Problem 1

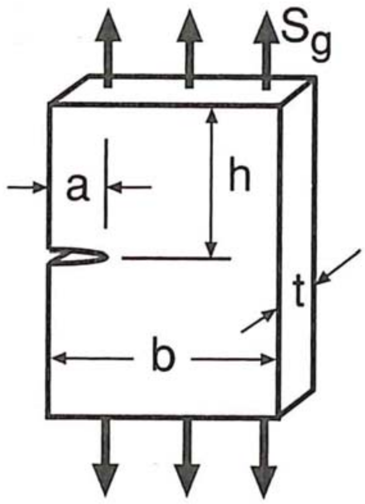


Fig. 1a

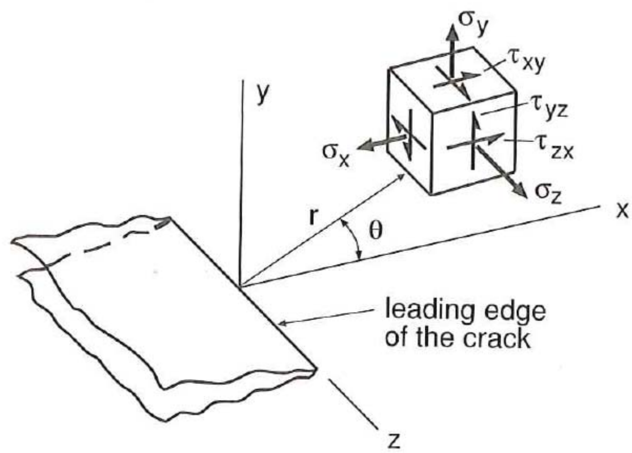


Fig. 1b

Consider a single-edged cracked specimen (crack size *a*, specimen width *b*>>*a*, specimen's uniform thickness *t*, specimen height 2*h*) under mode I loading (*in-plane loading*, far field stress S_g); see Figure 1a. The specimen is made of a linear elastic isotropic homogeneous material (elastic modulus *E*, Poisson's ratio ν). The stresses σ_x, σ_y, and τ_{xy} *near the crack tip* are given by the following expressions (see Fig. 1b):

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

These equations are based on the theory of linear elasticity; specifically, the strain in the z-direction, ε_z, is given by Hooke's law:

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

1) Consider the semi-plane corresponding to θ = 0. Describe the stress state of the specimen away from the crack (*r* ~ *b*-*a*), and near the crack tip (*r* ~ 0):
    a. At the center of the specimen (*z* = 0);
    b. At the edge of the specimen (*z* = *t*/2).

    Assume plane strain conditions ($\varepsilon_z = 0$) at the center of the specimen (*z* = 0), near the crack tip (*r* ~ 0).

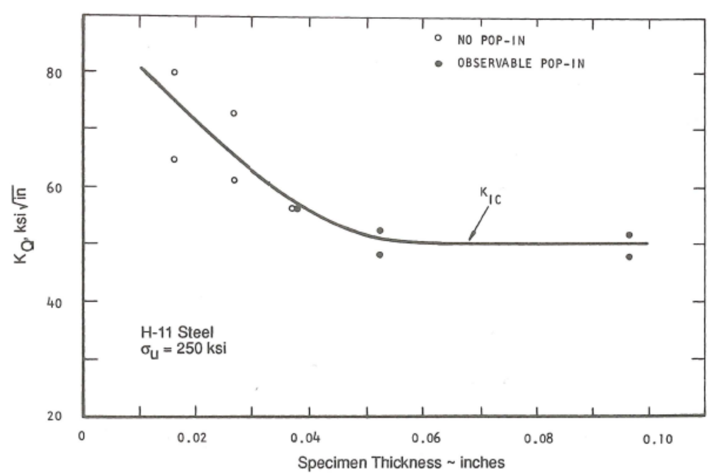
2) Consider the specimen is made of an elastic, perfectly plastic metal with a yield stress σ_0. Qualitatively describe the change in stress distribution ahead of the crack tip.

