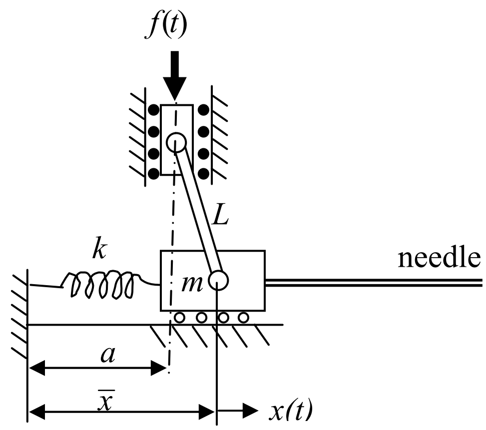
## Special Instructions:

**Please choose 3 out of the four problems given in this exam and clearly indicate which 3 problems you want to be graded. If you fail to clearly identify your choices or choose to do all 4 problems, the first three problems will be graded.**

**Problem 1:**

The schematic given below shows a proposed biomedical device to rapidly insert and retract a needle for obtaining tissue sample from a human organ. The sliding block and link which transmit the effect of *f(t)* is assumed to be mass-less.



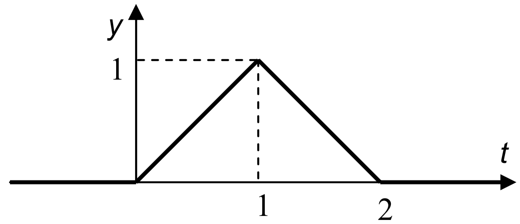

**a)** Derive a dynamic model to describe the motion of the mass *m* for a specified force *f(t)*.

**b)** Then obtain the transfer function from the linearized version of the equation about an operating point $\bar{x} > a$, where *a* is the un-stretched length of the spring (and at $\bar{x} = a$, the link is vertical.)

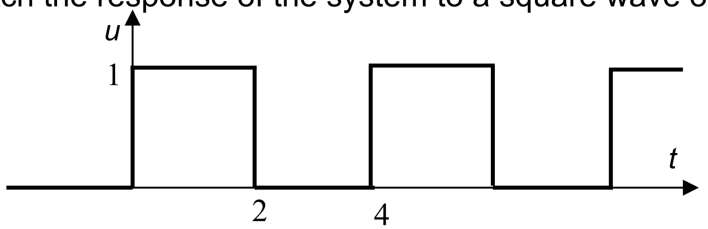
**c)** Discuss whether the proposed device will be able to rapidly insert and retract a needle with no overshoot as claimed. If not, suggest a simplest form of PID controller that will correct the problem. In either case, give the damping ratio is a function of the system parameters.

**Problem 2:**
The impulse response of a linear-time invariant system is given by

$$y_{\text{imp}}(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

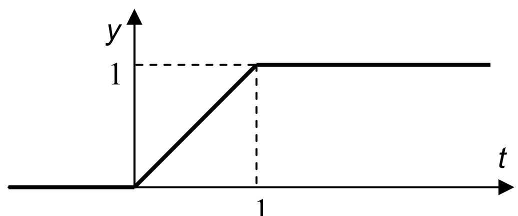



**a)** Is this system dynamic or static? Why?
**b)** Is this system causal? Why?
**c)** What is the transfer function of the system?
**d)** Determine the poles and zeros of the system. What is the multiplicity of each pole or zero?
**e)** Find and sketch the unit step response of the system.
**f)** Sketch the response of the system to a square wave of period 4 seconds shown:




**g)** Find the input signal that produces the following output:

$$y_{\text{imp}}(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \\ 0, & t < 0 \end{cases}$$




Depending on how you solve this problem you may or may not need to know that $\mathcal{L}(t^n) = \dfrac{n!}{s^{n+1}}$ and $\mathcal{L}(1(t-a)f(t-a)) = e^{-as}F(s)$.

