GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2003

Mechanics of Materials

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

1. A block of mass m moving with a velocity v_o hits squarely the prismatic member AB at point C. The cross sectional shape of member AB is square with sides of length c. The elastic modulus is E and $I = c^4/12$.

   (a) Determine the equivalent static force at point C in terms of the given parameters.

   (b) Determine the maximum stress in the member.

   (c) Determine the location of the maximum deflection of the member.

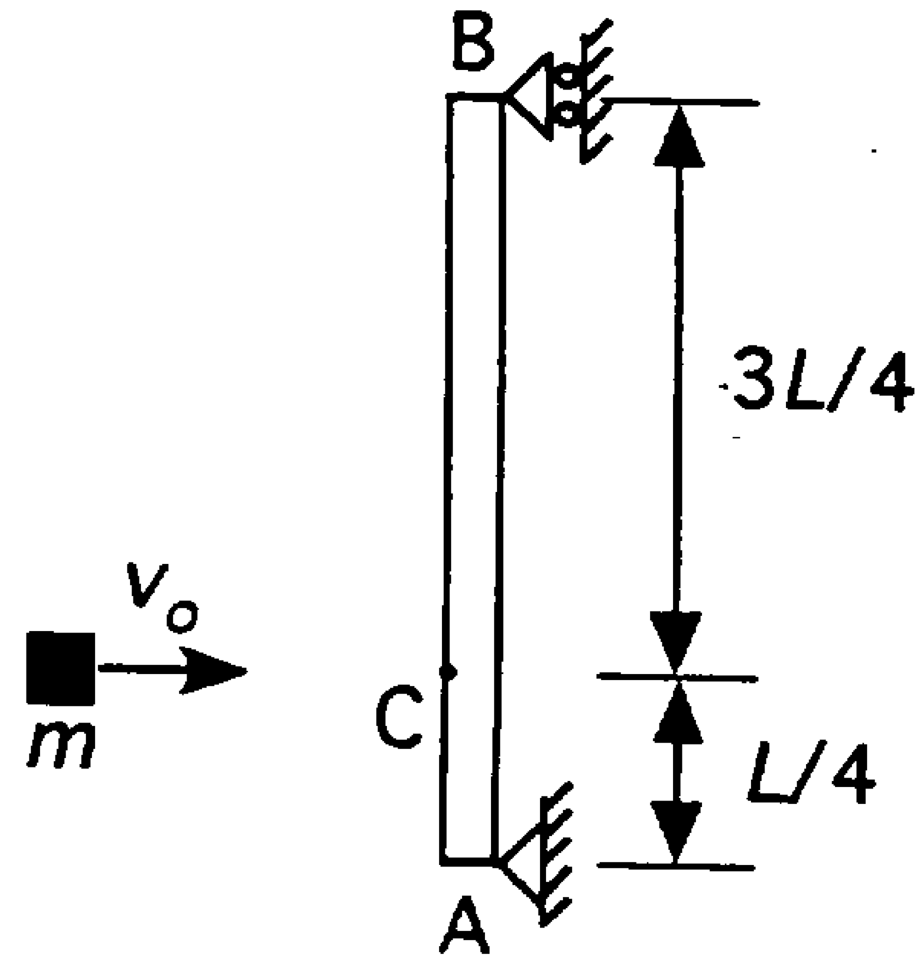
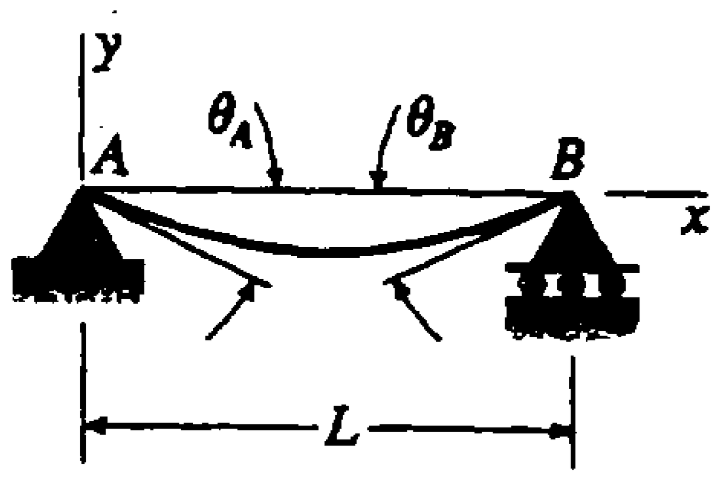
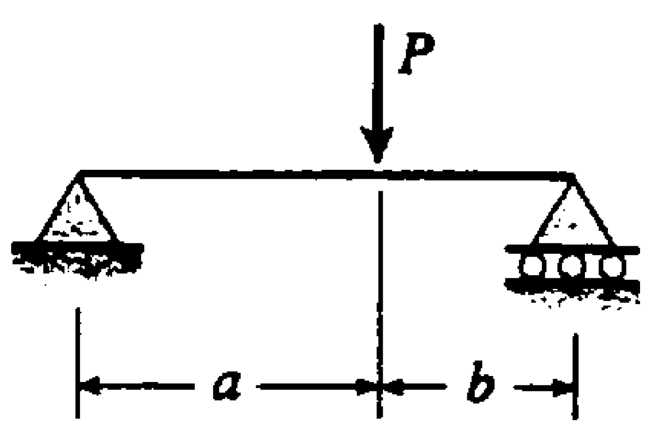


