

**Ph.D. Qualifier Examination**  
**Georgia Institute of Technology**  
**System Dynamics and Controls (SDC), Fall 2009**

**Problem 1**

The figure shows a hydraulic crane working on the CRC building at Georgia Tech. The crane is lifting material from the ground up to the roof of the building. The crane arrives in a compact form and then the arm extends via a telescoping hydraulic extension. The crane lifts the payload by reeling in a suspension cable. The extension arm can rotate in and out from the base. A third degree of freedom comes from a rotating base.

The machine is difficult to operate because the payload will swing like a pendulum and the extension arm can bend and oscillate. We want to understand the time response of this machine so that we can improve its design and add a control system to remove the dangerous payload swings and arm bending.

a) As a first step, ignore the bending of the arm and rotation of the base. Furthermore, assume the payload is suspended by a fixed length of cable. Develop a simple model that will predict the payload motion as a function of the arm movement.

b) Assume the payload is in the position shown in the figure. Sketch the response when the arm rotates quickly 5 degrees toward the vertical.

c) Change your model to account for the changing suspension length.

d) Add the effect of the arm bending to your model.

e) Sketch the response of this more complete model when the arm rotates quickly 5 degrees toward the vertical.

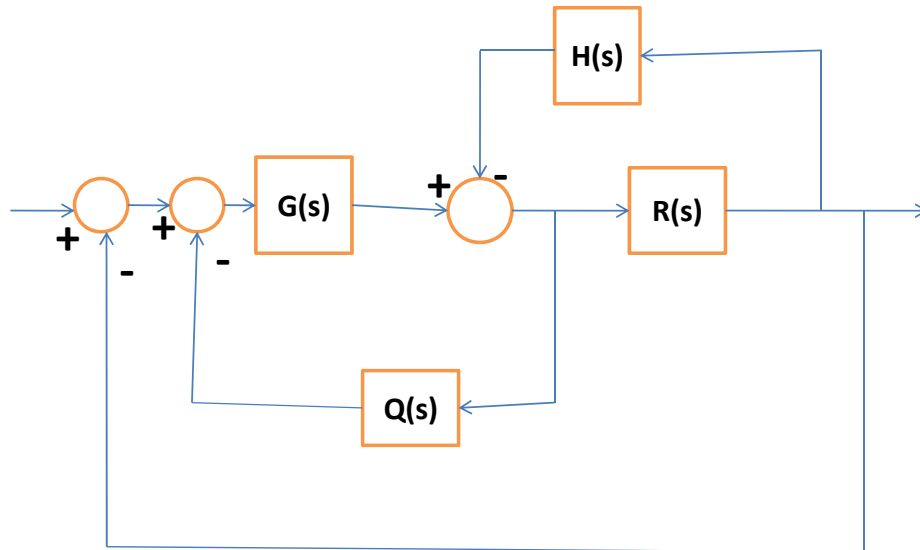
f) Sketch the response of the payload when a gust of wind hits it. Plot two separate cases on the same graph: i) The payload is near the ground and the arm is extended at a 45 degree angle, ii) The payload is near the roof and the arm is at a 10 degree angle relative to the vertical.



**Figure: Hydraulic Crane**

## Problem 2

Consider the following system:



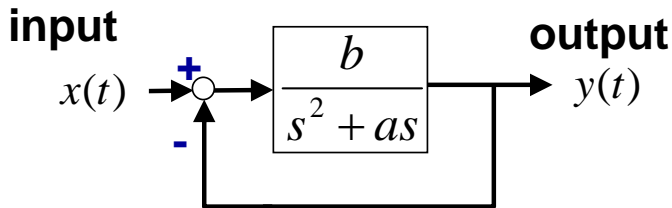
Here  $G(s) = \frac{1}{s+a}$ ,  $R(s) = \frac{1}{s^2+s+1}$ ,  $H(s) = \frac{1}{s+2}$ ,  $Q(s) = 1$ , and  $a$  is a constant parameter.

- Determine the range of  $a$  so that the system is stable.
- Sketch the root-locus plot for the range of  $a$  where the system is stable. In the plot, one needs to show the starting position for each branch. One also needs to indicate sufficient information to help to understand the plot.

