GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2004

SYSTEM DYNAMICS & CONTROL

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

✱ Please sign your **name** on the back of this page —

George W. Woodruff School of Mechanical Engineering

Fall 2004 System Dynamics and Controls

Doctoral Qualifying Examination

INSTRUCTIONS

There are 4 questions attached, please solve 3 of the four questions as completely as possible. State all assumptions, and make sure that you clearly indicate the thought processes that you employed to arrive at your answer. Answer only 3 questions. If you answer 4 questions, only the first 3 will be graded.

TRANSFER FUNCTION ANALYSIS

The dynamic system of interest with input u(t) and response y(t) has the transfer function

$$\frac{Y(s)}{U(s)} = \frac{s+2}{(s+1)^2 + a^2}$$

Define a system to have overshoot to a step input at t = 0 if y(t), its response, reaches a maximum for a value of t where $0<t<\infty$. For the system of interest we are concerned with the response of the system for values of a where $-\infty < a^2 < \infty$.

a) For what values of a is the response oscillatory?

b) For what values of a is the response unstable?

c) For what values of a does the response have overshoot?

Justify your answers analytically. For your convenience a brief table of Laplace transforms is attached.

Laplace transform pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	e^{-at}	$\frac{1}{s+a}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
8	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
9	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
13	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
14	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
15	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
16	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$

Root Locus

Consider the system shown in Figure 1, where the characteristics of the plant $KG_p(s)$ is given by the root locus in Figure 2. Design a controller $G_c(s)$ so that the system can operate

(a) with a settling time that is one-quarter of the original system (where K=4.2), and

(b) with a static velocity error constant at least 30 times larger than that of the original system (in order to ensure that the steady-state error to a ramp input is within an acceptable limit).

For the compensated system that you design, sketch the modified root locus and the response to a unit step and list the following results: the dominant complex poles, settling time, static velocity error constant, and maximum overshoot.

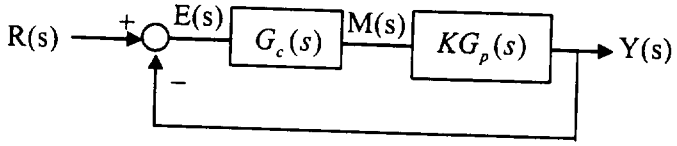


Figure 1

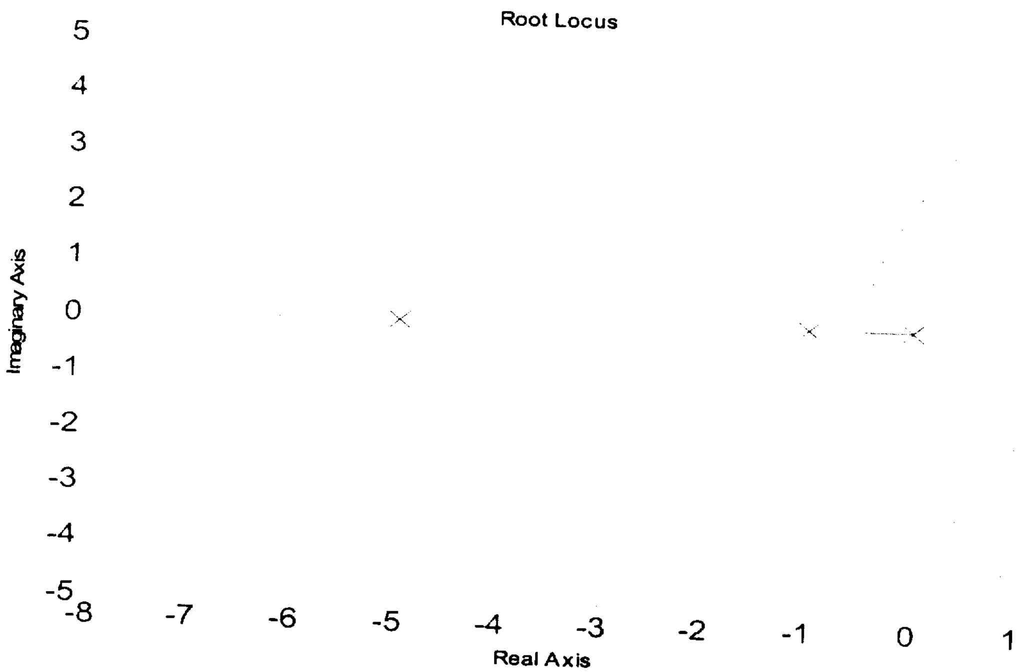


Figure 2

FREQUENCY RESPONSE

Please use the following transfer function for this problem

$$\frac{Y(s)}{R(s)} = \frac{s+10}{s(s+0.1)^2(s+1000)}$$

Part a

Sketch the Bode plot for the transfer function. *Please show gain in dB.*

Part b

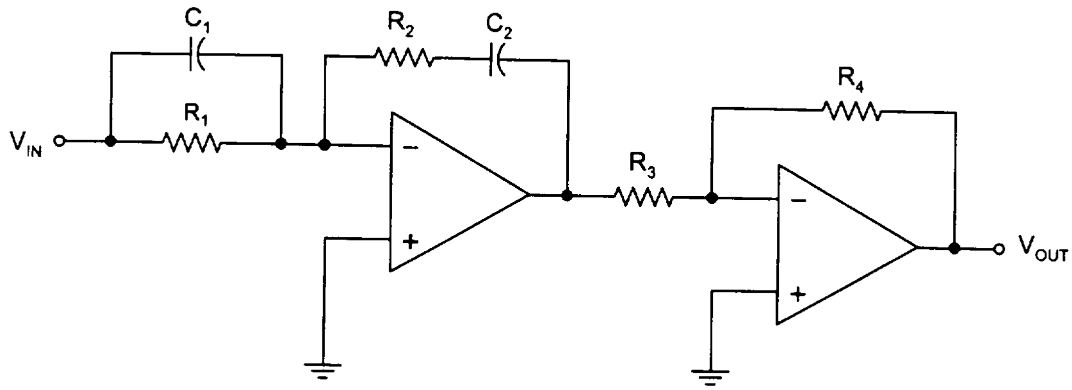
Determine the gain and phase margin for this system. Show them on your Bode Plot.

Part c

Determine, to a reasonable approximation, y(t) when r(t) is given by

$$r(t) = 10\sin(0.1t) + 20\sin(t+0.1) + \sin(200t) + 0.1\sin(6000t)$$

MODELING



a. Determine the transfer function between the input voltage, V_{in}, and the output voltage, V_{out}.

b. Can the analog electronic system shown in the above figure physically generate the transfer function that you derived for part a? This is a yes or no answer.

c. Justify your answer to part b.