GEORGIA INSTITUTE OF TECHNOLOGY

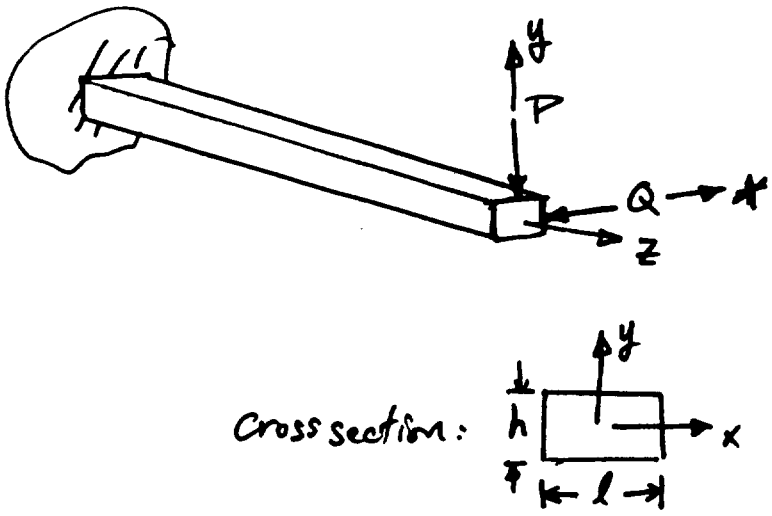
The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1995

Mechanics of Materials
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --



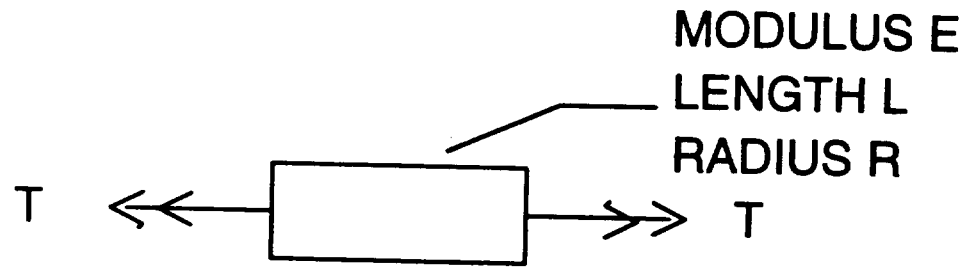
1. (a) A long, slender cantilever beam is subjected to bending about two axes as shown in the Figure above. Assuming infinitesimal strain, which are the nonzero stress and strain components in rectangular coordinates? Determine the condition for the onset of yielding in the beam in terms of the applied loads **P** and **Q**, based on the distortion energy or von Mises yield criterion for an initially isotropic, linear elastic material. At which location(s) does yielding first occur?

(b) If the beam remains linear elastic up to the point of failure, determine the failure condition for the critical location in the cross section of the shaft in terms of **P** and **Q**, assuming the maximum normal stress (Rankine) theory holds.

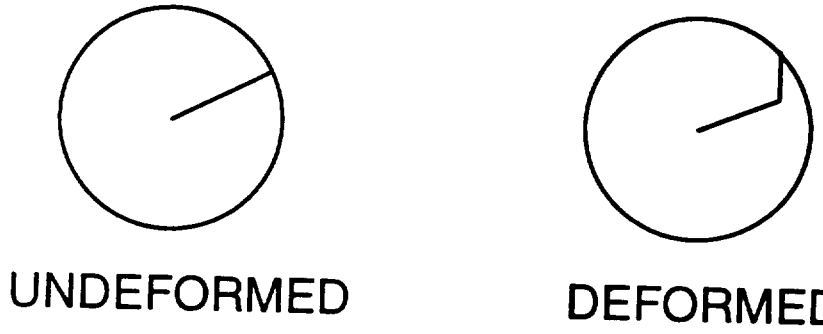
(c) If all you know is that the bending stress in the beam varies linearly through the thickness with respect to both x and y directions in a given beam cross section, what does this imply regarding nonlinearity, isotropy and homogeneity of material behavior?

(d) If the beam material yields during application of **P** and **Q**, is it permissible to superimpose the solutions for each of these two loads considered individually? Why or why not?

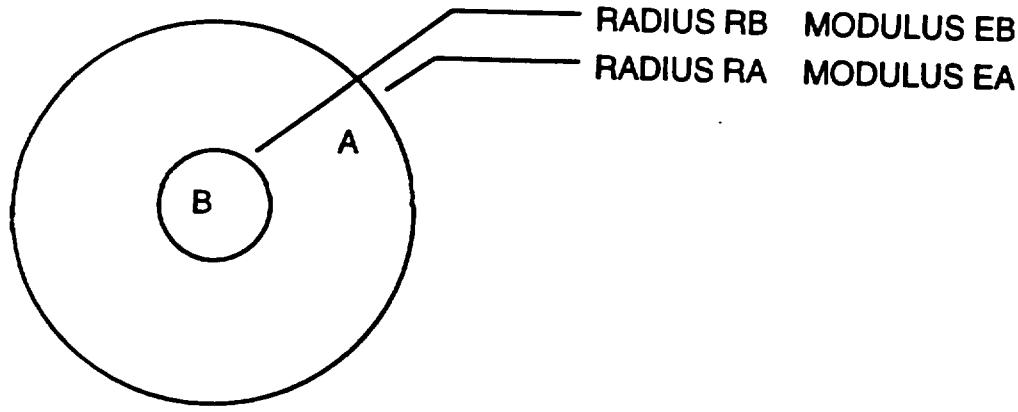
2. A uniform circular cylindrical shaft made of homogeneous, isotropic, linearly elastic material is subjected to a torque T as shown below.



