Mechanics of Materials PhD Qualifying Exam (written) Spring 2010

Answer any 3 of the 4 questions below. Do NOT do all 4!!

1. When a nuclear fuel rod (cylindrical) undergoes fission, thermal gradients develop between the interior and the periphery of the fuel due to the heat generated from fission.

(a) Determine non-zero strain and stress components.

(b) Estimate the maximum temperature difference between the centerline and the periphery that the fuel rod can withstand in the case of a sudden change in temperature.

Assume it is an isotropic ceramic with coefficient of thermal expansion $10.2 \times 10^{-6}\ K^{-1}$, Poisson's ratio 0.30, Youngs modulus 210 GPa and Ultimate Tensile Strength 147 MPa (Tension) and 2100 MPa (Compression)?

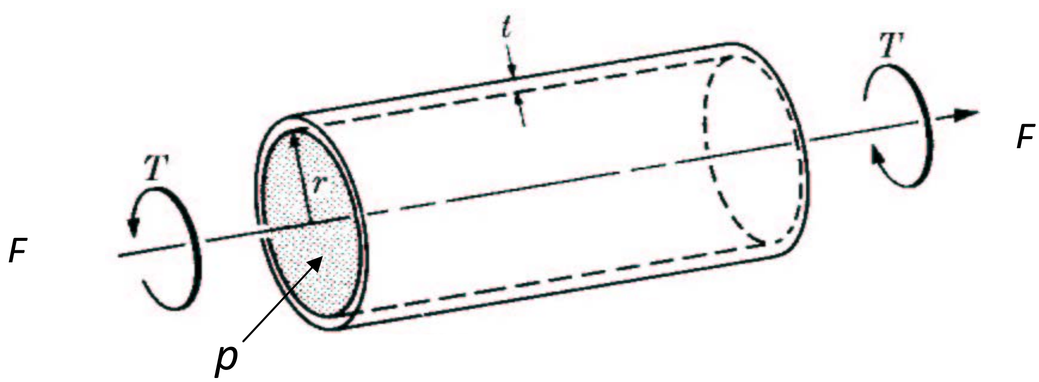
2. A thin walled pressure vessel is subjected to an internal pressure. The inner diameter of the vessel is 100 cm and the thickness is 45 mm respectively. The wall of the vessel is made from an aluminum alloy with E = 71 GPa, ν = 0.33, σ_y = 350 MPa and K_{Ic} = 25 MPa $\sqrt{m}$.

- Find the maximum allowable pressure (p_o) before yielding occurs. Assume the material yields according to Mises theory.
- While the pressure increased linearly from 0 to 0.5 MPa, the cylinder suddenly burst. Upon careful examination of the fracture surface, an elliptical flaw, whose major axis is oriented normal to the circumferential direction, was found. The flaw size was 1.5 mm. If $K = \frac{1.12}{\sqrt{Q}}\sigma\sqrt{\pi a} \sim \frac{1.8}{\pi}\sigma\sqrt{\pi a}$ where a = crack size and σ is the macroscopic stress, find the magnitude of the pressure at which failure took place.

3. A thin-walled polycrystalline copper tube with closed ends (assume isotropic linear elasticity with E = 130 GPa, ν = 0.34) has an internal diameter of 600 mm and a wall thickness of 30 mm. The uniaxial yield strength of copper is 70 MPa. It is subjected to combined in-phase ($p = \lambda F$, where λ is constant) cyclic loading consisting of:

- An internal pressure ranging from p = 0 MPa to p = 5 MPa
- an axial force ranging from F = 0 N to $F = 4\text{x}10^6$ N
- torque T = 0 N-m always.

For the purpose of this analysis, ignore any stress concentration due to the end caps on the tube.



