GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

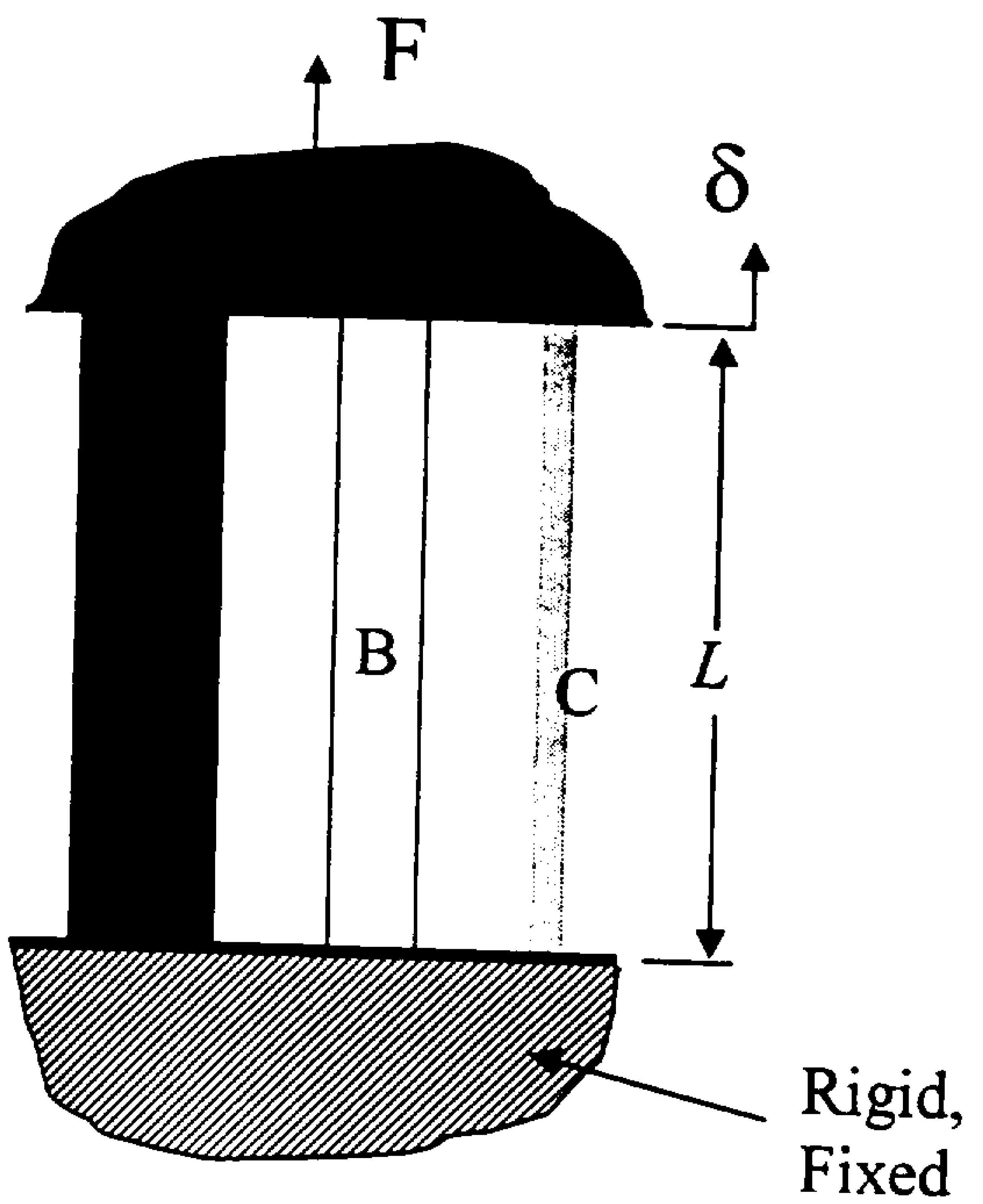
Ph.D. Qualifiers Exam - Fall Semester 2003

