George W. Woodruff School of Mechanical Engineering

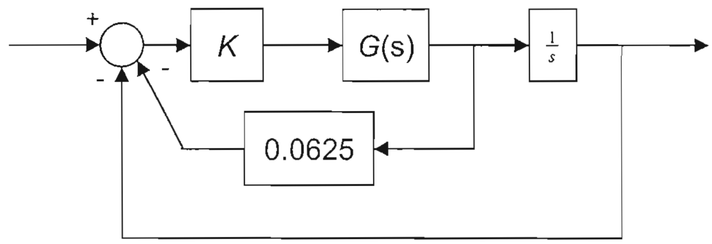
Spring 2005 System Dynamics and Controls

Doctoral Qualifying Examination

INSTRUCTIONS

There are 4 questions attached, please solve 3 of the four questions as completely as possible. State all assumptions, and make sure that you clearly indicate the thought processes that you employed to arrive at your answer. Answer only 3 questions. If you answer 4 questions, only the first 3 will be graded.

ROOT LOCUS



In the velocity-feedback control system shown above, $K > 0$, and the plant transfer function is

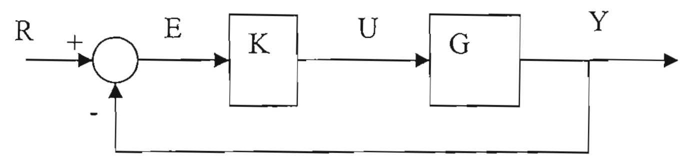
$$G(s) = \frac{1}{(s^2 + 18s + 162)(s + 9)}$$

(a) Draw the root locus, with any break-away/break-in points, asymptote intercepts, and imaginary axis crossings accurate to within ±1 rad/sec, and angles of departure/arrival and of any asymptotes accurate to within ±10°. Show all calculations.

(b) For what value of K will the dominant closed-loop pole be located at s = -1 rad/sec? Estimate the settling time of the closed-loop system step response at this value of the gain.

FREQUENCY RESPONSE

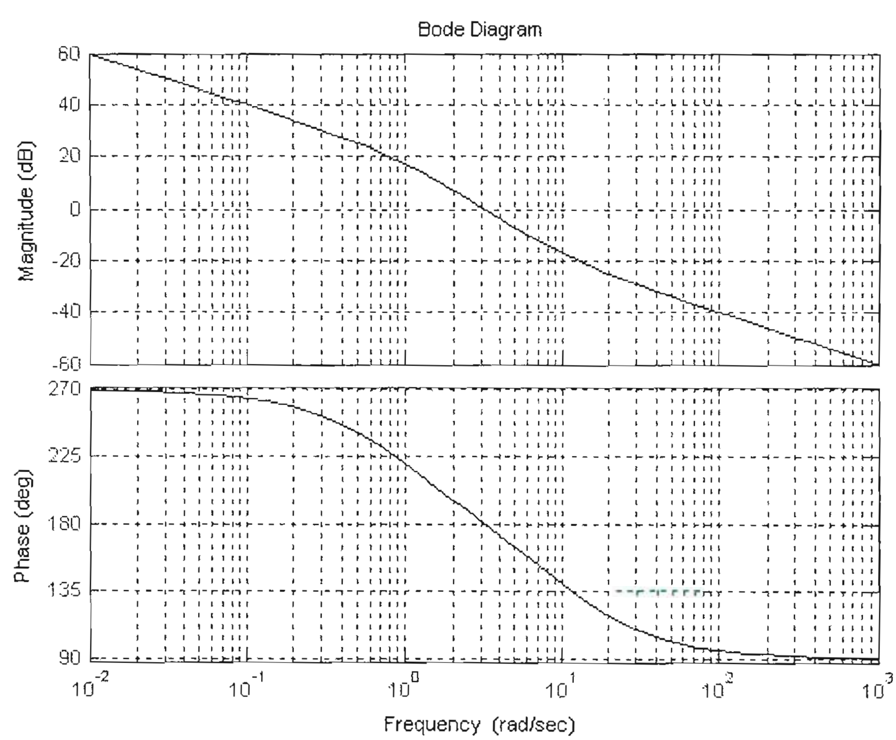
Consider the feedback system shown in the block diagram below.



The Bode diagram of the plant *G* (*s*) is given in the graph below. Note that in the phase plot 270=-90, 180=-180, and 90=-270 degrees.

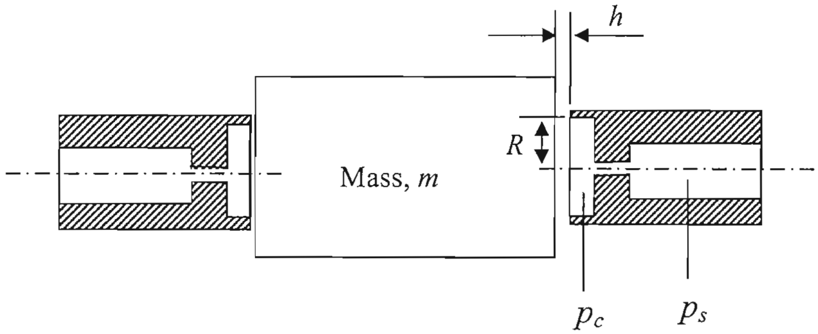
Answer the following questions based on the Bode diagram:

a) For what values of the proportional control gain K is the closed-loop system stable?
b) Find the steady-state error in response to the unit ramp reference input r(t)=t as a function of K. For what values of K are your answer valid?
c) Repeat part (b) for the sinusoidal reference input r(t)=sint
d) Estimate the transfer function G(s).
e) Design a feedback controller (P, PD, or PID) such that the compensated system has no steady-state error to a constant reference input with a phase margin of at least 45 degrees and a cross-over frequency of at least ω_c=10 rad/sec.



MODELING

The figure (shown below) illustrates a non-contact bearing system consisting of a pair of pocketed-orifice bearings, each of which is supplied with constant air pressure p_s.



The pocketed-orifice bearing can be modeled as a dynamic system consists of an air capacitor (with pressure p_c) and two orifice resistors (a fixed orifice between p_s and p_c, and a variable orifice between p_c and the ambient). You may assume that the flow through the orifice is proportional to the square root of the pressure difference across it.

(1) Derive a linearized model to describe the gap motion $h(t)$.

(2) Determine the design criteria such that the non-contact bearing system will function as a self regulator of the gap h.

CONTROL DESIGN

A simplified model of a classical naval nuclear reactor is given by the following transfer function:

$$H(s) = \frac{(s+4)}{(s+1)(s-1)}$$

Your mission is to design a controller, P, PI, PD, PID, Lead, Lag, Lead-Lag, etc... to meet the following specifications:

1. Use the simplest controller possible (so if a PI works, using a PID will be incorrect).
2. Use the lowest gains possible.
3. The real part of the poles must be less than -6.
4. The imaginary part of the poles must be zero (*i.e.*, your closed loop system should have poles on the real axis.)

Since this design is going to be used for a nuclear reactor, the Navy has requested that you design the controller using the root locus method (drawing a detailed root locus) and then validate your design by analyzing the closed-loop transfer function.

Part A

Design the controller

Part B

In no more than 10 sentences, discuss the relationships between your answers in **Part A** and the closed-loop performance of the system.