TABLE G-2 DEFLECTIONS AND SLOPES OF SIMPLE BEAMS

	 EI = constant	v = deflection in the y direction (positive upward) $v' = dv/dx$ = slope of the deflection curve $\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward) x_1 = distance from support A to point of maximum deflection $\delta_{max} = -v_{max}$ = maximum deflection (positive downward) $\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise) $\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise)
5		$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \quad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \quad (0 \le x \le a)$ $\theta_A = \frac{Pab(L+b)}{6LEI} \quad \theta_B = \frac{Pab(L+a)}{6LEI}$ If $a \ge b$, $\delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI}$ If $a \le b$, $\delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$ If $a \ge b$, $x_1 = \sqrt{\frac{L^2 - b^2}{3}}$ and $\delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$

2.(a) Provide the definition, an example, and the stress-strain relationship for the two-dimensional plane stress problems.

(b) Provide the definition, an example, and the stress-strain relationship for the two-dimensional plane strain problems.

(c) Consider the bending of a cantilever. Is this a plane strain problem, a plane stress problem, or neither? Justify your answer.

(d) If the cantilever is made of isotropic elastic material, express the flexural rigidity of the beam in terms of the Young's modulus and Poisson ratio.

(e) If the cantilever material is anisotropic, such that the elastic properties in the axial direction of the beam differ substantially from those along the thickness of the beam, would the expression in part (d) still hold? Why (why not)?

