# GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Spring Semester 2002**

System Dynamics & Controls
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

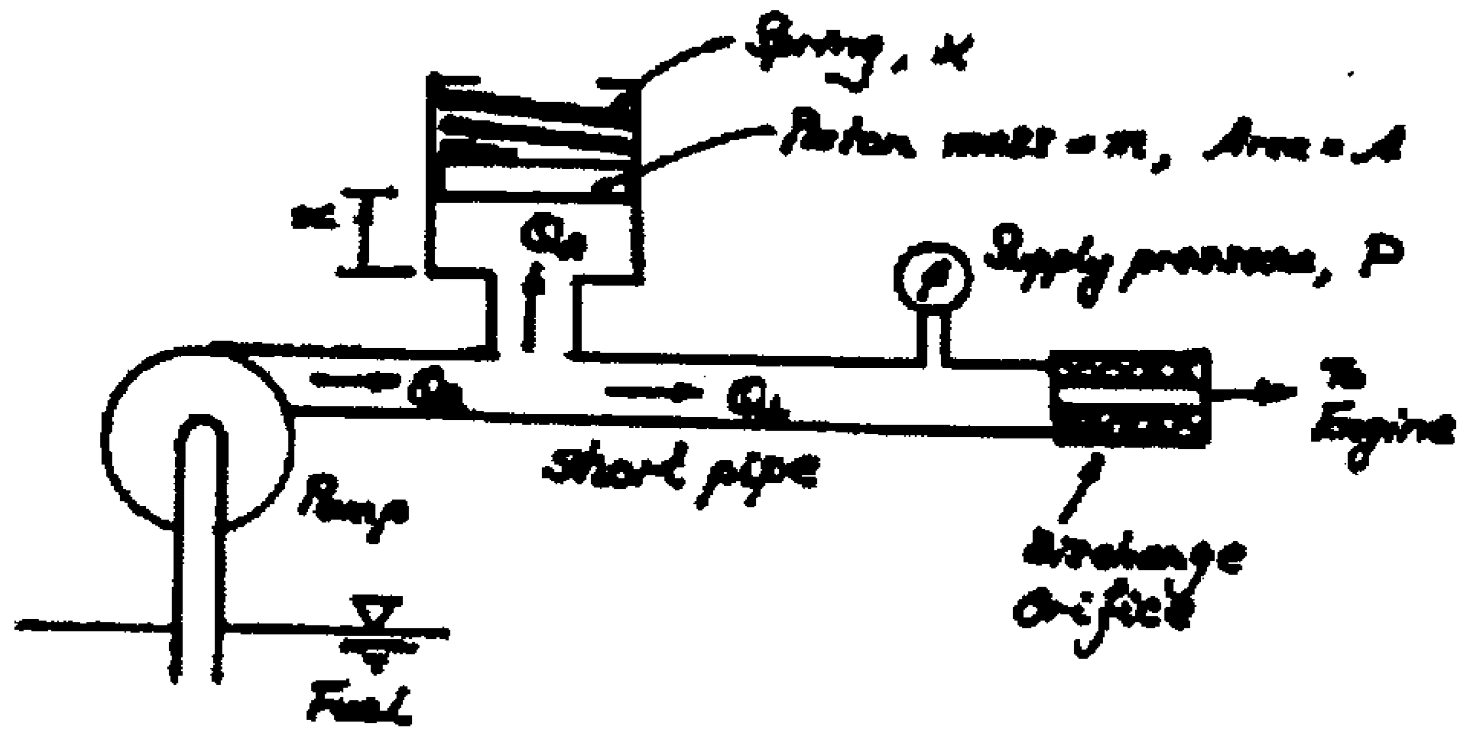
## **Special Instructions:**

1. Problem 1 will be graded for all students.

2. You must choose *2* out of the last three problems (problems 2-4) to be graded. If you fail to clearly indicate your choices or choose to do all 4 problems, the first three problems will be graded.

3. **Problem 1:**

The following figure shows a system for supplying fuel to automotive engine using an electrical fuel pump.




An "accumulator" (that is a piston moving against a spring in the cylinder) is provided to smooth fluctuations in the supply flow, Qs. The flow through the orifice is proportional to the square root of the pressure difference across the orifice.

**a)** Derive the transfer function P(s)/ Qs(s) about an equilibrium operating point. (Give all the equations required to define the operating point).