3) Using the Tresca criterion for yielding, and using the stress equations near the crack tip, calculate the distance from the crack tip r_y (for $\theta = 0$) below which the material has yielded:
    a. At the center of the specimen ($z = 0$);
    b. At the edge of the specimen ($z = t/2$).

    Calculate as well σ_y (for $\theta = 0$) for $0 < r < r_y$ in each case.
    <u>Tresca criterion</u>: yielding occurs when the maximum shear stress exceeds $\sigma_0/2$.

4) The actual plastic zone size is $r_p = 2r_y$, as a result of internal force redistribution. Plot σ_y (for $\theta = 0$) as a function of r:
    a. At the center of the specimen ($z = 0$);
    b. At the edge of the specimen ($z = t/2$).

5) Based on the previous plots, interpret the following graph, giving the fracture toughness (value K_Q at which specimen fractures) as a function of specimen thickness for a steel.



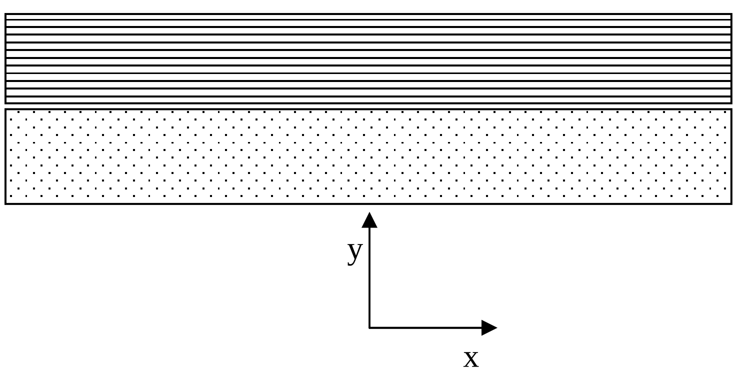
PROBLEM 2

Two unidirectional (epoxy reinforced with continuous unidirectional fibers) laminas are chemically bonded upon heating at T=200 ^{o}C. Each lamina has thickness t, and contains 63 vol% of E-glass fibers with the fiber orientation as shown in the schematic. Once bonding is achieved the laminate is taken back to ambient temperature. Using the schematic and the data provided answer the following:

a) Calculate the thermal strain (along the x-direction) for each of the laminas if they were not constrained (bonded to each other)
b) Assuming strong adhesion at the interface draw the stress profile for the laminate (bilayer)
c) Will it bend? Which direction? Where is the neutral axis (with respect to the interface, do not provide an equation)
d) Propose alternative design to lower (thermal) residual stresses but without altering significantly the in plane mechanical properties

Coefficients of Thermal Expansion of Various E-Glass–Epoxy Laminates

Laminate	Fiber Volume Fraction (%)	Direction of Measurement	Coefficient of Thermal Expansion, 10^{-6} m/m per °C (10^{-6} in./in. per °F)
Unidirectional	63	0°	7.13 (3.96)
		15°	9.45 (5.25)
		30°	13.23 (7.35)
		45°	30.65 (12.08)
		60°	30.65 (17.03)
		75°	31.57 (17.54)
		90°	32.63 (18.13)



Problem 3

Consider a hook of circular cross-section with dimensions shown in the figure. For all parts, neglect the effect of curvature on the stress distribution in section 1-2.

(a) Determine the maximum load P that can be supported without exceeding the yield strength of 30 ksi in section 1-2. (1 ksi = 1000 psi = 1000 lbf/in^2)

(b) What are the principal stresses at points 1 and 2?

(c) If the material is elastic-perfectly plastic, sketch the approximate stress distribution in section 1-2 when plastic collapse (i.e., complete plastic deformation) occurs. For this part, you do not need to do any calculations.