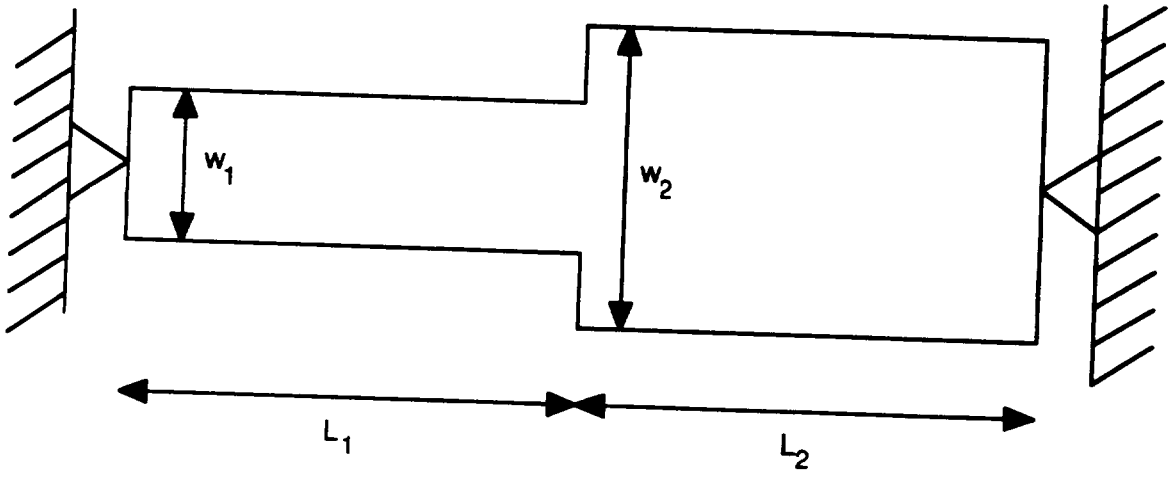
a) Sketched below is a cross section of the above shaft showing a radial line in the undeformed body on the left and the same line after deformation (exagerated for clarity) on the right. Make an argument based on physical reasoning that the line in the figure on the right should really be strainght, i.e., that straight lines remain straight for the problem stated.



b) Suppose that the shaft above is not homogeneous but is made instead of two materials, A and B, as indicated in the sketch below. Determine the stress distribution in the shaft. Be sure and state your assumptions CLEARLY and to justify them by physical reasoning.



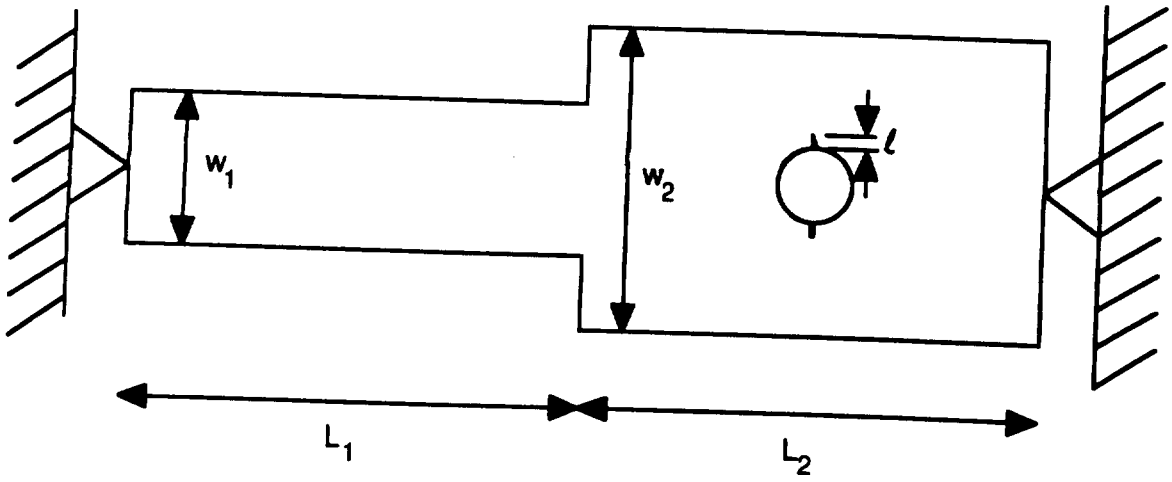
3. Consider the plate of thickness t with dimensions shown in the figure below. The plate is made from a homogeneous, isotropic, linearly elastic material (E, ν, α). A close fit exists at both of the rigid supports at room temperature. Determine the nominal values of the stress in portions 1 and 2 if the temperature of the plate is reduced by ΔT.



If L_1 and L_2 are sufficiently long relative to w_1 and w_2, respectively, why is it OK to neglect the complexities at the transition where the width changes?

Is the total strain in each portion zero? Please show why or why not?

Now consider a hole of radius c with stress concentration factor k_t in the wider plate. A pair of cracks of length ℓ emanate from the hole as shown in the figure below. If the same ΔT is applied, what are the stress intensities for these cracks? Consider the case when ℓ is very small and when ℓ is large. How could one determine whether a crack of length ℓ is small or large for this case?



4. A compressed air tank is supported by two cradles as shown, designed to permit free expansion/contraction of the tank (i.e. they do not exert any longitudinal force on the tank). The cylindrical body of the tank has a 30-inch inside diameter, d, and is fabricated from a steel plate having a thickness, t, of 1/2-inch. The plate is butt welded along a helix forming an angle of 25° with a transverse plane. The end caps are spherical and also have a uniform thickness of 1/2-inch. For an internal gage pressure, p, of 200 psi, determine:

a) Are the tank walls under state of "plane stress"? Why or why not?

b) Find the principal normal and principal shear stresses in the spherical caps.

c) Find the stresses in the directions perpendicular and parallel to the helical weld.