Ph.D. Qualifier Examination
Georgia Institute of Technology
System Dynamics and Controls, Spring 2010

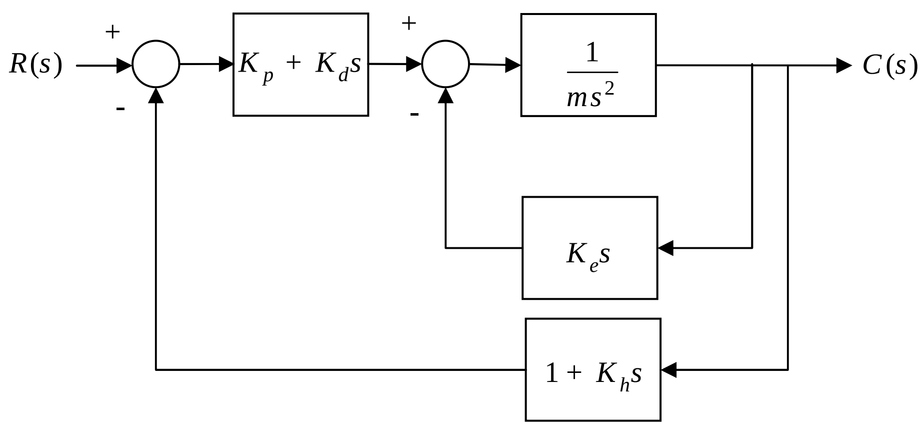
Choose three problems. If choose four problems, the top three scores will be used.

[1] The figure below shows the block diagram of a dynamic system. The error signal is defined as $E(s) = R(s) - C(s)$ in the Laplace domain.

a) Determine the governing differential equation in terms of $e(t)$ and $r(t)$.

b) Given a unit step input, what is the governing differential equation for $t > 0$?

c) Will the system reach the specified input? Explain.



[2] Figure 1 (right) shows a person using an aerial lift to work on a building in Atlantic Station. The mechanism uses a telescoping mechanism to lift up the operator. Figure 2 (right) shows the hydraulic piston that changes the angle of the extended arm. The operator moves the machine by pushing on/off buttons that open valves to change the angle and extend the arm.

Figure 1: Aerial Lift.

Figure 2: Hydraulic Piston.

a) Create a simple model that can predict the endpoint response when the operator pushes 1 button to change the angle.

b) Sketch the endpoint response to a button press that lasts 2 seconds? Plot the input pulse and the response on the same graph.

c) As the arm is extended, the oscillation frequency of the endpoint will change. Sketch the relationship between arm length and oscillation frequency.

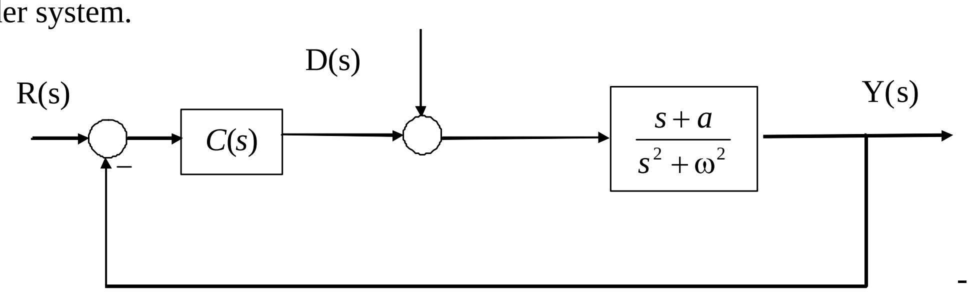
d) Sketch the relationship between the frequency and the mass of the people and tools that are in the basket at the end of the extension arm.

e) Suppose a gust of wind hits the machine when the arm is fully extended. Sketch the response to this disturbance and state how it differs from when a person pushes a control button for a short period of time.

f) Suppose the tires shown in Figure 2 have low air pressure. Create a more advanced model to capture any additional effects that this might cause.

g) Sketch a pulse response of your more advanced model.

[3] The block diagram shown below depicts a feedback control system for an undamped 2^{nd} order system.



a) Assuming a proportional controller ($C(s)=K_p$) , derive the expressions for the closed-loop poles of the system as K_p approaches infinity in terms of K_p, a , and ω. Prove your assertion.

b) For what values of K_p is the closed-loop system in (a) stable? What is the fastest settling time that can be achieved? For full credit you must consider all the possibilities for a.

c) Assuming $a>0$, design a feedback controller of your choice such that the closed loop system has zero steady-state error in response to a constant reference input r and constant input disturbance d. Using root-locus analysis, explain how the gains of the modified controller can be chosen to achieve a critically damped closed-loop system.

[4] Consider the following transfer function:

$$C(s) = k\frac{1+\beta s}{1+\alpha s} \quad (k > 0, \alpha > 0, \beta > 0)$$

a) Find the condition for α and β such that C(s) works as a phase-lead compensator.

b) Find the condition for α and β such that the maximum phase lead created by C(s) is 45 [deg]. Use $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$ if necessary.