

Georgia Institute of Technology

M.E. Ph.D. Qualifier Exam  
Fall Semester 2003

DEC 11 2003

# GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff  
School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Fall Semester 2003**

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System Dynamics & Controls  
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

\* Please sign your name on the back of this page —

**Systems and Controls Qualifier**

**Fall, 2003**

**Answer any 3 of the following 4 problems.**

1. A simplified representation of a gantry robot is shown in Figure 1. The main body of mass,  $M$ , is propelled along a horizontal track by a traction force,  $f$ . The main body contains an actuator for rotating the arm, which will have a grasping device like a hand at its end for picking-up objects. The actuator applies a torque,  $T$ , to the arm. The arm, hand and grasped object have a total mass,  $m$ , and a moment of inertia,  $J$ , relative to its mass center at point  $C$ . Find the equations of motion for  $x_1$  and  $\theta$  in terms of the given quantities, and the inputs  $f$  and  $T$ .

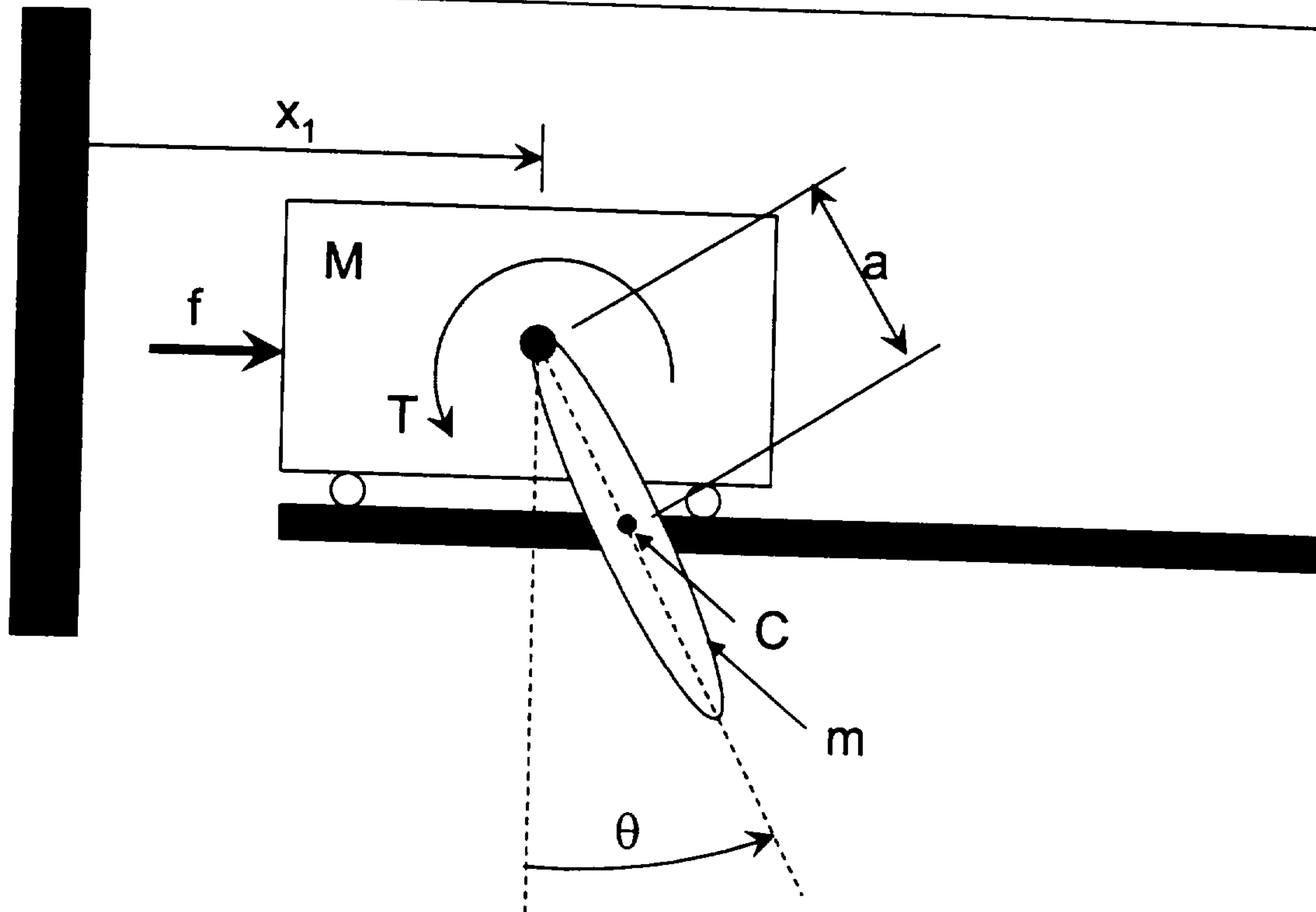


Figure 1: Position Control System with Proportional Gain.

2. Figure 1 shows the block diagram of a position servo that consists of an electromechanical (EM) actuator and a PID controller. Figures 2 and 3 show the Bode plots for the EM actuator and the closed-loop system respectively, which were obtained during the tuning of the PID controller.

- (a) Determine the transfer functions for the EM actuator and the closed loop system. What is the open-loop transfer function for the system? Would you operate the system with this set of PID gain values? Explain.
- (b) Determine the relationship(s) among the PID gains ( $K_p$ ,  $K_i$  and  $K_d$ ) so that the system is always asymptotically stable. Discuss the effect of the proportional gain on the gain margin of the position servo for a given set of derivative and integral gains.

