GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2004

System Dynamics & Controls

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

✱ Please sign your **name** on the back of this page —

Please **p r i n t** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

Systems and Control Qualifier: work any 3 of 4.

1. A motion system with actuation that can be modeled as a force source acting on a rigid mass is very simple to control since the open loop system is

$$M\ddot{x} = F(t) \qquad (1)$$

For example, position and velocity feedback

$$F = k_p x + k_v \dot{x}$$

can theoretically place the closed loop poles wherever desired. Consider a scheme to measure acceleration of the real actuation system with an accelerometer and use this measurement as a feedback control system. The actuation system can be approximated by

$$V(s) = \frac{K_a}{\tau_a s + 1} U(s) + N(s)$$

where *V(s)* = the actuator velocity
U(s) = the actuator input
N(s) = a noise that represents all disturbances

And the accelerometer is represented as

$$A(s) = G_{ACC}(s) Y(s)$$

where *Y(s)* = position of the accelerometer.

(a) Draw the block diagram that represents this actuator with proportional acceleration feedback of gain K_1.

(b) If the accelerometer is ideal, giving a signal exactly proportional to its acceleration, Draw the root locus as the proportional gain K1 varies. You can use $\tau_a = 0.1$ seconds. For this case what is the "optimal" gain K1 to achieve the ideal situation proposed by equation (1)?

(c) Find the transfer function from noise input *N(s)* to acceleration output for the diagram in (a).

(d) In practice the accelerometer is often constructed as a spring-mass system where the position of an internal mass yields the output measurement. In this case

$$G_{ACC} = \frac{K_{ACC} s^2}{s^2 + \frac{b}{M}s + \frac{k}{M}} \qquad (2)$$

Thinking of the function of an accelerometer, how would you pick the parameters in the transfer function to get the correct steady state response and good transient response? What are realistic limitations due to?

(e) Using the model for an accelerometer in equation (2) how would the root locus plot previously created in (b) be modified. Sketch the new root locus. Use a natural frequency of 1000 rad/sec and a damping ratio of 0.5 for the accelerometer.

(f) Do you see any reasons the original hypothesis for using acceleration feedback should not work based on the models proposed here?

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2. Figure 1 shows the block diagram for the attitude control of a satellite with structural resonance. Figure 2 compares the Bode plots of G for two different designs:

Design (a): Actuators and sensors are on separate platforms.

Design (b): Actuators and sensors are on the same platform.

(1) From the Bode plots (without deriving the transfer function for G), compare the two designs from the perspective of stabilizing the system with a PD controller. Explain your reasons in terms of stability margins.

(2) On the basis of your arguments in (1), design a PD controller $K(Ts+1)$ for the better design so that the phase margin is no less than 45°. Give the values of K and T and the corresponding cross-over frequency.

(3) Discuss the sensitivity of T on the closed-loop step response.