Within the section of interest shown above, parts (a)-(d) below pertain to the given loading conditions:

a) At the peak applied stress, determine the value of the *maximum principal stress* σ_{max}

b) At the peak applied stress, determine the value of the *minimum principal stress* σ_{min}

c) Determine the factor of safety on initial yielding of the tube based on the Tresca yield criteria

d) Using the maximum principal stress criterion, estimate the fatigue life for the tube given the completely reversed uniaxial stress-life relation

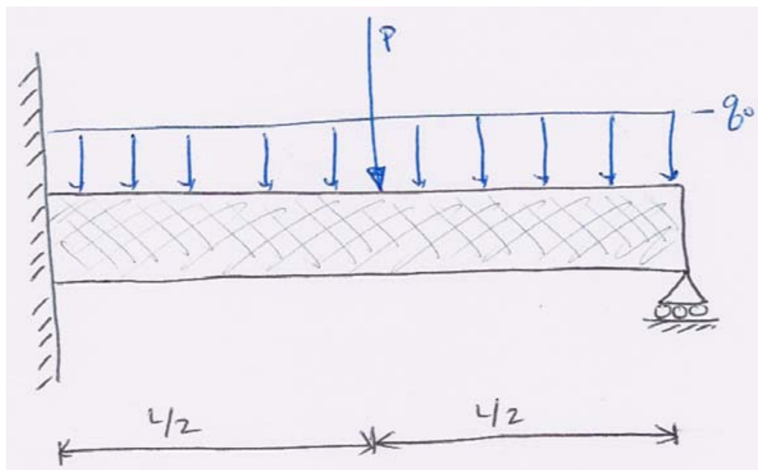
$$\frac{\Delta\sigma}{2} = 1000\left(2N_f\right)^{-0.15}$$

State all assumptions and approximations.

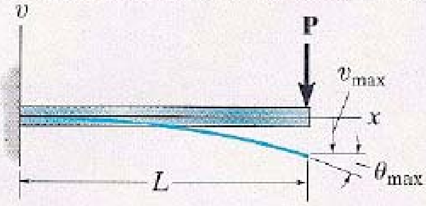
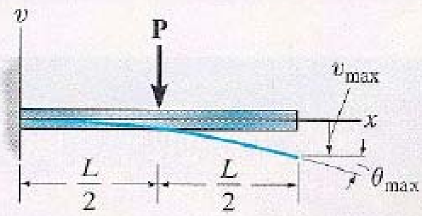
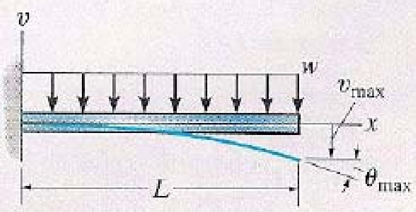
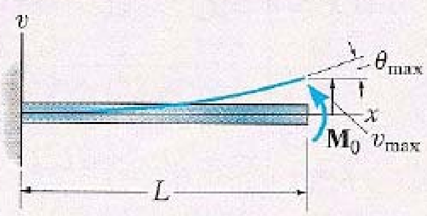
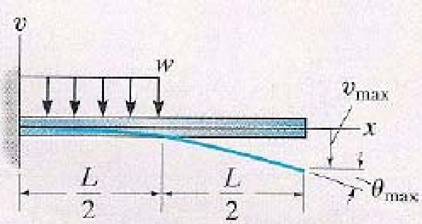
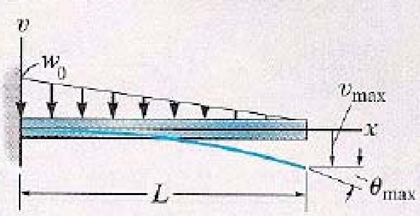
$$\sigma_{xx}(x,y) = \frac{-M(x)y}{I}, \qquad \sigma_{xy} = \frac{V(x)Q_p(y)}{Ib}, \qquad EI\frac{d^2v}{dx^2} = M(x)$$

4. Find the deflection curve ($v(x)$) and the components of stress $\sigma_{xx}(x,y)$ and $\sigma_{xy}(x,y)$ in terms of applied loads (q_o and P), geometry (L and the moment of inertia, I), and the Young's modulus (E) for the cantilever beam with applied point force P and uniformly distributed load q_o, shown. Note the beam equations below and that we have provided a table showing deflection curves of some simple cases.

$$\sigma_{xx}(x,y) = \frac{-M(x)y}{I}, \qquad \sigma_{xy} = \frac{V(x)Q_p(y)}{Ib}, \qquad EI\frac{d^2v}{dx^2} = M(x)$$



Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{6EI}\left(\frac{3}{2}L - x\right) \quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{24EI}\left(3x - \frac{1}{2}L\right) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0L}{EI}$	$v_{max} = \dfrac{M_0L^2}{2EI}$	$v = \dfrac{M_0x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{192EI}(4x - L/2)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$