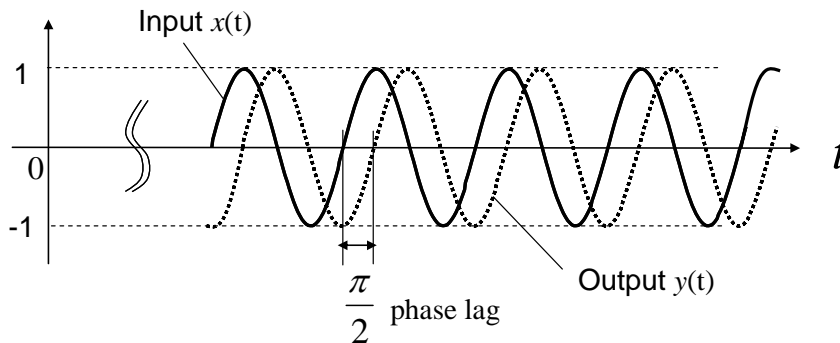
### Problem 3

Consider the steady-state response of the following unity feedback system,  $y(t)$ , to a sinusoidal input,  $x(t) = \sin \omega t$ , with the amplitude of 1 (one) and the frequency of  $\omega$ . The open-loop transfer function of the system is  $P(s) = \frac{b}{s^2 + as}$  where both  $a$  and  $b$  are positive parameters.

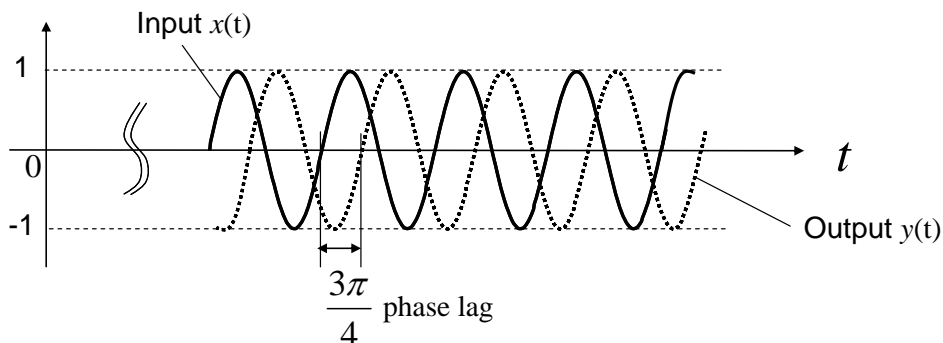
*Hint:* You can use  $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}$  and  $\mathcal{L}[\cos \omega t] = \frac{s}{s^2 + \omega^2}$ , if necessary.



- (a) **Determine condition(s) for  $a$  and  $b$**  such that there exists a frequency of  $\omega$  for  $x(t)$  by which i) the amplitudes of the input and output become the same, and ii) the phase lag of the output sinusoidal becomes  $\pi/2$  (rad) as shown below. **Obtain the frequency  $\omega$**  as a function of  $a$  and/or  $b$ .



- (b) **Determine condition(s) for  $a$  and  $b$**  such that there exists a frequency of  $\omega$  for  $x(t)$  by which i) the amplitudes of the input and output become the same, and ii) the phase lag of the output sinusoidal becomes  $3\pi/4$  (rad) as shown below. **Obtain the frequency  $\omega$**  as a function of  $a$  and/or  $b$ .



### Problem 4

A robotic vehicle is modeled as a uniform beam of length  $2a$  with mass  $m$ , inertia  $J$ , and suspension springs  $k_1$  and  $k_2$ . The wheel input displacements are  $u_1$  and  $u_2$ , and the body motion is a vertical displacement  $y$  and a small roll  $\theta$ .

- Assuming all initial conditions are zero, model the system using Laplace transforms. For uniformity, leave your answer in a form that has *no fractions*.
- What condition(s) is (are) sufficient to decouple the output displacements for all inputs?
- Determine the transfer functions that relate the vertical displacement to the input displacements. What is the order of these functions?

