

DEC 17 2002

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2002

System Dynamics and Control

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- Please sign your name on the back of this page—

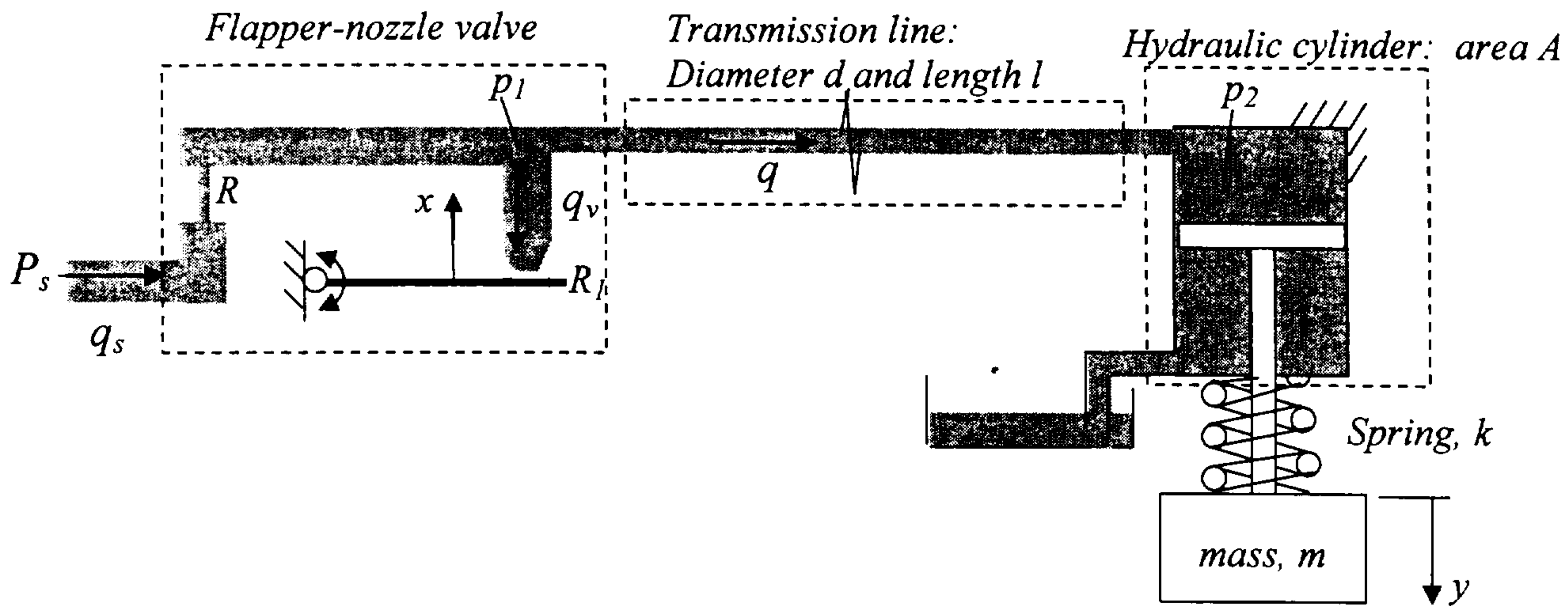
Answer three of the four questions

Please **print** your name here.

**The Exam Committee will get a copy of this exam and will not be notified
whose paper it is until it is graded.**

Problem 1:

The following actuator (used in a hydraulic position servo) consists of three basic components; namely, a flapper-nozzle valve, a fluid transmission line, and a piston-cylinder. The valve transforms the flapper displacement signal x to fluid pressure that actuates the piston and thus positions the mass.



- (1) Derive a dynamic equation that characterizes the flow through the circular transmission line for a given pressure drop, $p_1 - p_2$. Determine the effect of the transmission line geometry (d and l) on the bandwidth of the transmission. You may assume the flow is laminar and at steady state, the pressure drop per unit length along the line is given by $\frac{32\mu}{d^2} v$ where μ is absolute viscosity of the fluid, and v is the mean velocity of the flow.
- (2) The flapper-nozzle valve consists of an orifice R and a variable orifice R_v . The volumetric flow rate through an orifice is proportional to the square root of the pressure drop Δp . For the variable orifice, the flow rate is given by $q = c(\bar{x} - x)\sqrt{\Delta p}$ where \bar{x} is the distance of the nozzle from the un-deflected flapper. Derive a transfer function that relates the displacement (y) of the mass to the input displacement x for a specified supply pressure P_s . Clearly define the equilibrium operating point about which your derivation is valid.