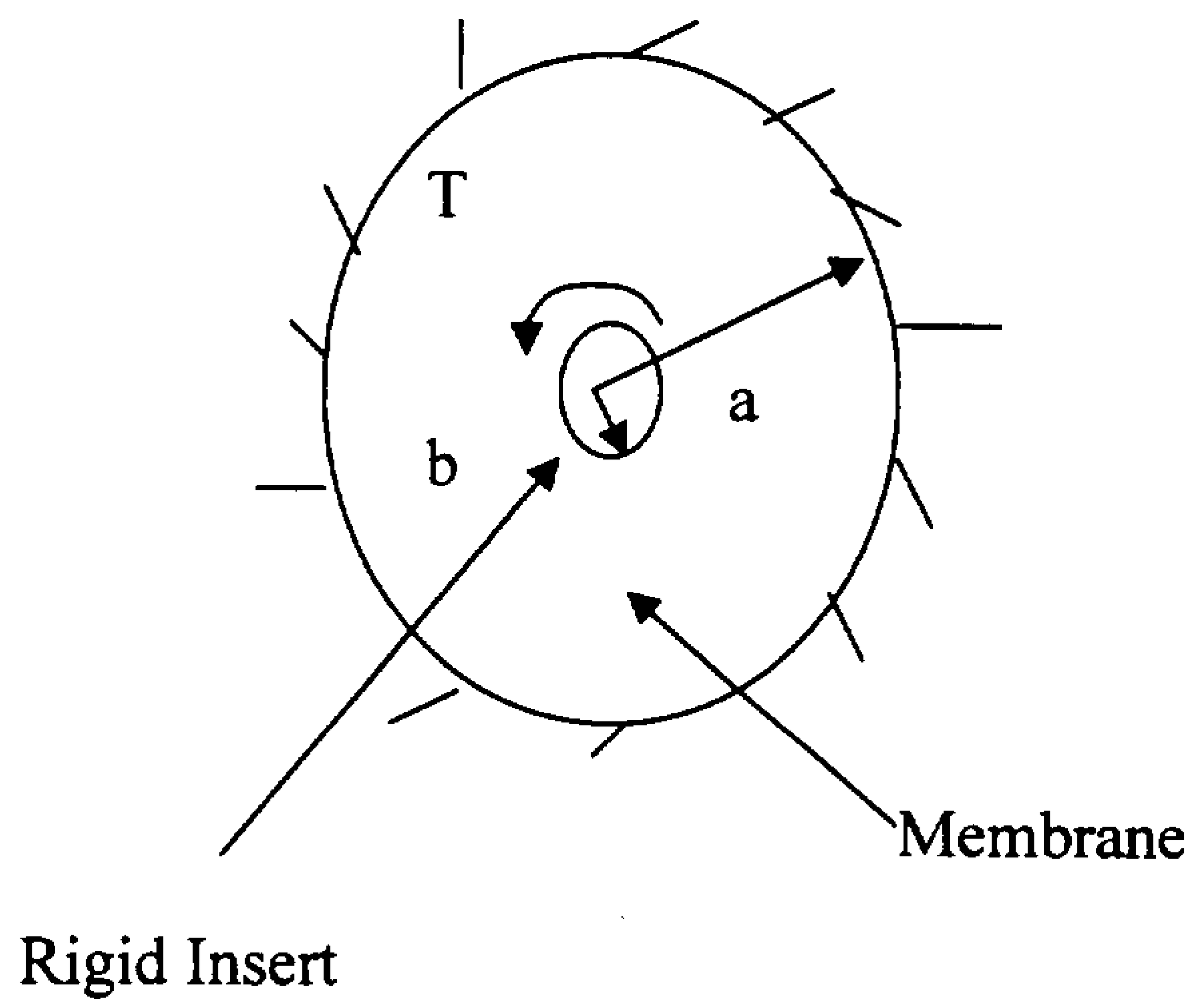
3. A thin, linear elastic circular membrane is stretched uniformly and then clamped at its outer edge as shown. The radius of the membrane is a. A rigid circular insert of radius b<a is attached to the membrane as shown and used to apply a torque T.

a) State clearly the usual assumption(s) made in analyzing the torsion of linear elastic uniform circular cylindrical shafts subjected to a constant torque T.

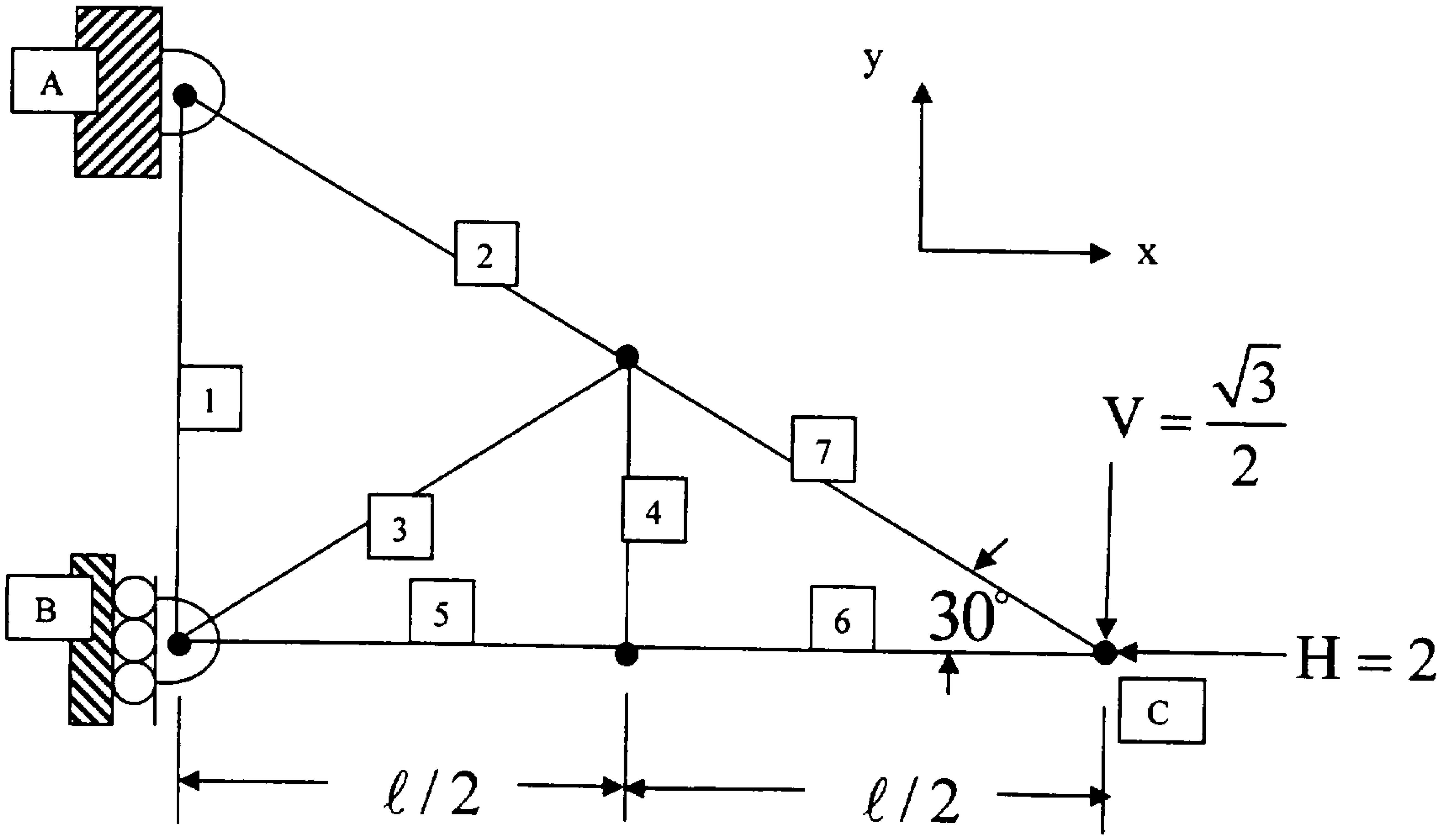
b) Assume that these assumptions hold for the problem described. Calculate the relationship between the torque T and the rotation of the insert θ.