**Problem 3:**

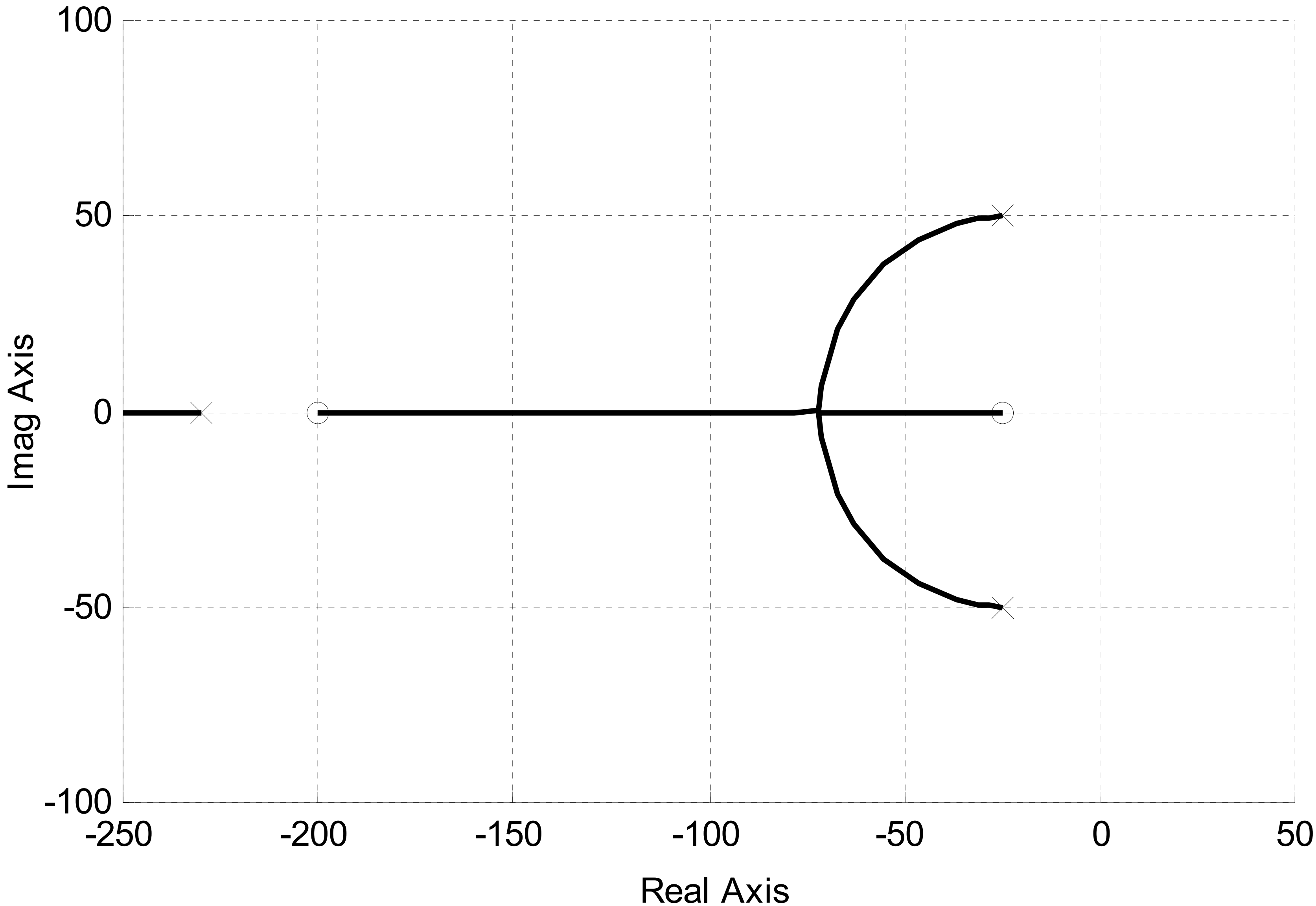
You are designing a control system for a plant described by the following transfer function:

$$G(s) = \frac{23(s+200)}{s^3 + 280s^2 + 14{,}625s + 718{,}750}$$

The specifications for the closed-loop system step response are $t_{s,2\%} \leq 0.1$ and P.O. $\leq$ 16%. You will use a velocity-feedback controller, with a derivative time $T_d = 0.04$. The resulting root locus for the system is shown on the next page.

**a)** Assuming only the dominant poles of the closed-loop system contribute to its step response, find the minimum value of the proportional gain $K_p$ for which both specifications will be satisfied.

**b)** Justify the assumption made in part (a), i.e., determine whether any CLTF zeros or non-dominant poles are expected to have a non-negligible effect on the closed-loop response. If the assumption is not valid, discuss which of the specifications will be adversely affected.


100
50
0
-50
-100
-250
-200
-150
-100
-50
0
50
Real Axis
Imag Axis

**Problem 4**:
The frequency response for a plant has been experimentally determined and is given in the figure as a Bode diagram. The plant input is *u(t)* + *d(t)*, where *u* is the control input and *d* is a disturbance. The plant output is *y(t)*. *y(t)* is measured and compared to $y_{ref}(t)$ and the error *e(t)* is fed back into the plant such that $u(t) = K_P e(t) + K_I \int_0^t e(\tau)d\tau$.

**a)** Diagram the system with standard block diagram conventions and transfer function representations. Use $G_P(s)$ to represent the plant.

**Answer the following (b)-(f) for the closed loop system as described.**

**b)** What is the maximum $K_I$ for stability assuming $K_P = 0$?

**c)** What is the gain margin assuming $K_I = 0$.

**d)** What is the steady state error for a unit step input to $y_{ref}$? If your answer depends on the values of the gains $K_I$ and $K_P$ you should explain.

**e)** What is the steady state error for a unit step input to *d*? If your answer depends on the values of the gains $K_I$ and $K_P$ you should explain.

**f)** For a disturbance input *d(t) = sin(10t)*, what is the amplitude of the response *y(t)* of the closed loop system at steady state? Assume $K_I = 0$ and $K_P = 1$.