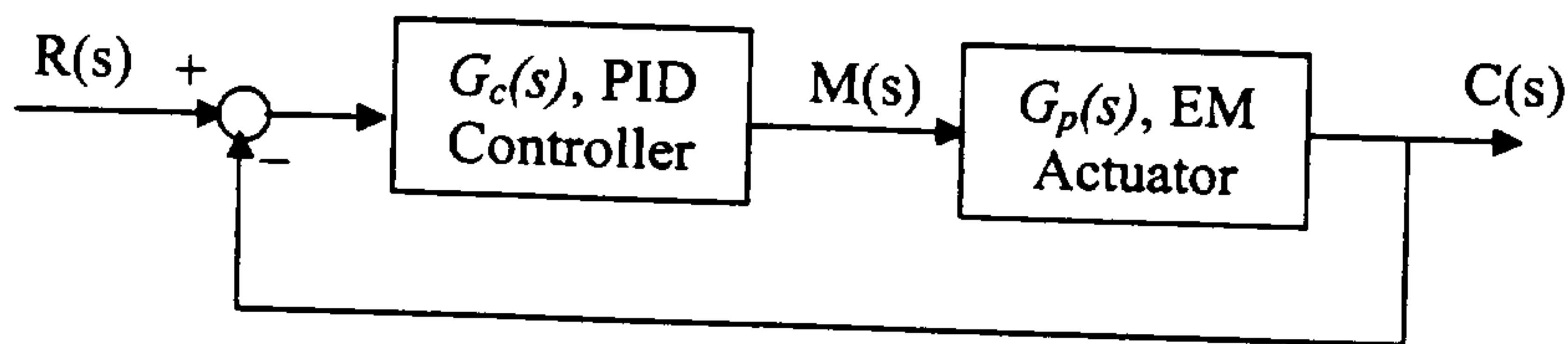


Figure 1

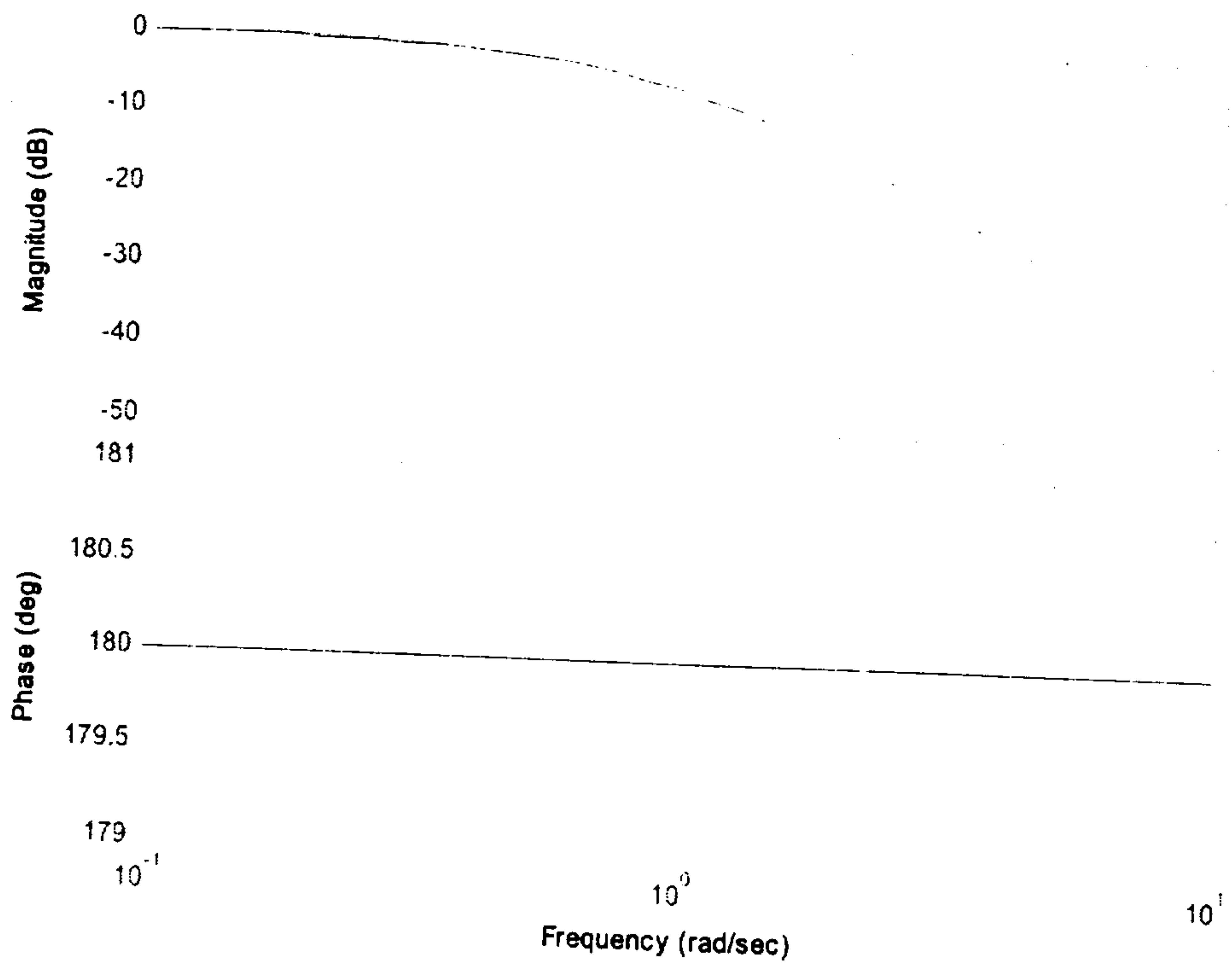


Figure 2 Bode plot of the EM actuator

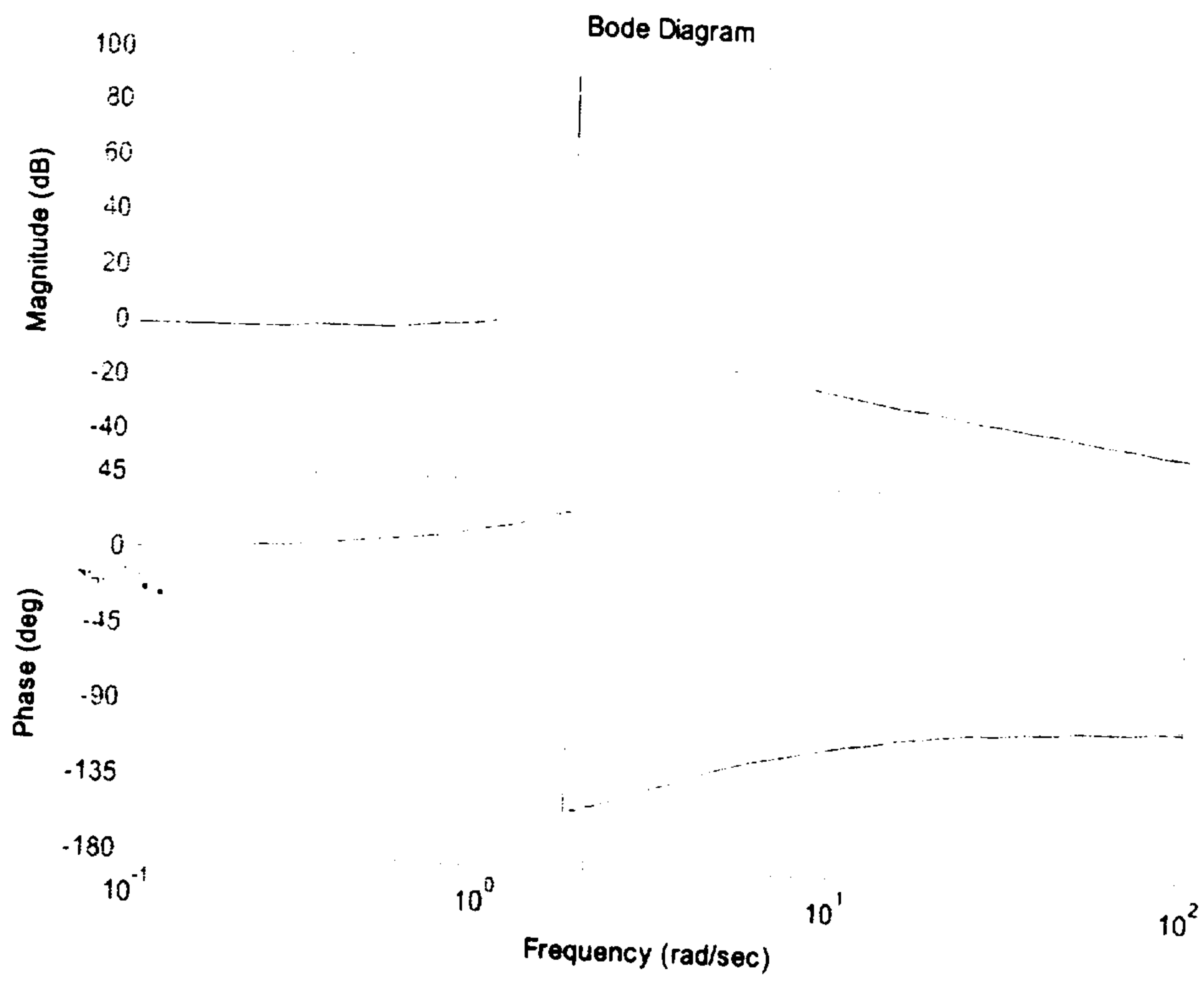


Figure 3 Bode plot of the closed-loop system

3. The electric heater shown (drawing not to scale) is used to control the temperature of liquid entering a storage tank. The fluid flows at a constant velocity of  $v$  m/sec. The addition of heat is represented as a first order lag so that the temperature increase just downstream of the heater  $\Delta T_t$  relative to the inlet temperature