Problem 2:

Gantry cranes like the one shown in Fig. 1 are used in many places to move heavy payloads. Unfortunately, the payload often oscillates for a long time because the system is very lightly damped.

In an effort to reduce the payload oscillation, a crane manufacturer is developing a feedback control system that uses a camera to measure the payload position. During initial testing they are simply concentrating on motion in the horizontal direction (the x direction) and neglecting hoisting of the payload. In this case, the crane can be reasonably modeled with the simple two-mass and spring system shown in Fig. 2. The company initially tries a PD control law, so that:

$$F_c = K_p(x_d - x_2) + K_d(\dot{x}_d - \dot{x}_2)$$

where x_d is the desired position and x_2 is the payload position.

During initial testing without a payload, the crane trolley responds as shown in Figure 3. The corresponding payload response is very good compared to the natural response which is very lightly damped. Unfortunately, when a payload is attached to the end of the crane, the trolley responded as shown in Figure 4.

- 1) Derive the transfer function for the trolley position as function of the desired position, x_1/x_d , for the system shown in Fig. 2 using the forcing function given above.
- 2) For the case without a payload, the system can be approximately modeled as having no second mass, that is $m_2=0$. Plot a root locus for this case and use it to explain the step response shown in Fig. 3. Use some very simple values for the system parameters, such as $m_1=1$, etc., in order to plot the root locus.
- 3) For the case with a heavy payload, plot the root locus. Compare it to the root locus for the no-payload case and explain the response shown in Fig. 4.
- 4) Based on your results for 2) and 3) suggest a new control scheme that would make the system perform better.

