M.E. Ph.D. Qualifier Exam
Fall Semester 1999

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 1999

System Dynamics and Controls
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

1. A disk drive read/write head is moved by pivoting a small beam about a vertical axis. The read/write head is mounted at the opposite end of the beam from the pivot and has a small additional mass. The torque to rotate the beam is provided by a small electric torque motor which provides torque proportional to the current flowing through the coils of the stator. To suitably design and control the disk drive, a dynamic model is required. In fact, several dynamic models of varying complexity and accuracy are needed. You are to model the system below being careful to communicate the essential assumptions and definitions.

The phenomena that may be of importance in the dynamic behavior are:

1. Friction at the pivot
2. Inertia of the beam and head
3. Disturbance forces on the head
4. Actuator dynamics
5. Beam flexibility
6. Control inputs are either
   a) Current into the coil
   b) Voltage across the coil
7. Outputs of possible interest are
   a) Angular position of the beam at the pivot
   b) Position of the head along the curving track that it follows

a) Derive a first model incorporating aspects 1, 2, 3, 4, 6a, and 7a

b) Extend the first model above by incorporating aspects 6b.

c) A coworker has produced a model with flexibility supposedly of the form

$$M\ddot{\theta} + D\dot{\theta} + K\theta = Bu$$

Where M is a symmetrical mass matrix, D is a damping matirix and K is a spring matrix. θ is a 2x1 vector of variables. B is the coefficient of an input vector u. Convert this model to state space form.

d) A transfer function has been produced by the same coworker from the state space matrix which is:

$$\frac{Headposition}{Motorcurrent}(s) = \frac{A(s^2 + c)}{s^2(s^2 + b)}$$

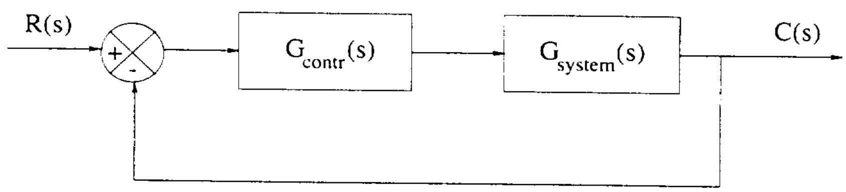
i. What can be determined about the friction used in this model?

ii. c<0. What term is used to describe this system? What are the consequences of this fact?

iii. If b>0 is to be compared to an experimental response to check the realism of the model, what type of experiment would you propose and what aspect would you look for in the response and how would you relate that to b?

Problem 2

Consider the following block diagram:



where

$$G_{\text{system}}(s) = \frac{3}{(s+1)(s+2)}.$$

(a) Your boss wants you to design a controller for this system. The design specifications are as follows:

(1) The closed-loop system must be stable and

(2) the steady-state error for a unit step should be as small as possible.

A P-, PI- and PD-controller are available. Which one is the most suitable for this task?

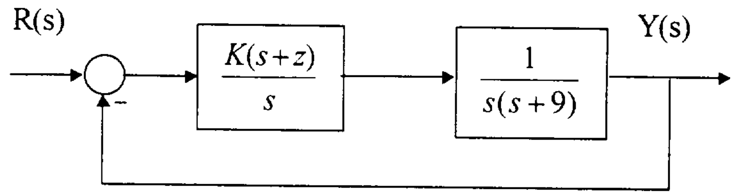
Provide a short explanation:
Why did you choose this controller? Can you use this controller to satisfy both design specifications simultaneously? Why or why not? (NO calculations necessary!!!)

(b) Calculate by hand the steady-state error for a unit impulse, $r(t) = \delta(t)$, if you would use a simple P-controller.

(c) Now use the controller you chose in 2(b).

- Derive the conditions for which the closed-loop system is stable.
- From the results you obtain or from separate calculations, determine when the steady-state error obtained in 2(c) is meaningful.

3. The block diagram shown below depicts a servo control system with a PI feedback control:



a) Assuming z=6, sketch the closed-loop poles of the system as K varies from 0 to infinity. Graphically estimate the value of K for which the dominant closed poles have the largest damping ratio possible.

b) Find z such that the closed-loop system has a *triple* real pole for some K. Plot the corresponding root-locus.

4. Determine the controller gains for the controlled system given below, where $G_c(s)$ is a PI controller or $G_c(s) = K_p + K_i/s$. The dynamic model of the plant $G_p(s)$ was determined experimentally. The results are given below in the form of Bode plots.
   (a) Determine the value of K_i such that the velocity error constant is 2 sec^{-1}, and
   (b) Derive the necessary equation(s) to solve for the maximum value of the gain K_p for stability. (Note: Exact numerical solution is not required.)