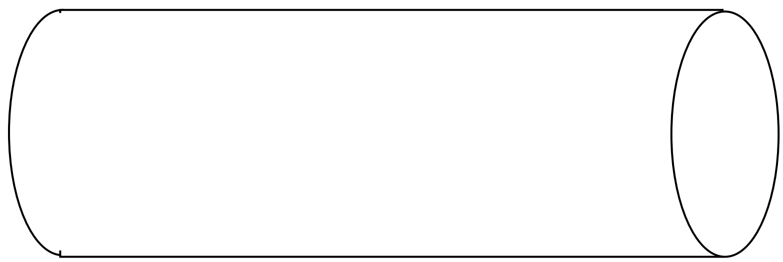
1. Given an elastic / perfectly plastic circular rod of radius r and length L that is loaded in torsion (being twisted about its axis)

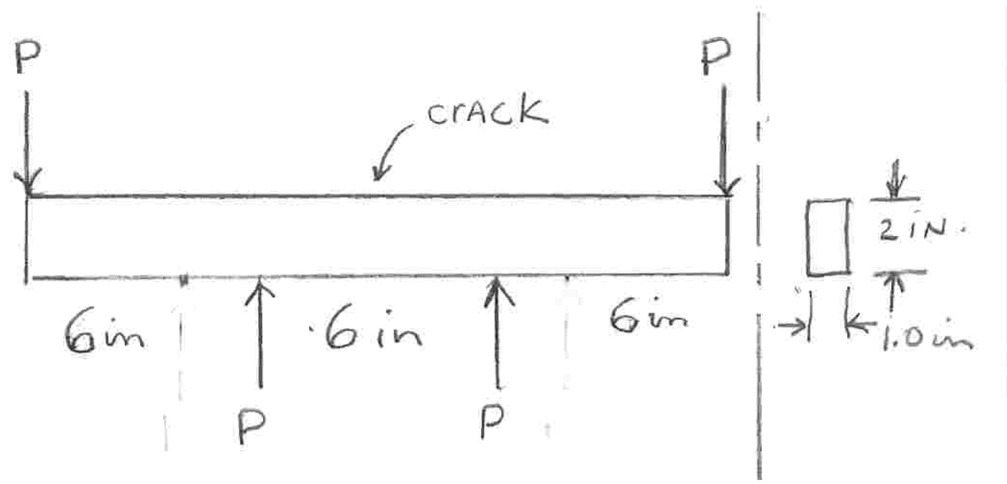


a. Utilize geometry to develop an expression for the strain in the rod as a function of the radius, length, and angle of twist. Give an expression for the maximum strain and show where it occurs. (You must label the changes in geometry associated with twisting the rod on the above sketch. Clearly show the geometry change along the length and along the radius.)

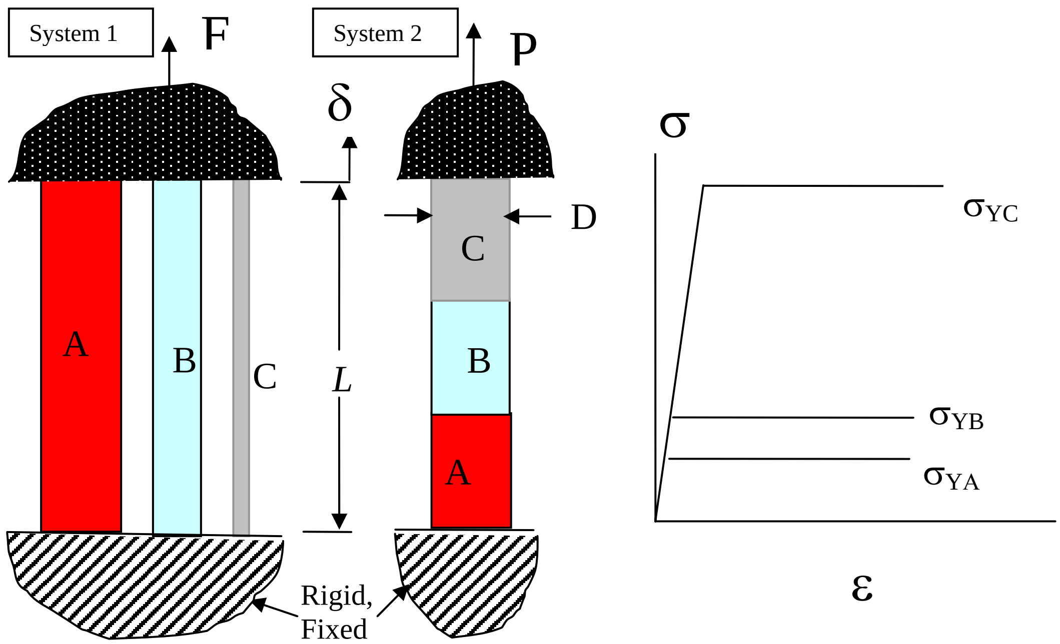
b. If the material is loaded so that exactly half of the volume has undergone plastic deformation: Sketch a circular cross section and indicate on the sketch what material has undergone plastic deformation. What is the stress distribution as a function of radius? Give an expression or expressions for the shear stress as a function of radius and twist angle.

c. Integrate the moments of the shear stress over the cross section to obtain a relation between the applied torque and the shear stress.