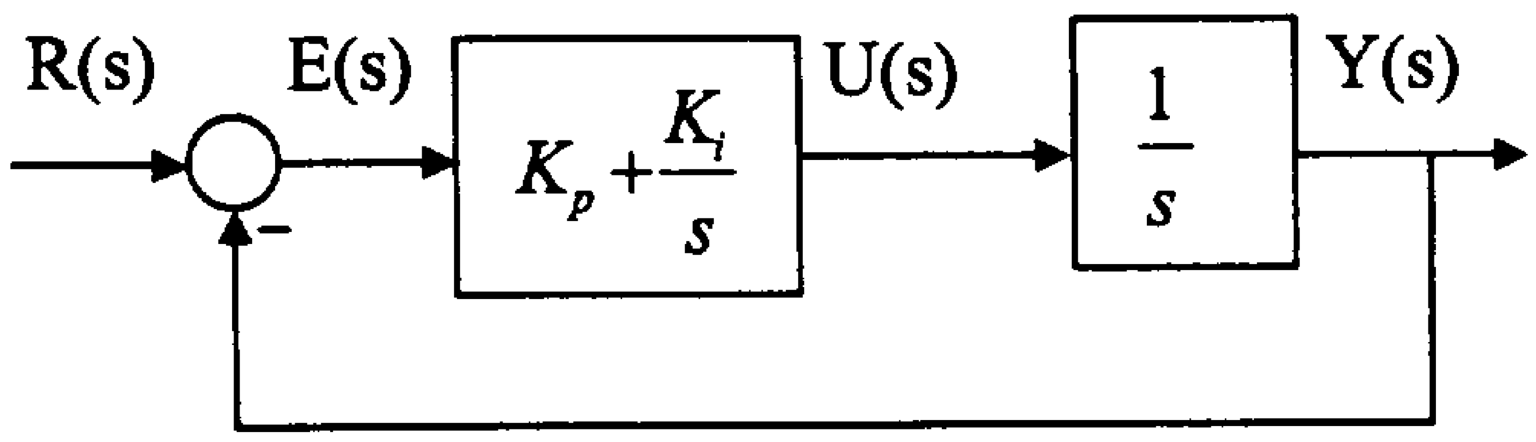
**b)** Derive an expression for the damping ratio for the system.

**c)** Assume that the system is critically damped. Provide a neat sketch to describe P(t) when Qs(t) is experience a step change of magnitude Qo.

**d)** If the short pipe is replaced by a long pipe, show how would you account for the inertia of the fluid in the transfer function derived in part (a).

**Problem 2:**

The block diagram shown below depicts a Proportional-Integral (PI) feedback controller of a single integrator system where u(t), y(t) and r(t) represent the input, output, and the reference input.




Throughout the problem we shall assume that the PI gains ($K_p$ and $K_i$) are chosen such that the system has a *critical* ($\zeta$=1) closed-loop damping ratio.

**a)** What is the relationship between the PI control gains in order to satisfy the critical damping assumption?

**b)** What is the steady state error due to a step or ramp reference inputs. Why?

**c)** Derive an expression for y(t) and u(t) in response to a unit step reference input r(t)=1(t). Compute the output maximum *overshoot* percentage.

**d)** For what values of the PI gains does the input magnitude remain below a given saturation limit $u_{max}$ in response to a step reference input of magnitude $r_{max}$.

**e)** Derive a neat expression for the *amplitude* of the error e(t) in response to a sinusoidal reference input r(t)=sinωt. What is the maximum amplitude of the error for all possible frequencies ω?

Depending on how you solve this problem you may or may not need to know that

$$L\,(t^n e^{-at}) = \frac{n!}{(s+a)^{n+1}}$$

**Problem 3:**

Figure 1 depicts a simple position control system that has a single zero and two poles. All questions in this problem will address the system shown in Figure 1. *For all questions, do not use a calculator. All answers should be reduced as much as possible, but not in decimal format.* This question is not designed to bog you down in calculations. It is design to see if you can make use of the root locus and really understand its fundamentals.