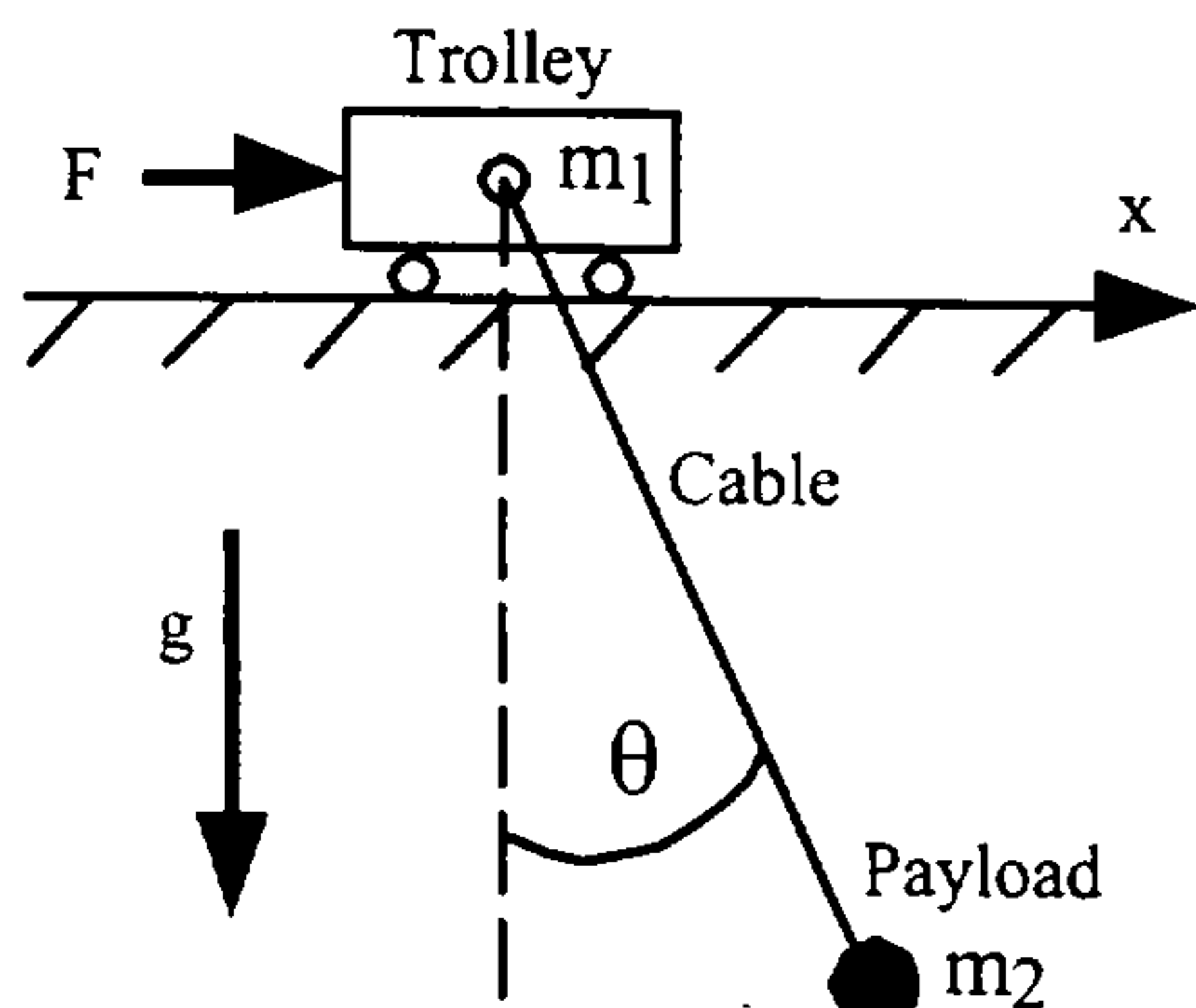


Fig. 1: Gantry Crane.

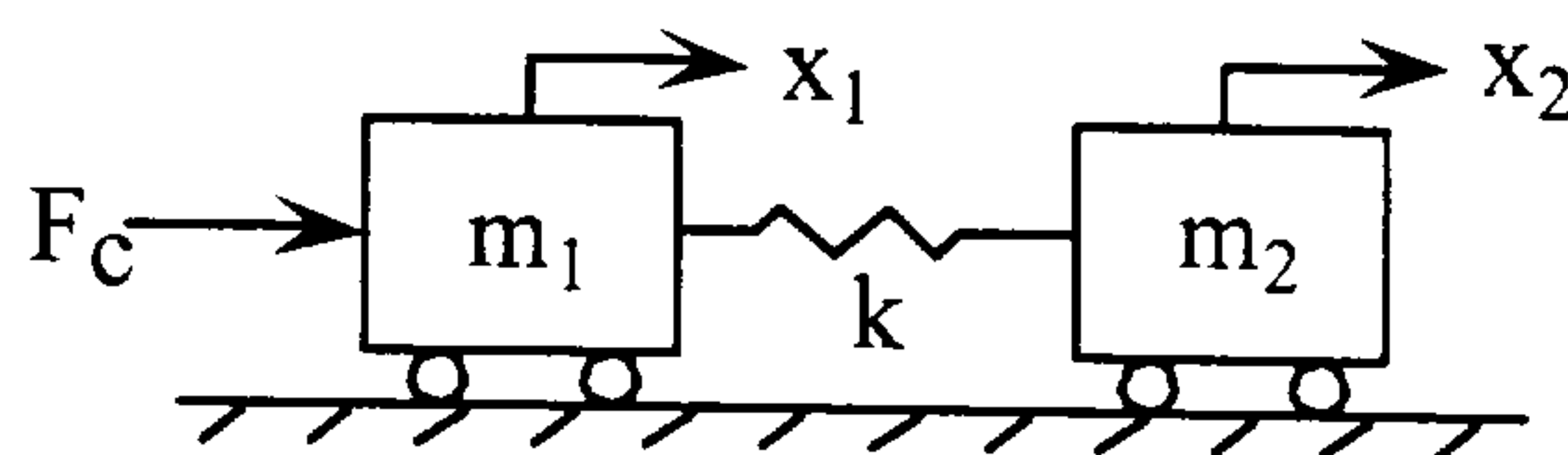


Fig. 2: Mass-Spring-Mass System.