2. A beam of 2014-T6 aluminum is subjected to 4 point bending. The beam is 18 inches long and is 2 inches deep and 1 inch wide. The upper side of the beam has a small "thumbnail" shaped crack in the center that is oriented in the width direction and 2a = 0.5 inch. The stress intensity factor geometric correction factor for the crack is 1.1. What is the maximum amount of load, P, that can be applied before the beam fails? E = 10.5 msi; S_{yield} = 44 ksi; K_{Ic} = 35 ksi(in)$^{1/2}$.



3.



In the diagram shown above, three materials (A, B, C) exhibit elastic-perfectly plastic behavior shown above (right) with yield strengths $\sigma_{YC} = 450\ MPa,\ \sigma_{YB} = 150\ MPa,\ \sigma_{YA} = 90\ MPa$ and Young's moduli of E_C = 200 GPa, . $E_B = E_A$ = 70 GPa. Systems 1 and 2 represent different structural arrangements of these materials to carry the load between plates.

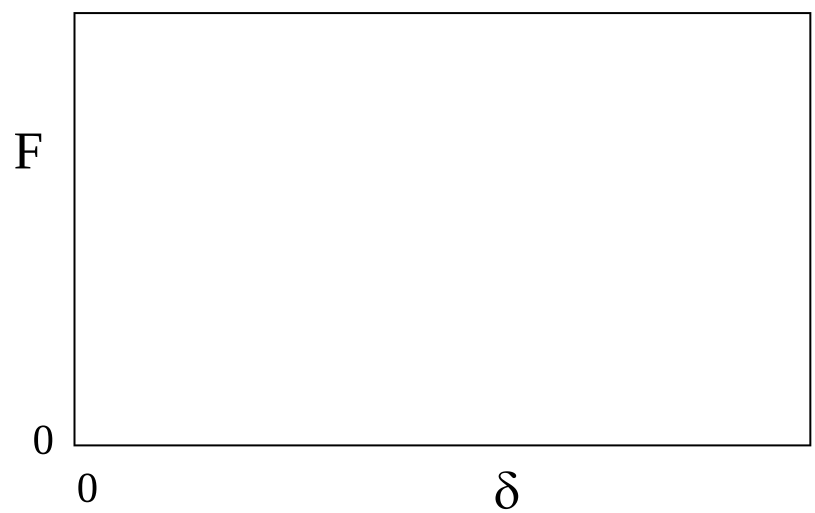
In System 1 there are three circular bars A, B, C of length L = 20 cm arranged in parallel in simple uniaxial tension between two rigid plates to which they are fixed. The lower plate is fixed and the upper moves. The diameters of the bars in upper left are D_A = 3 cm. D_B = 2 cm, and D_B = 1 cm. The total applied force is F and the displacement of all three bars relative to the initial length L, δ, is the same. Initially, there is no residual stress in any of the bars.

In System 2 the three circular bars are connected in series (equal volume fractions) to construct a circular bar of diameter D with length L = 20 cm, also loaded in simple uniaxial tension between two rigid plates to which they are fixed. The total applied force is P and the displacement of the upper plate is δ.. Initially, there is no residual stress in any of the bars.

(a) Determine the diameter D of System 2 such that the load P at initial yield is the same as that of F at initial yield for System 1. State all approximations.

(b) Determine the diameter D of System 2 such that the maximum load P the system can carry is the same as the maximum load F that System 1 can carry (i.e., the limit loads are equal).

(c) For D = 3 cm in System 2, please quantitatively construct the force versus displacement curves for both Systems 1 and 2.



(d) If the applied load on Systems 1 and 2 is removed after initial yielding of either system, will either system have a residual stress? Please explain.

(e) What concepts are being explored in this simple problem? Check all that apply

( ) equilibrium
( ) strain-displacement relations
( ) stress-strain relations
( ) compatibility relations
( ) fracture theory
( ) plasticity theory
( ) bending theory
( ) torsion theory
( ) thermal constraint effects