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# GEORGIA INSTITUTE OF TECHNOLOGY

## The George W. Woodruff
## School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Spring Quarter 1995**

SYSTEMS DYNAMICS AND CONTROLS
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

**-- Please sign your name on the back of this page --**

1. In many situations it is desired to support an object with a magnetic field as shown in Fig. 1. The spherical mass M is magnetic and is therefore attracted upward by the magnetic field induced by $I$. Hence, there is a position $x=d$ at which the mass is supported against gravity by the magnetic field. Feedback is introduced by using the optical system to detect the position of the mass. The photomultiplier and amplifier are adjusted to give a current $i'$ as nearly as possible proportional to deflection $x'$ from the equilibrium position $x=d$.

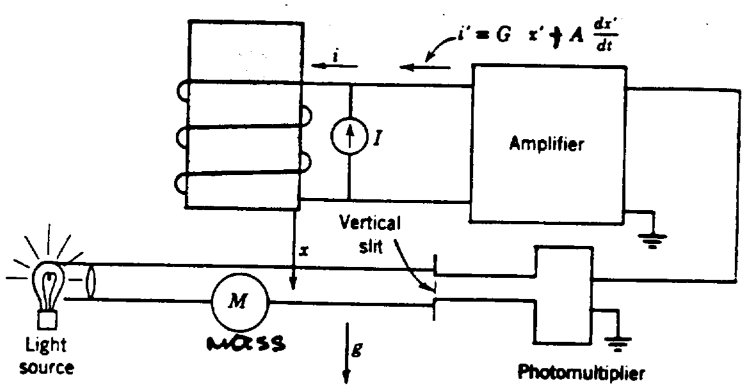



Figure 1

Measurement of the inductance $L$ as a function of the position of the mass shows that

$$L(x) = L_1 + \frac{L_o}{1+x/a}$$

where $L_1$, $L_o$, and $a$ are positive constants. Model the dynamic system by using the following steps:

(a) Obtain a constitutive relationship describing the external force $f$ which requires to hold the mass M in equilibrium as a function of the displacement $x$.

(b) Given the current $I$, the equilibrium position $d$ is determined. Perturbation $x'$ from the equilibrium lead to a perturbation current $i'$; that is

$$x = d + x'$$

$$i = I + i'$$

Obtain a linearized equation of motion about an operating point $(d, I)$.

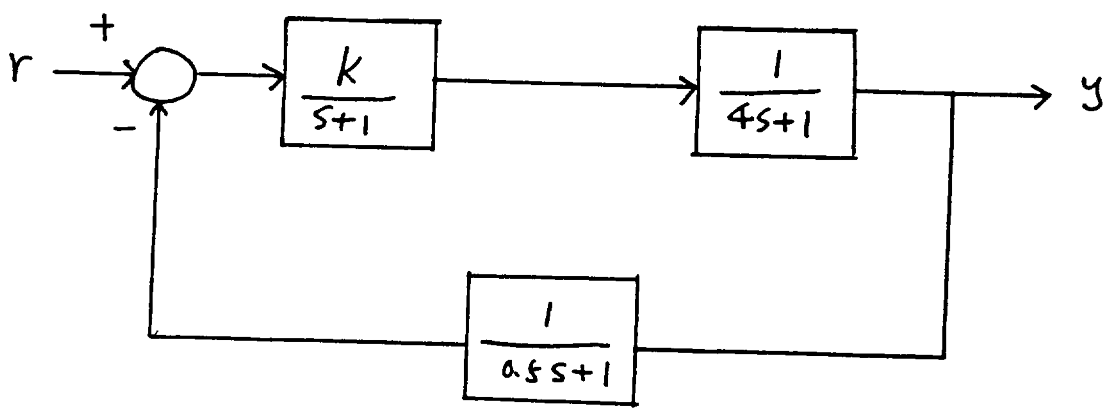
(c) If $i' = Gx' + A(dx'/dt)$ where $A$ and $G$ are adjustable amplifier gains, determine the range of $A$ and $G$ for the system to be stable. What would be the natural frequency and the damping ratio for the system if $A = 0$ ?

2. Consider the following system.

(a) Determine the gain $K$ required to keep the steady-state error to be less than 7% of the reference setting.

(b) With the gain determined in part (a), use the Nyquist criterion to investigate the stability of the system.

(c) Estimate the gain and phase margins of the system. Is this a good design? Explain.



System Dynamics and Controls Qualifying Exam

3. A flexible shaft has a transfer function from the torque, applied at one end of the shaft, to the angle of rotation at some point along the shaft. The transfer function consists of an infinite number of poles and zeros. In this problem you are to analyze the feedback control of that system by root locus techniques. A proportional control is used to control the system with a gain K between zero and + infinity and with a unity gain negative feedback.

(a) The first case to consider is shown in the figure with two open loop poles at the origin and an infinite number of alternating open loop zeros and poles placed on the imaginary axis. (This is the case with measurement and torque at the same point on the shaft and no friction in the system.) Sketch the root locus as K varies for this case and describe in a few words the nature of the system response and its stability properties. How would this change if the angular velocity were measured and fed back instead of the angle.

(b) The second case introduces friction at the point of application of the torque. The new arrangement of open loop poles and zeros is shown in the second figure. Sketch the root locus as K varies. The sketch will be approximate, but show the essential stability properties in the sketch. How have they changed from case (a).

(c) The third case results from moving the sensor from the end of the shaft to the middle. The alternating structure of the poles and zeros is no longer observed. This is indicated in the third figure. Again, sketch the root locus as K varies, showing the essential stability properties in the sketch. (Hint: concentrate on what happens for K just greater than zero or for K very large.) How have they changed from cases (a) and (b).