$$\Delta T_t(s) = \frac{B}{\tau s + 1} U(s).$$

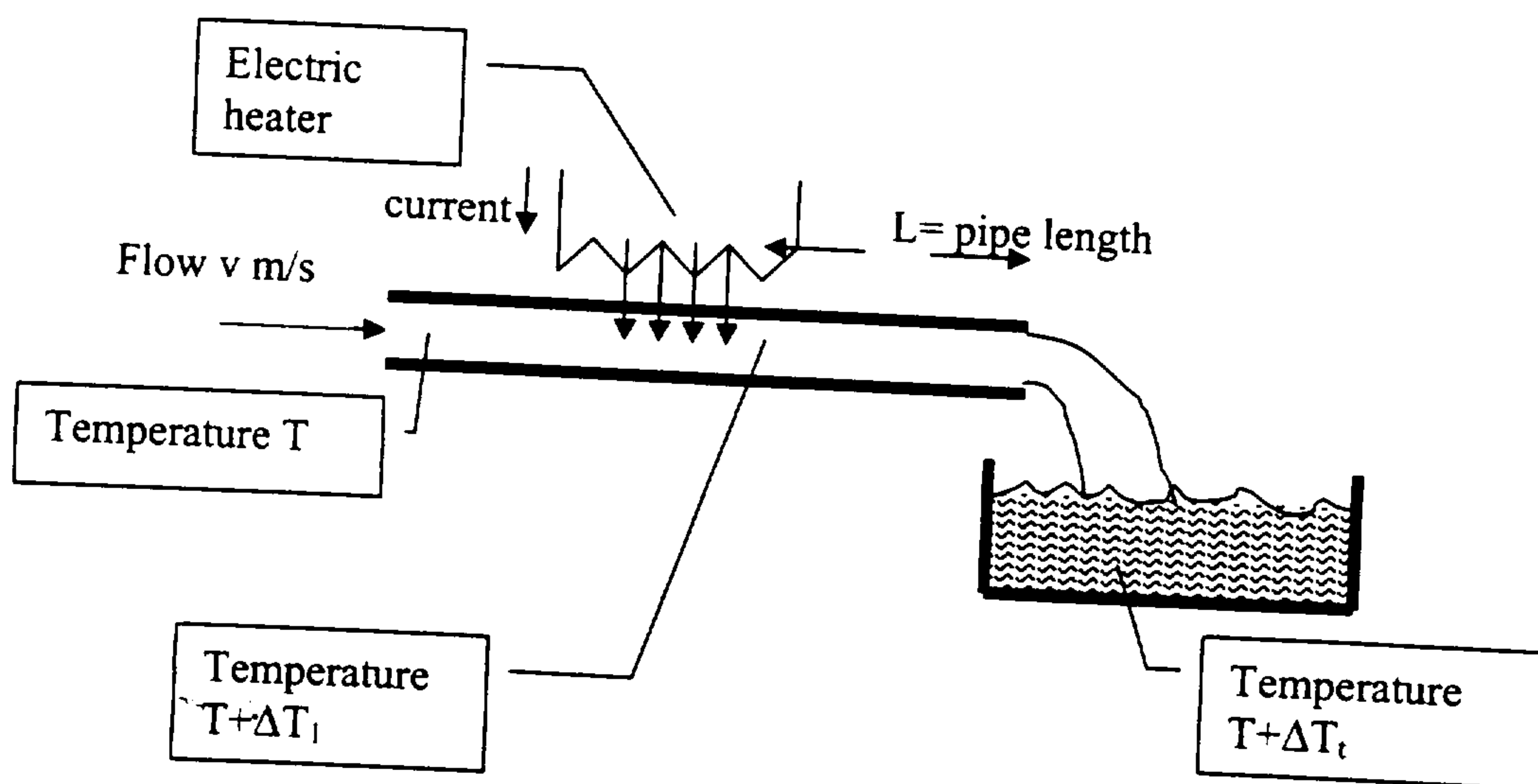
A feedback loop creates  $U(s) = K \{ \Delta T_{desired} - \Delta T_t \}$

(a) Consider (dimensionally consistent) numerical values  $B = 1$ ,  $K = 1$ ,  $L = 0$ . Explore, using the root locus technique, the system behavior as  $\tau$  is varied through physically meaningful values. Sketch the root locus and the step response of the closed loop system for two significantly different values of  $\tau$ .

(b) Construct a block diagram for the complete system when temperature used for feedback is  $T_t$  measured in the tank. Assume the tank is thoroughly mixed and perfectly insulated, and that  $L$  may be significant. Show transfer functions in terms of the symbols already defined

(c) Construct the root locus for the system of (b) as  $K$  is varied and  $B = 1$ ,  $\tau = 2$  sec,  $L = 0$ . What value of  $K = K_{critical}$  yields a critically damped response? Is a rise time faster than  $K = K_{critical}$  accurately predicted by this analysis for this physical system? Why or why not?

(d) What distortion will you observe in the root locus of part (c) as  $L$  increases to a significant value? Justify your answer with a rigorous argument.



4. You are a member of a team designing a control system for a satellite. The satellite will orbit earth while it scans various portions of outer space, looking for signs of intelligent life. Since the search might take a while, the satellite must continually obtain power from the sun. To do this, it is equipped with large solar panels. In order to maximize power absorption, the satellite routinely reorients itself so that the solar panels are pointing at the sun. Given the solar panels are very large and thin, they have a tendency to vibrate. A simple model of the satellite is shown in Figure 1.