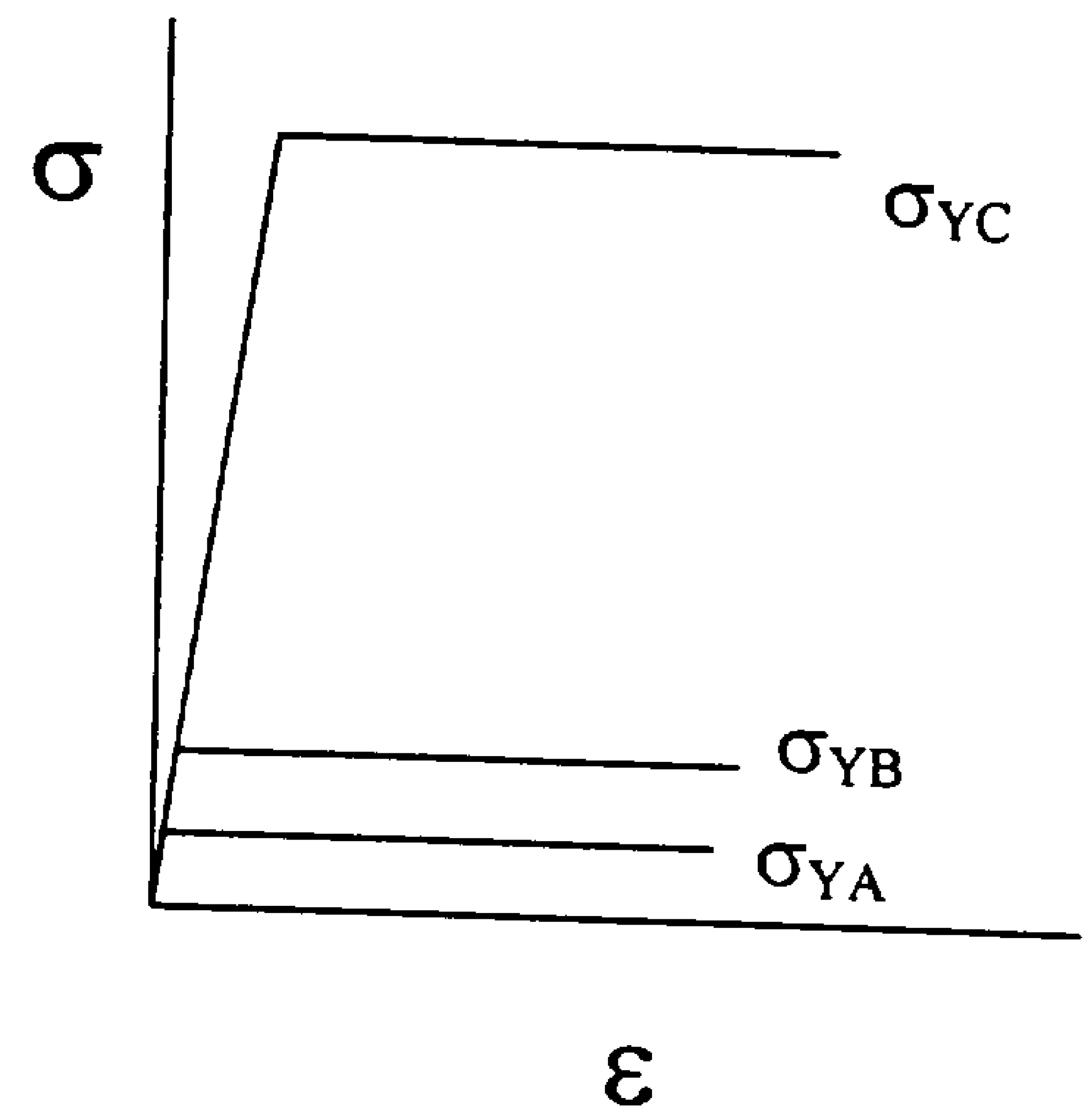
Mechanics of Materials
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

