M.E. Ph.D. Qualifier Exam
Fall Quarter 1998

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1998

System Dynamics and Controls
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Please **p r i n t** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

Problem 1

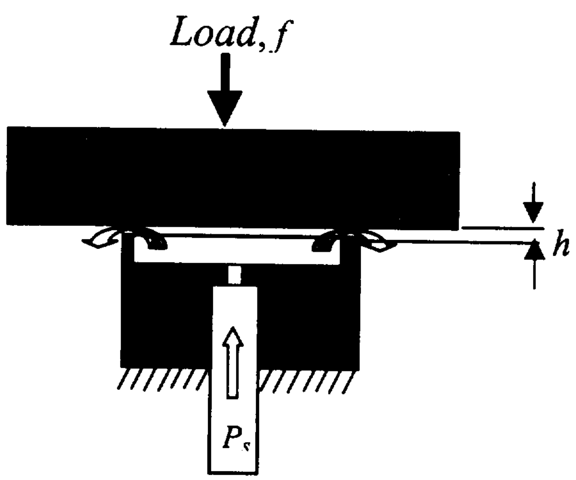
The figure below shows a precision table (mass m) supported by an air bearing. Air enters the bearing from a pressure source P_s, passes through an orifice, momentarily settles in a cylindrical pocket (diameter d and air capacitor C) to attain a pressure P_p, then exhausts to atmospheric condition through an annulus restriction. At steady state, the pressure supply $\bar{p}_s$ is required to maintain a nominal gap $\bar{h}$ under a specified load $\bar{f}$. Both the orifice and the annulus restriction can be characterized by a flow-pressure relationship in the following form:

$$q = ka\sqrt{\Delta p}$$

where q is the mass flow rate through the restriction; k is a constant for the particular restriction; a is area through which the air flows; and Δp is the pressure difference across the restriction.

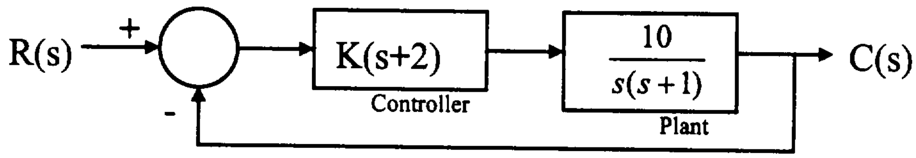
a) Derive a linearized dynamic model to describe the gap between the table and the air bearing.

b) Determine the steady-state error when the table is experienced a step change in load.



Problem 2

The following block diagram represents a plant and controller (PD). This system will be the subject of the following parts of this exam question.



a) Sketch the root locus diagram for this system, using K as the locus parameter. Based on your root locus sketch, can this system be made to respond to a step input with a 2% settling time of 2 seconds and no oscillation by choosing some positive real value for the parameter K? Please explain how you came up with your answer.

b) Can the complete response of this system to a step input be determined from the root locus diagram? What information does the root locus diagram provide and how is this related to system response?

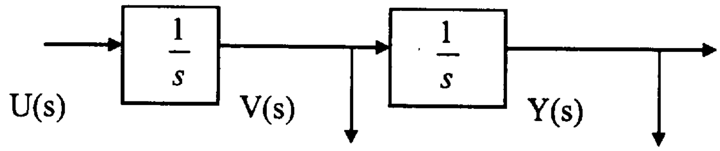
c) If the controller was placed in the feedback loop, rather than the feedforward loop, how would this have effected the root locus diagram that you sketched in Part A? How would your interpretation of the root locus change?

d) Let's say that you've chosen a point on the root locus diagram and you need to determine the value of K that would yield that point. How would you calculate that value of K from your sketch in Part A? How would you deal with the DC gain of the plant?

e) Explain in words and illustrations, if needed, how you would determine the effect of varying both parameters of the PD controller (K and the location of the zero (currently at s=-2)) on the closed loop poles of the system given above. Refer back to the root locus diagram and make full use of the root locus technique to answer this question.

Problem 3

Consider the double integrator system shown below where u is the input force and v is the output velocity and y is the position output, both of which are assumed to be measurable.



a) Design a causal (i.e., no pure differentiation) feedback control system (by completing the above feedback diagram) such that the closed-loop position system meets the following specifications:

- A step change in the reference position causes no overshoot.
- The velocity error (i.e., $1/K_v$, K_v: Static velocity error constant) is less than one percent.

b) Determine the phase-margin and the bandwidth of the control system you designed in **(a)** and comment on the significance of each.

c) Suppose that the reference position $r = A \sin \omega t$. Find the steady-state amplitude of the position error, $e=y-r$, in terms of A and ω.