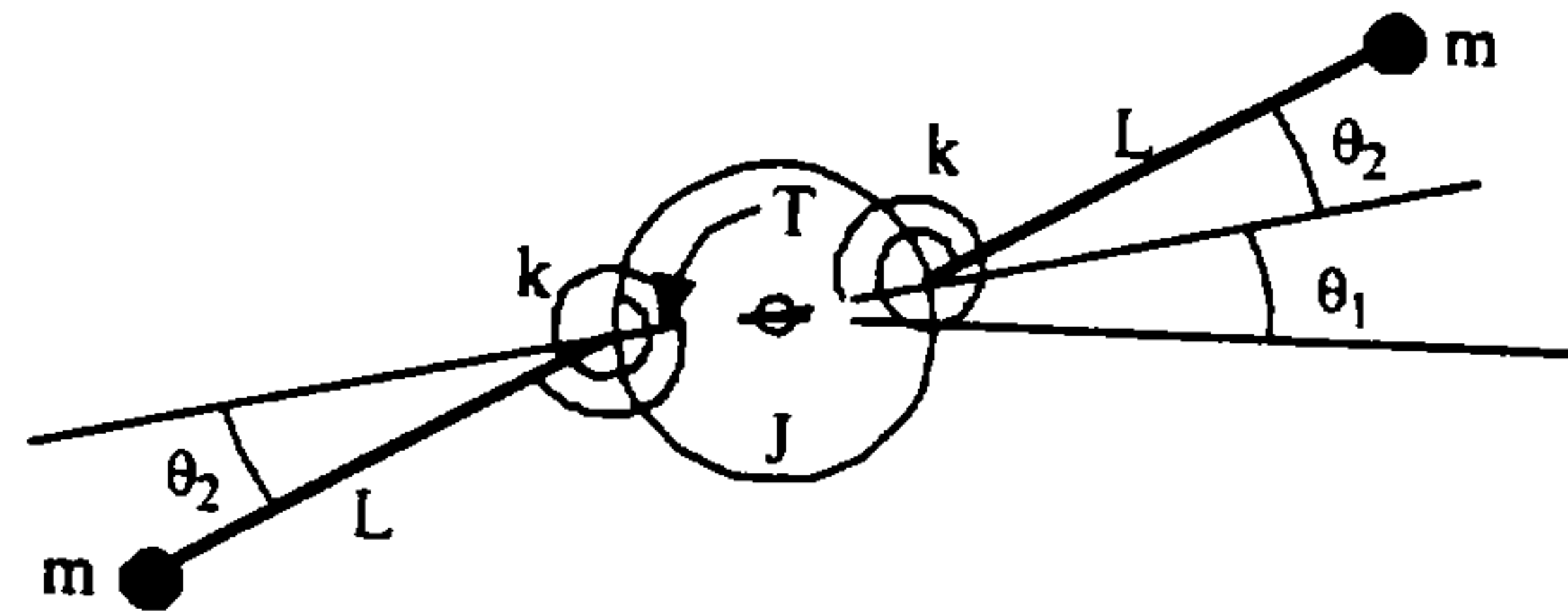


Fig. 1: Satellite Model.

The satellite can be moved by using either its on-off reaction jets, or its momentum wheels. The momentum wheels simply rotate disks of inertia within the body of the spacecraft. These controlled rotations cause the satellite to rotate in the opposite direction (conservation of angular momentum). Gyroscopes can be used to sense the velocity and position of the satellite body and strain gauges give the deflection of the solar panels relative to the spacecraft body.

- 1) Assume that the satellite thrusters are fired so that they result in a pulse of torque being applied to the satellite body. Plot the time response of the body,  $\theta_1$ , and the deflection angle,  $\theta_2$ .
- 2) The solar panels will actually have a lot of additional vibration modes that are much higher than the low modes captured in Fig. 1. How will they effect the response?
- 3) The reaction wheels can create virtually any torque vs. time profile that you can imagine (the maximum torque value is limited). Plot a "good" reaction wheel torque profile so that the satellite moves a small distance from one configuration (initial angle) and stops in some other configuration.
- 4) How would the reaction wheel torque profile change for long move distances?
- 5) Sketch a block diagram representation of a control system that you would design for this satellite. Label all blocks and signals. Explain why you chose each block.