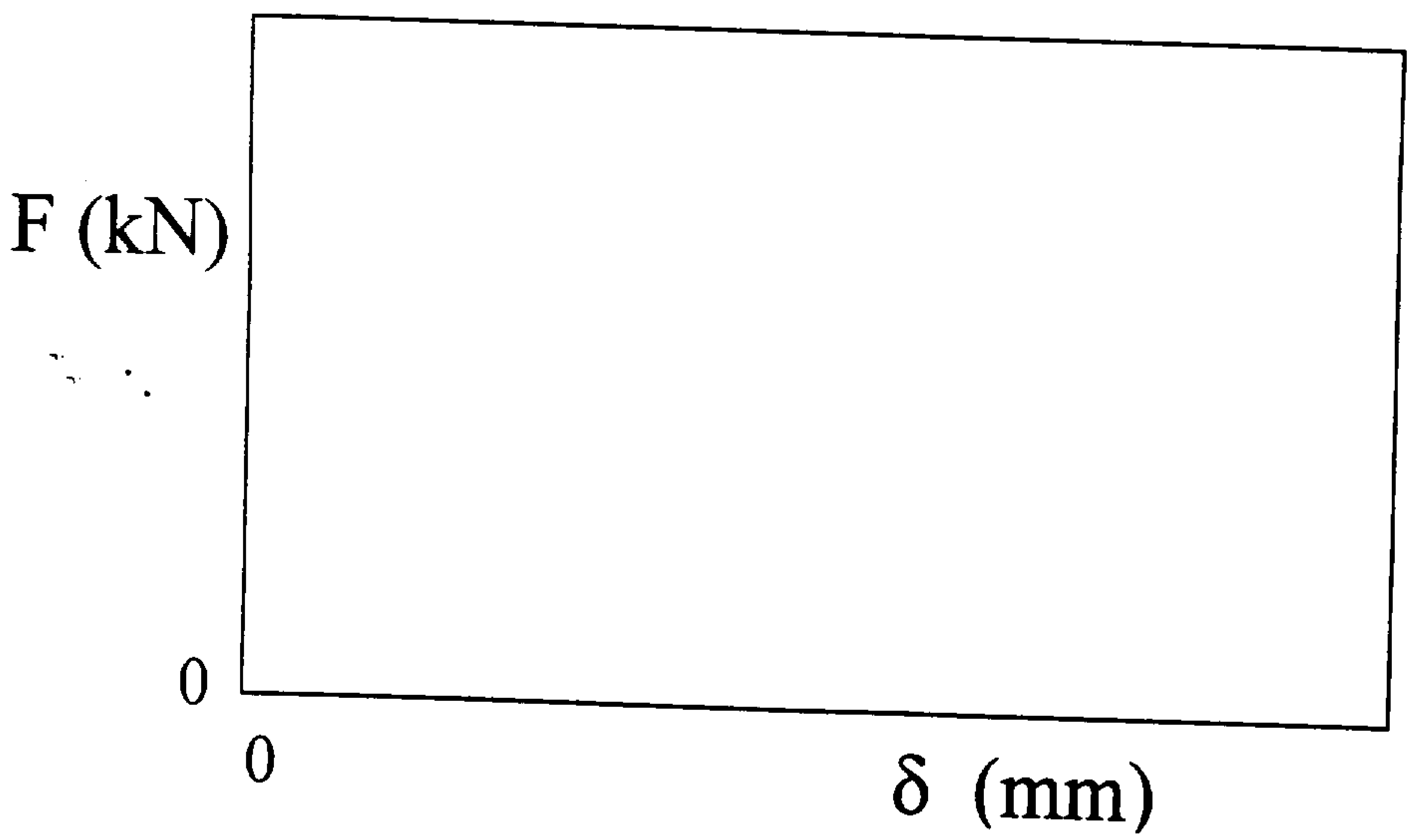
* Please sign your **name** on the back of this page —

Problem 1: In the diagram shown below, assume that the three circular bars A, B, C of length $L = 10$ cm are loaded in simple uniaxial tension between two rigid plates. The areas of the bars are A: 5 cm^2, B: 3 cm^2, C: 1 cm^2. The total force is F and the displacement of all three bars, δ, is the same. Initially, there is no residual stress in any of the bars. The three bars have elastic-perfectly plastic behavior as shown above,
with $\sigma_{YC} = 400\ MPa = 4\sigma_{YB} = 8\sigma_{YA}$ and equal Young's modulus, E = 100 GPa.





