure drop across the pipe.

   (a) Show that the pressure difference required to accelerate a fluid in a pipe is given by $\Delta P = \frac{\rho l}{a}\frac{dq}{dt}$ where ρ is the density of the fluid and q is the flow through the pipe.

   (b) Derive the differential equation(s) for the system, which relates h_1, h_2 and q_B to q_{in}.

   (c) Obtain the transfer function that relates the output flow-rate at B to the supply flow-rate q_{in}. Outline the steps for deriving the transfer function. State any assumptions made clearly.



The forward-path (or feedforward) transfer function of a unity-feedback control system is

$$G(s) = \frac{K(s+\alpha)}{s^2(s+3)}.$$

Determine the values of α so that the root loci ($-\infty < K < \infty$) will have zero, one, and two break points, respectively, not including the one at $s = 0$. Construct the root loci for $-\infty < K < \infty$ for all three cases.

A lead compensator is of the form

$$G_{lead}(s) = K\frac{s+z}{s+p}$$

where p, z and K are parameters that need to be specified by the control designer. One difficulty with using the lead compensator is that if the gain, K, is not chosen properly, then the 0 dB crossover frequency shifts and the cross over frequency is no longer located at the maximum phase lead of the controller. Your task is two fold:

1. Derive the frequency at which the peak phase lead, ω_{max} occurs
2. Determine K as function of p and z such that the gain of the lead compensator is unity at ω_{max}.