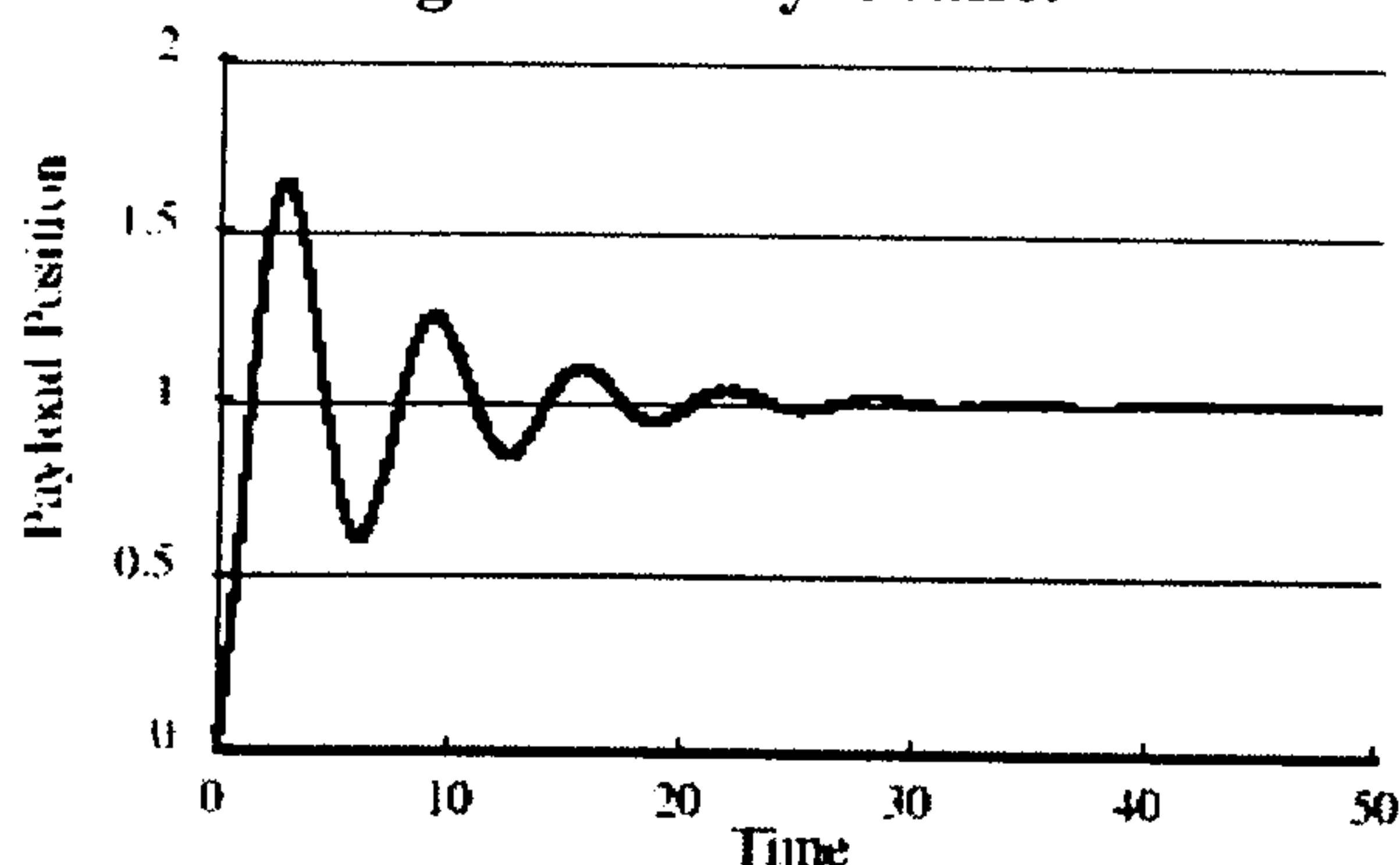


Fig. 3: Response with No Payload.