(a) Determine and plot the force vs. displacement curve for the three bar system shown above in tension, starting at zero load, F. Neglect any end effects of the bars (i.e., any bending effects). Please clearly label axes and indicate coordinates at points were each member yields.



(b) If the force is increased to the point at which the last bar just begins to yield and then the load is suddenly completely removed, please draw the unloading response on the F-δ graph in part (a). Is the unloading linear elastic or does it involve yielding? Explain.

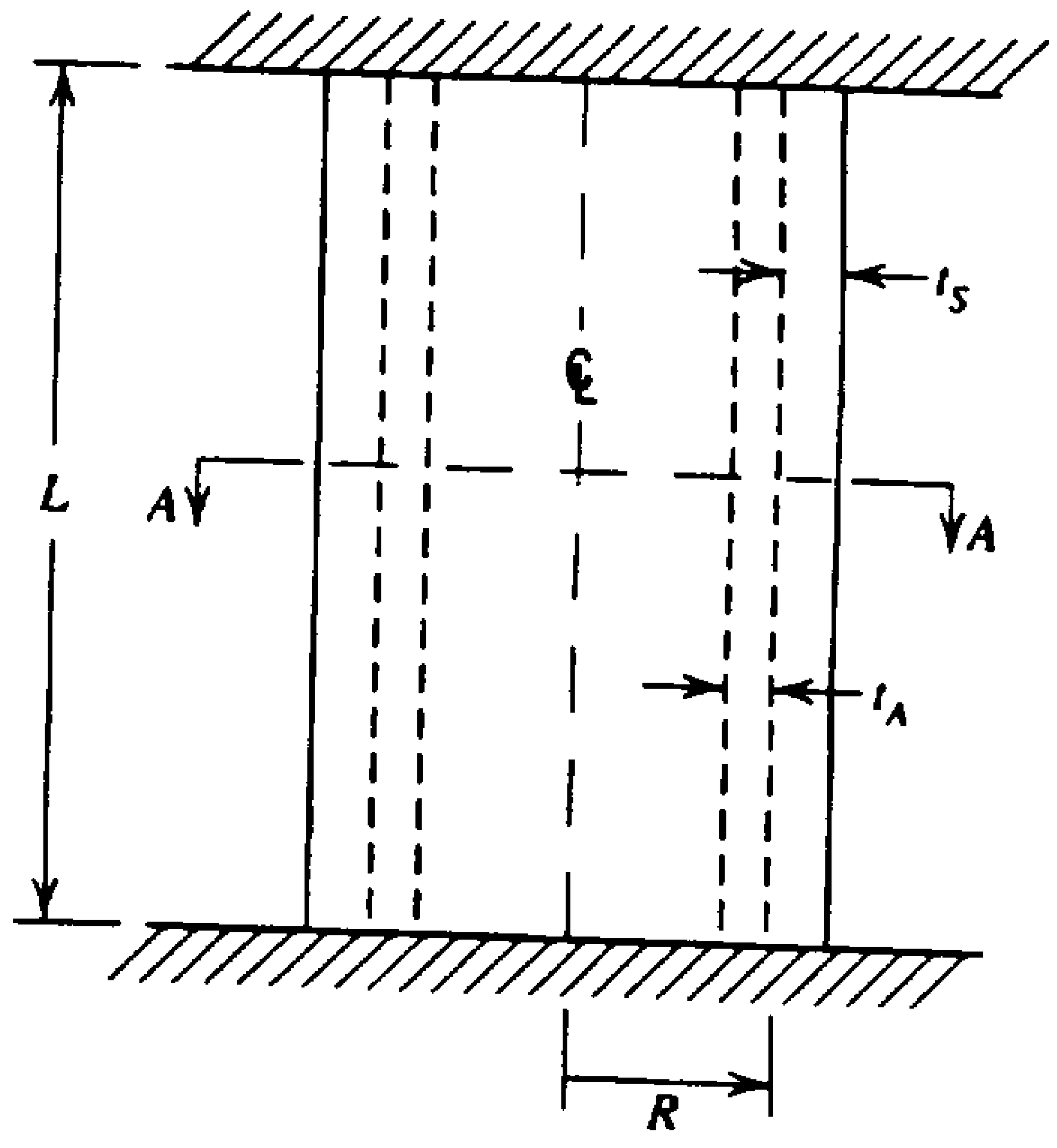
(c) Will there be a residual stress in any of the bars upon unloading as in part (b) to zero applied load? Explain and analyze.