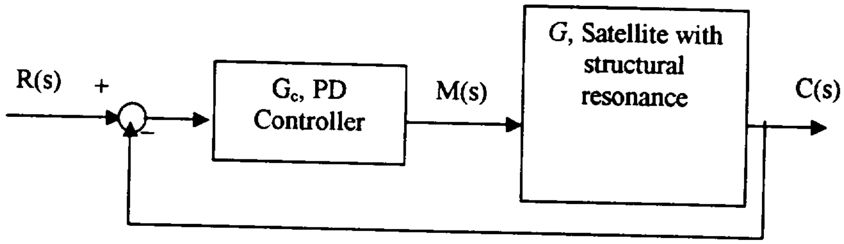

Figure 1

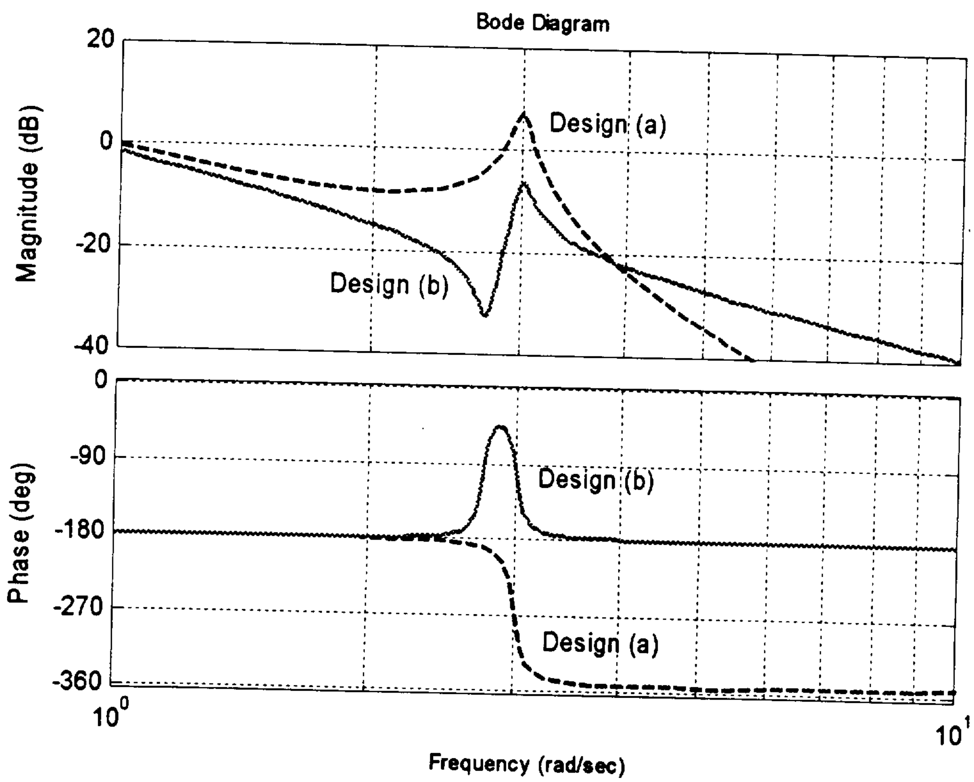

Figure 2 Comparison of two designs

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3. All parts of this question refer to the lumped parameter model shown in Figure 1. The two masses, m_a and m_b, are connected to the pivot point by massless links of lengths a and b, respectively. The entire system rotates together about a frictionless pivot. The angle of rotation is given by the angle θ as shown in Figure 1. Note θ is zero when the links are horizontal. The system also has a linear damper with damping coefficient, c, and a spring with a spring constant, k. For this problem, an external force, F, acts upon mass, m_b, in the vertical direction.

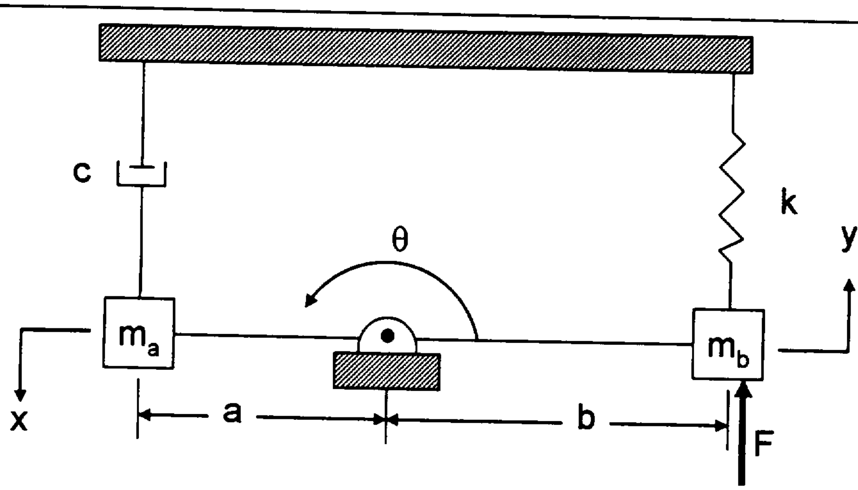

Figure 1: 2 Mass Pivot System with Spring and Damper.

Part a

Determine the transfer function between the force, F, and the angle θ. You may assume small angle motions of the system.

Part b

Clearly this system mimics a rotational system. Draw the equivalent rotational system, and provide the relationships between the equivalent rotational system's parameters and that of the system shown in Figure 1. (Hint: the equivalent rotational system will have some rotational damping. How is the rotational damping parameter related to the parameters in Figure 1 such as c?)

Part c

Is the equivalent rotational system that you developed in part b "completely" equivalent to the linear system shown in Figure 1? Justify your answer. Please limit your discussion to one page (one side of a page).