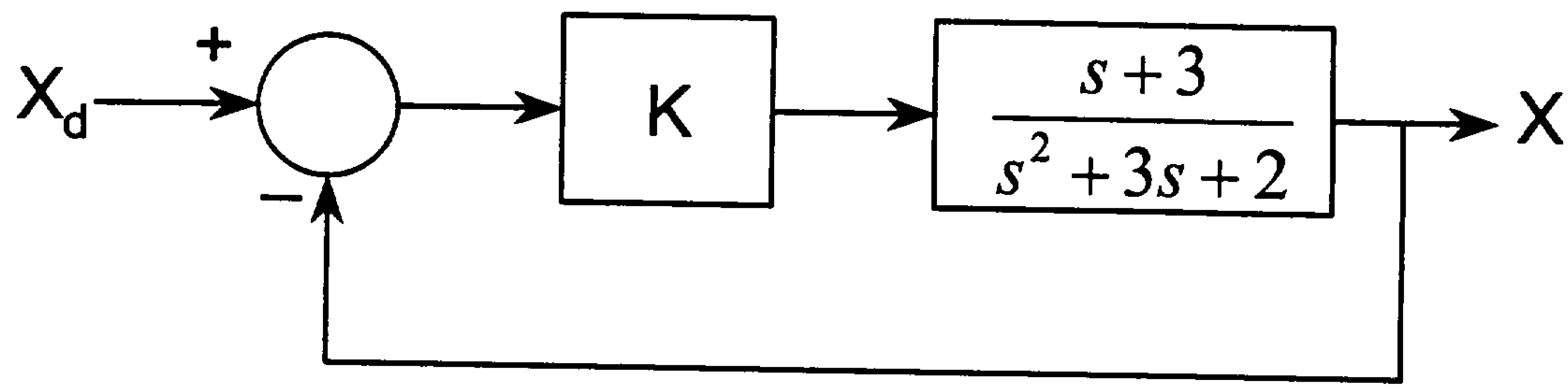



Figure 1: Position Control System with Proportional Gain.

**a)** Draw the root locus for the systems shown in Figure 1 as a function of K. Highlight all of the important aspects of the plot. Compute all break points and values of K that yield those breakpoints.

**b)** Clearly, the roots take a circular trajectory about the zero in your root locus plot. **Compute the radius of that circle.**

**c)** Determine the minimum damping ratio that you can achieve with this system, and the corresponding gain value for that minimum damping ratio.

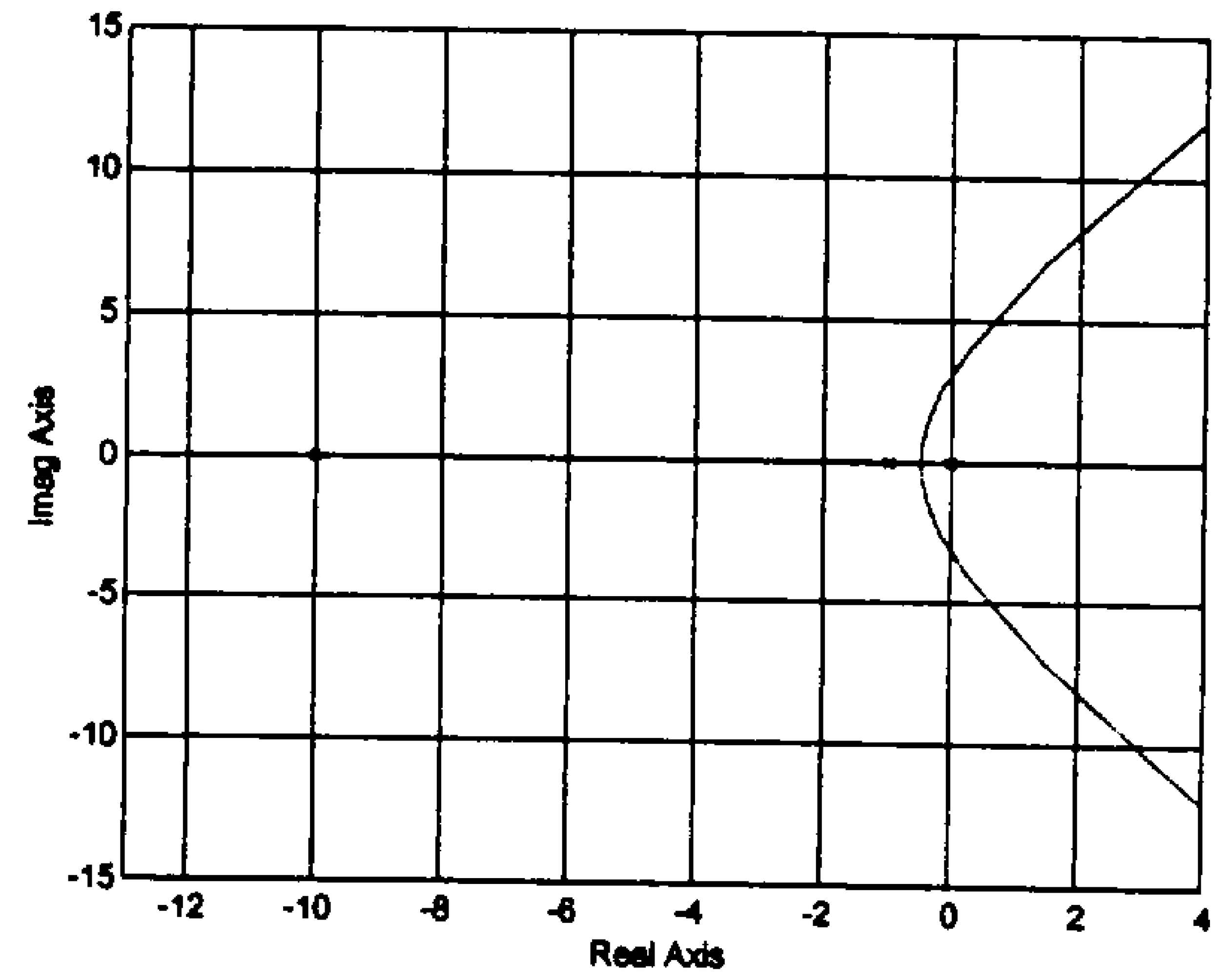
**d)** Clearly you can compute the percent peak overshoot, $M_p$, for the minimum damping ratio that you determined in part C using your standard second order response relationships. Will the system shown in Figure 1, exhibit the $M_p$ that you could compute? In three sentences or less discuss why or why not, and indicate if you expect the $M_p$ to be larger or smaller and why you expect this.