(d) As in part (a), assume that we start with no residual stresses in any of the bars. If the system shown above has no external load (F = 0), but the bars remain attached to the plates (δ the same but non-zero, in general) and the coefficient of thermal expansion α of bars A and C is the same, but that of bar B is zero, upon increasing temperature of this system (assume temperature is uniform and the same in all bars at each point during the heating process), which of the three bars A, B and C will be in tension and which in compression? Please set up the complete system of equations to solve for the forces in the bars as a function of temperature change. **Neglect yielding.** Can all three be in tension or compression? Explain.

(e) What concepts are being explored in this simple problem? Check all that apply

( ) equilibrium
( ) strain-displacement relations
( ) stress-strain relations
( ) compatibility relations
( ) fracture theory
( ) strengthening due to hard particles
( ) ductility theory
( ) bending theory
( ) torsion theory
( ) thermal constraint effects

Problem 2 Consider a composite cylinder of length L formed from an inner cylinder of Al alloy ($E_A = 70\ GPa$, $\nu_A = 0.3$, $\alpha_A = 20.0 \times 10^{-6}\ 1/°C$) with outer radius R and thickness t_A, and an outer cylinder of steel ($E_S = 210\ GPa$, $\nu_S = 0.3$, $\alpha_S = 10.0 \times 10^{-6}\ 1/°C$) with inner radius R and thickness t_S. This composite cylinder is supported snugly in an upright, unstressed state between rigid and frictionless supports. An inner pressure $p = 600\ kPa$ is applied to the cylinder, and the entire assembly is subjected to a uniform increase of temperature $\Delta T = 100°C$.