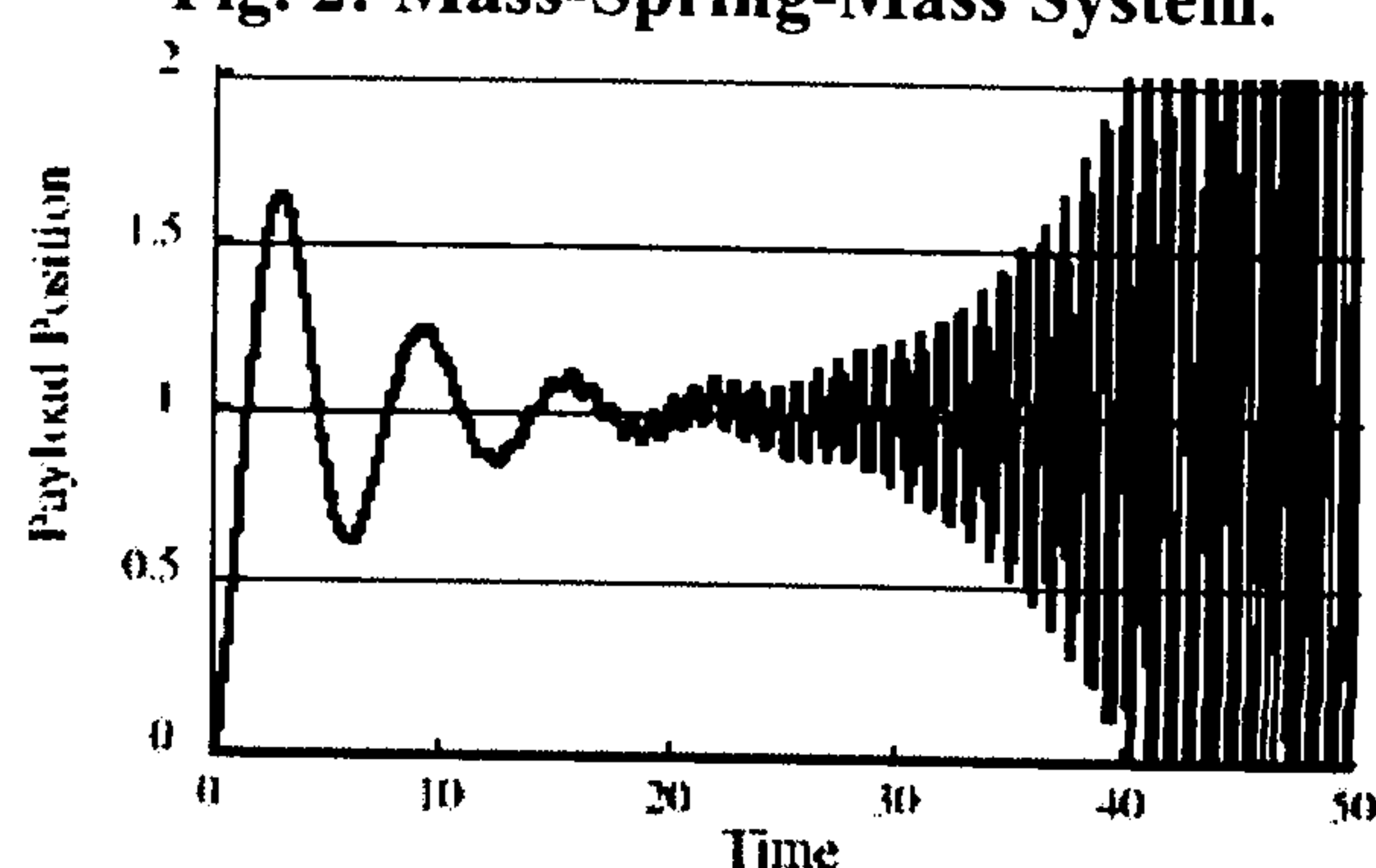


Fig. 4: Response with Payload.



Problem 3:

In this problem you are asked to design a position (y) controller for a single degree of freedom system with acceleration (u) used as control input. It is assumed that both velocity (dy/dt) and position (y) are measurable and can be used for feedback control. Design a feedback controller without using any differentiators that satisfies the meets the following specifications:

- a) The step response of the closed-loop system has no overshoot and settles down in less than 0.2 sec.
- b) The control system has a phase margin of at least 60 degrees.
- c) The amplitude of the steady-state output (y) due to a sinusoidal input disturbance $d=\sin\omega t$ is less than 0.001 for all real frequencies ω .

Compute the corresponding values of the settling time, phase margin, gain margin, and the amplitude of the steady-state error (specified in c) in terms of ω .



Problem 4:

A position feedback control system incorporates an actuation subsystem $G(s)$ and a flexible load. The position z of the flexible load is well approximated by the spring-mass-damper in the sketch with a transfer function $F(s) = Z/Y$. The motion subsystem is known only in terms of the Bode plot of $G(j\omega)$ given graphically in the following. $G(s)$ may include lumped elements, distributed parameter elements and/or time delay. The controller is a simple proportional control but incorporates some time delay of T seconds. The system can be assumed linear and the necessary parameters are given below.

$k = 100,000 \text{ N/m}$, $m = 10 \text{ kg}$, $b = 40 \text{ N/(m/s)}$, $K = 10$, $T = .001 \text{ sec}$.

- Model the load and find and sketch carefully the Bode Plot of $F(s)$ on the blank plot that follows. Remember the axes if necessary for the scale of the problem.
- For what range(s) of k/m will the closed loop system be stable if the ratio b/m remains constant? **IMPORTANT:** Carefully explain your logic and the steps you must take so that the conclusion will be justified even though the result may be very rough due to the inexact graphical nature of the data provided here.
- What is the order of the closed loop system?