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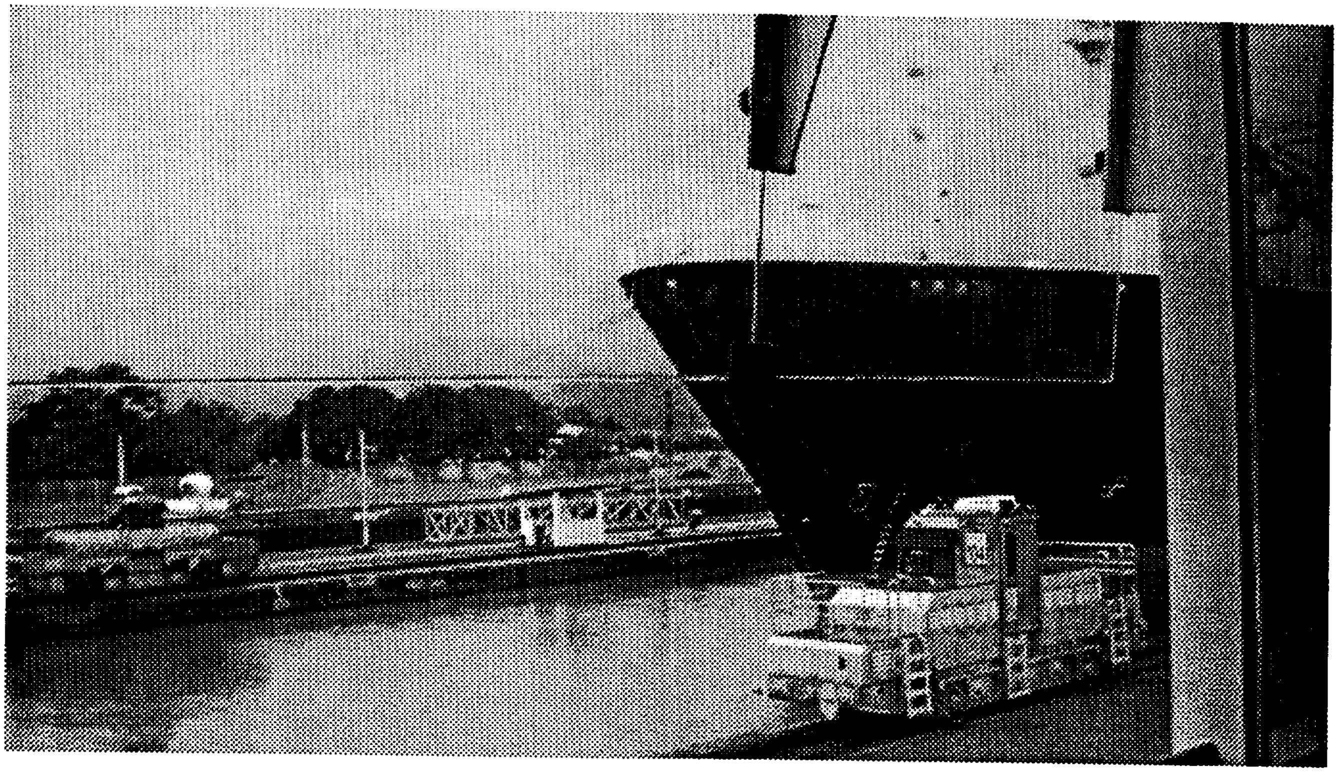

Figure 1: A Ship Being Positioned in the Panama Canal.

4. As cargo ships travel through the Panamá Canal, they must be raised up from sea level to the height of Lake Gatún which is about 27m above sea level. They then proceed across the lake and are lowered back to sea level on the opposite side.

To accomplish the lifting and lowering of such huge masses, the ships go through a series of locks. The locks are analogous to bathtubs that can be drained and filled with water. Given that the canal was complete in 1915 and cargo ships have gotten progressively larger, the big ships barely fit into the locks. To position and control the ships within the locks, they are attached by cables to Mitsubishi "Mules", as shown in Figure 1. The mules are small locomotives with powerful winching mechanisms. Two or three mules are attached to each side of the ship.

1) Sketch a simple model for the positioning control of the system taking into account the mass of the ship and mules, the cable and winch flexibility, the horsepower limitations, and the effect of the water.
2) What kind of control system would you implement to help position the ships?
3) How would you get numerical values for your modeling parameters?
4) What kind of sensors would be needed?
5) Suppose the mules started pulling at full force, how would the ship respond?
6) Sketch the tension in the cables based on your answer to part 5.