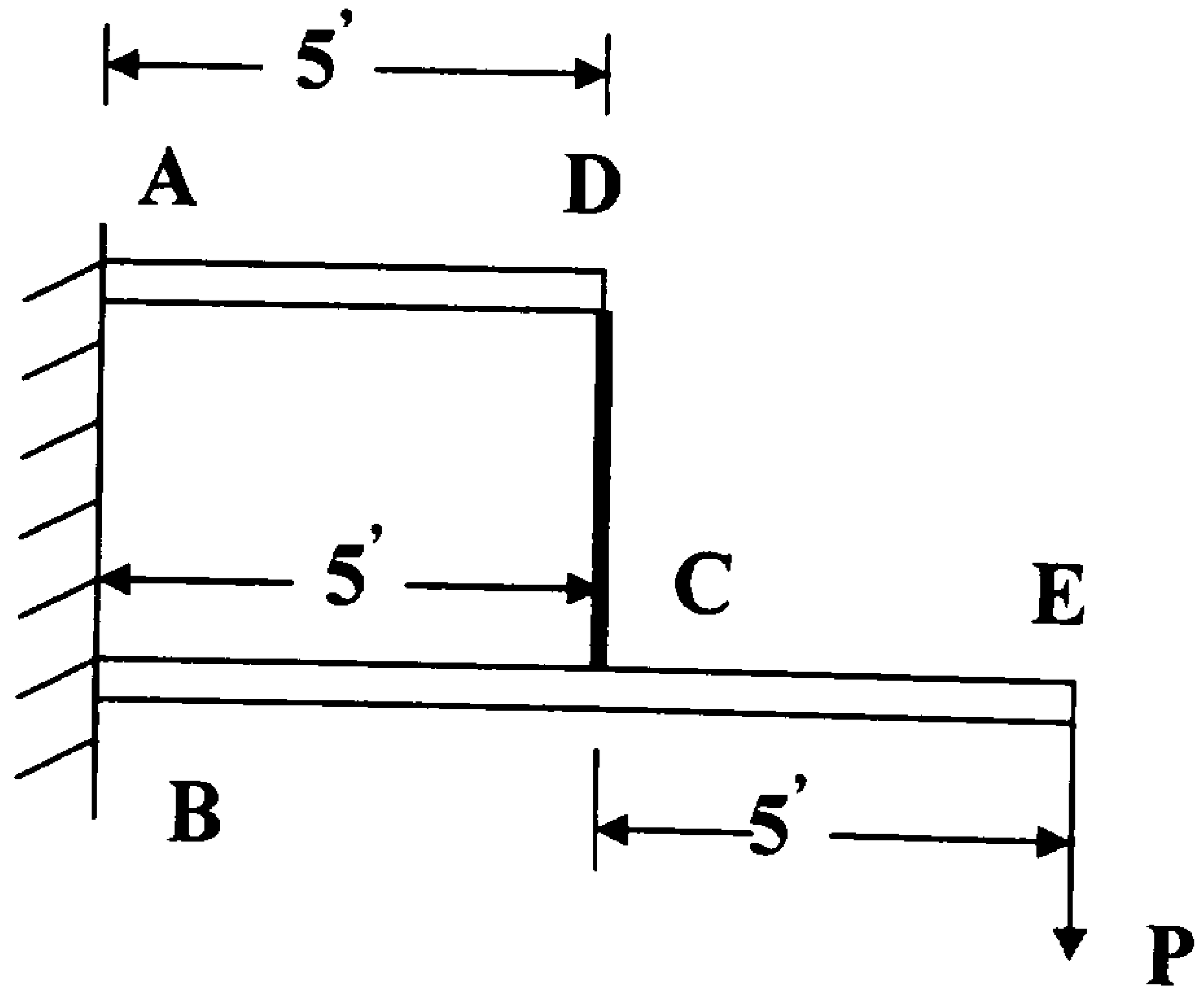


Determine the longitudinal stresses (σ_{LA}, σ_{LS}) and circumferential stresses ($\sigma_{\theta A}$, $\sigma_{\theta S}$) in both the Al alloy and steel cylinders at cross section A-A for the case $t_A = t_S = t = 0.02\ R$, and $L >> R$.

Problem 3 Two cantilever beams, AD and BE are connected by a 12.5 ft long taut steel rod (E_{steel}=30x10^6 lb/in^2) as shown in the figure. Both beams have identical EI = 9.3x10^9 lb-in^2 for bending in the plane of the paper and the rod has cross-sectional area 0.5 in^2. A load P = 12,000 lbs is applied as shown.

(a) Determine the deflection of point D.

(b) Is it possible to uniformly change the temperature of the rod so that the deflection at point D becomes zero? If so, find the temperature change required. (The coefficient of thermal expansion for steel is 6.5x10^{-6}/degree.)



Problem 4 A plate with a circular hole is subjected to compressive stress at its boundary as shown in Figure 1. The material properties are:

E=70 GPa
ν=0.35
K_{IC}=1.5 MPam$^{1/2}$

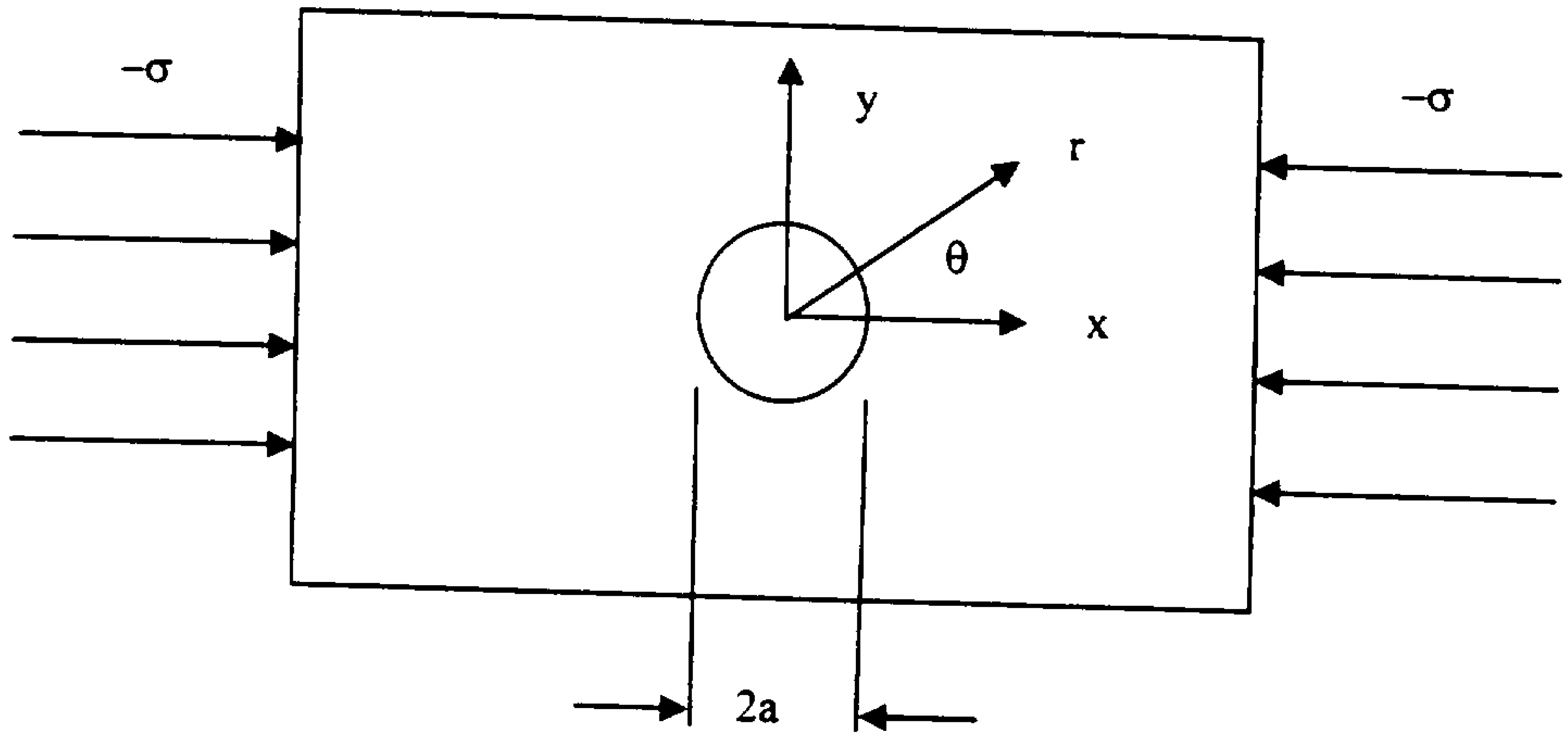


Figure 1. Plate with a circular hole subjected to remote uniaxial compression.

The stress distribution near the hole is given by:

$$\sigma_{rr} = \frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{r\theta} = -\frac{\sigma}{2}\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\sin 2\theta$$

The process of cutting the hole left surface damage consisting of many small cracks up to 2mm in length distributed all around the hole. At a certain load these cracks are observed to propagate in a particular location.

(a) Where are the observed cracks likely located, at what load, and why? (Assume the small cracks behave as through cracks.) Recall that for this geometry the approximate equation for the stress intensity factor is $K = s\sqrt{\pi a}$.

(b) Describe any restrictions on your calculations, e.g. size range of hole for which the equations are valid, effects of plate thickness on use of a critical stress intensity factor, etc. Will the crack arrest or will the specimen shatter once the crack begins to grow? Why?