c) What would happen if the membrane were not stretched before clamping and applying the torque?

d) How would your analysis change if the membrane was nonlinear elastic?



4. Truss Problem:



The simple truss structure shown above has a fixed pinned joint at "A" and a simply supported pin joint at "B". It is loaded by forces "V" and "H" at point "C". The material in the truss members or struts is assumed to be elastic-perfectly plastic with Young's modulus "E" and yield strength "σ_o". The yield strengths in tension and compression are equal. Each truss member is made of the same material. Show all work using appropriate equations and free body diagrams.

a. For the truss shown, what ratio of V/H leads to zero stress in the truss element #2?
b. For the truss shown, what ratio of V/H leads to zero stress in the truss element #4?
c. If the truss structure shown is to be equally likely to fail due to yielding in any given strut, please determine the cross-sectional area of each strut.
d. If all struts have the same cross-sectional area, which strut(s) yields first?
e. If all struts have the same cross-sectional area, derive the relationship between the applied load "V" and the vertical displacement at point "C".
f. How would the problem in part (d) change if strut #1 were removed?
g. How would the problem in part (d) change if all the joints were not pinned but instead acted as rigid connections between the connecting members? Please explain in words.
h. How would the problem in part (d) change if the structure were fixed at point "B" in addition to being fixed at point "A"? Please explain in words.