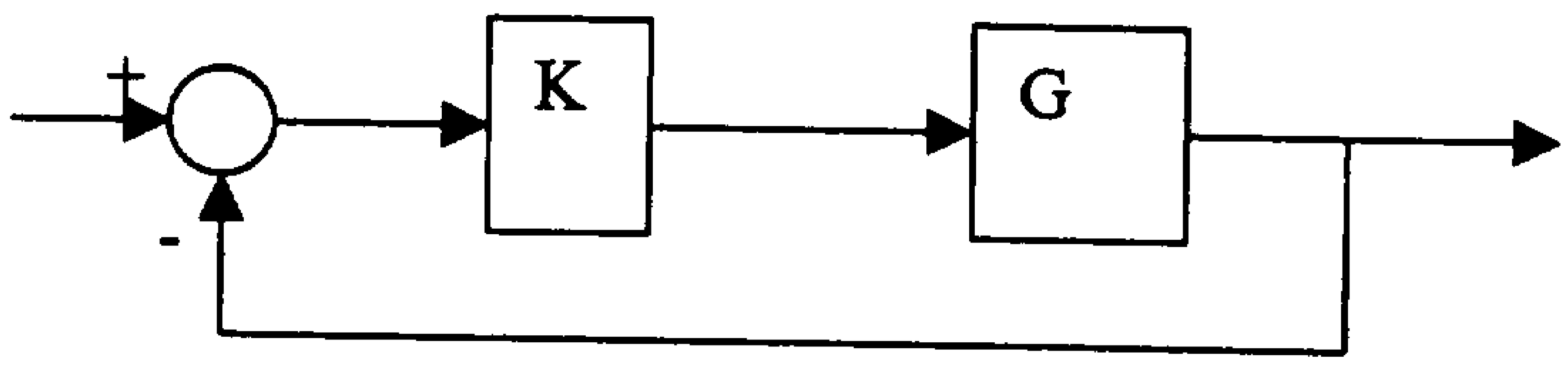
**e)** Clearly, there are segments of the root locus plot in which the poles are complex conjugate and in which they are purely real. In fact, you know that poles must either be purely real or must occur in a complex conjugate pair. In 3 sentences or less, explain why this is so. Do not go into a significant mathematical explanation; just provide a simple intuitive answer.