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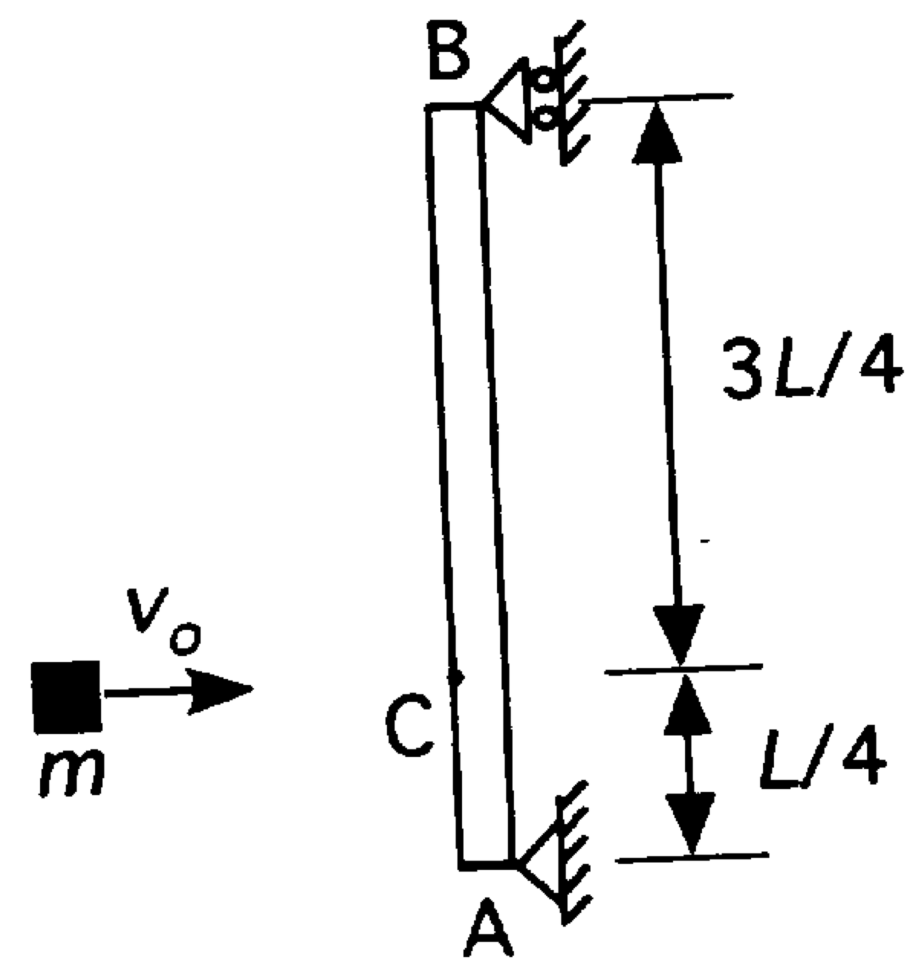
   (b) Determine the maximum stress in the member.

   (c) Determine the location of the maximum deflection of the member.



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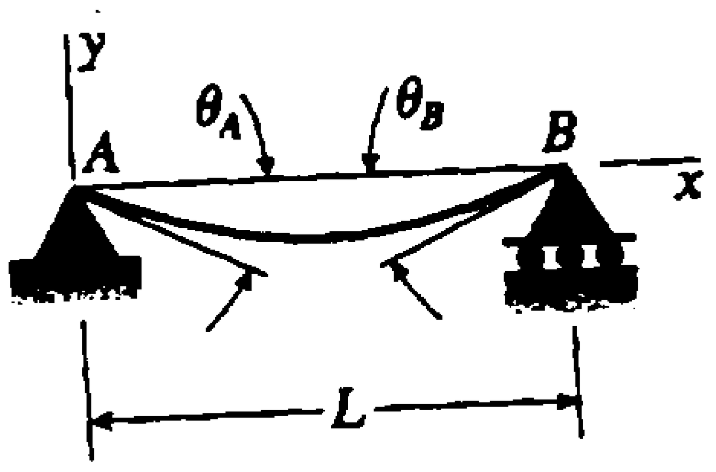

EI = constant

v = deflection in the y direction (positive upward)
$v' = dv/dx$ = slope of the deflection curve
$\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward)
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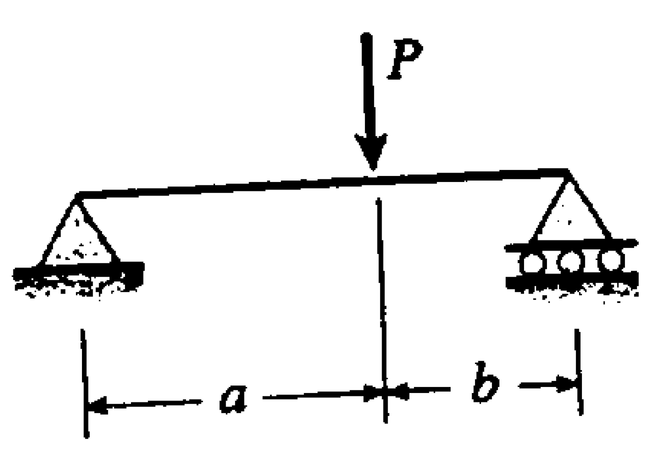

$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L+b)}{6LEI} \qquad \theta_B = \frac{Pab(L+a)}{6LEI}$$

$$\text{If } a \ge b, \quad \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI} \qquad \text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

$$\text{If } a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad \text{and} \quad \delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\,LEI}$$