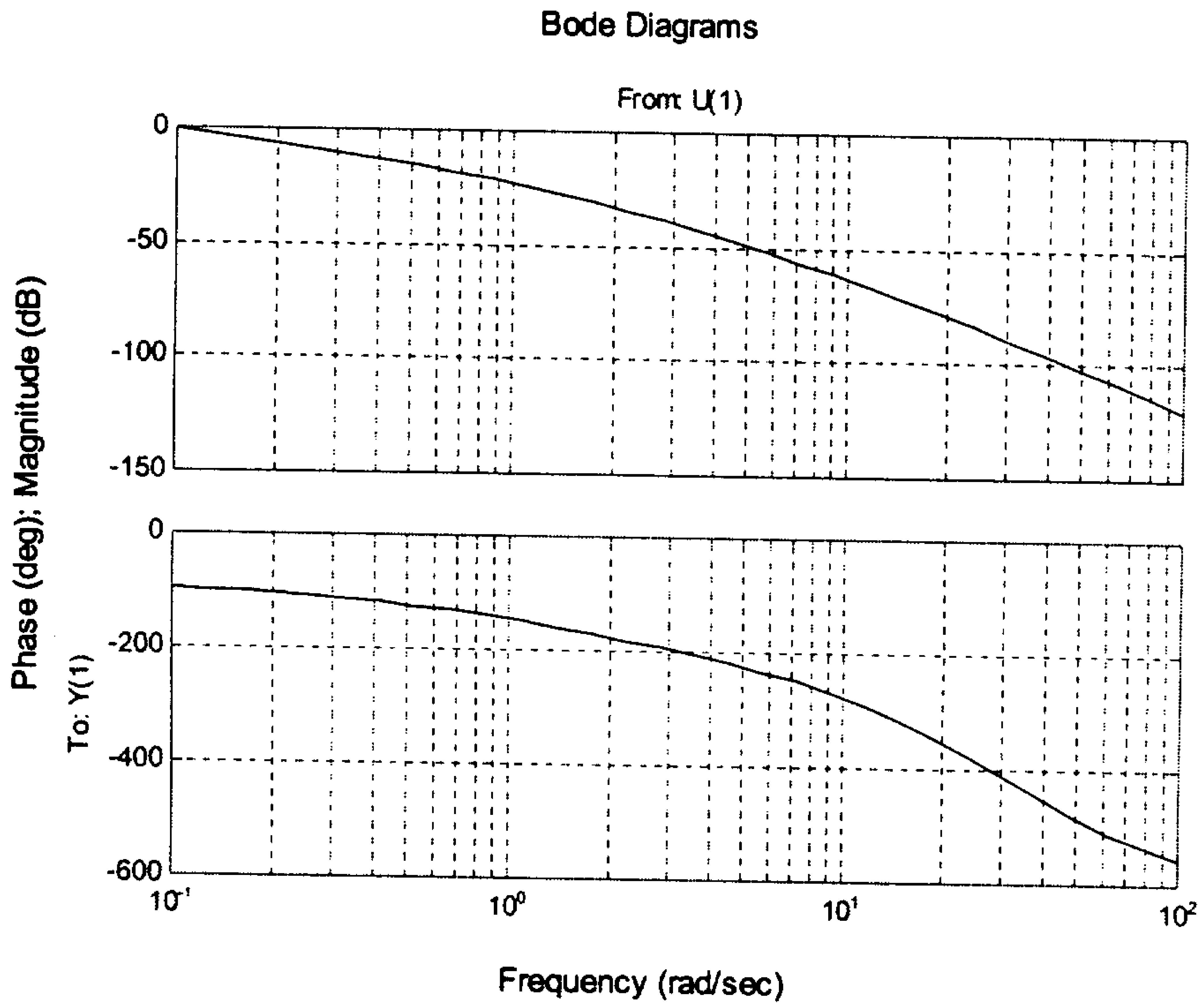
**f)** As K→∞, one pole will proceed off to -∞ along the negative real axis, and another pole will proceed to the zero located at −3. Determine the rate that the poles migrate to -∞ and −3 as a function of the gain, K.

**Problem 4:**

The system G is to be placed in a feedback loop. When modeled from engineering analysis the best model obtained is a linear third order plant $G1(s) = 1/(s^3 + 11s^2 + 10s)$. The root locus of G1 in the feedback as the loop gain K is varied is also given below. To verify the model an "open loop" frequency response has been obtained and the Bode plot of that experiment is given below.




**a)** Do you see any discrepancies between the model G1 and the experiments on G? Propose possible causes for this discrepancy. If we only consider linear models, what modifications to G1 would improve the match between experiment and model? Explain all answers.

**b)** Propose a lead compensator that would improve the gain margin and/or phase margin of the feedback control system.