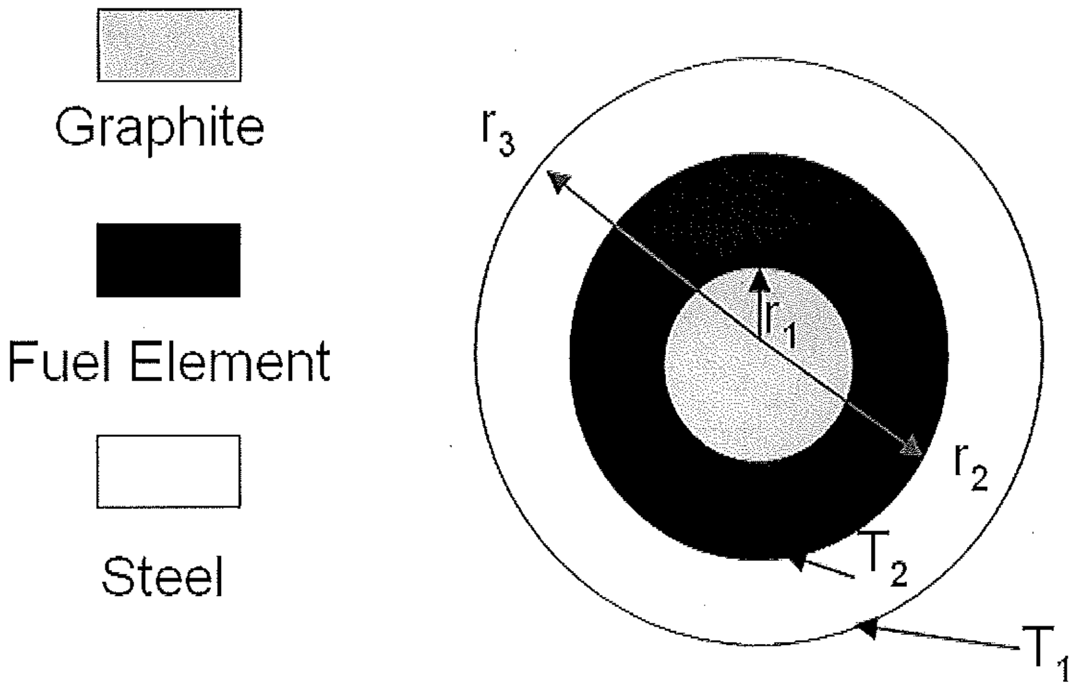
Ph.D. Qualifying Examination
Fall 2008
Heat Transfer

1.



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

A spherical steel nuclear fuel storage container shown above, has an internal spherical core of radius r_1 of graphite. The steel container is of internal radius r_2 and external radius of r_3. It is filled with fuel which generates heat at a uniform volumetric rate of $\dot{q}$. Thermal conductivity of the steel is $k_s = 15$ W/mK and that of graphite, $k_g = 3$W/mK.

(a) Starting from the heat diffusion equation, develop an expression for the temperature as a function of radius.

(b) Develop a general expression for the value of r_3 that would minimize the temperature T_2 for an external convection coefficient of h and temperature $T_{infinity}$. Find r_3 for h= 100 W/m^{2}K, ambient temperature of $T_{infinity} = 30$ °C, and dimensions $r_1 = 0.3$m and $r_2 = 0.9$ m.

2. A liquid flows between two parallel plates, which are at a distance 2a apart, with a fully developed velocity profile. The velocity distribution is given by:

$$v_x = v_c\left(1-\frac{y^2}{a^2}\right)$$

The two plates are subjected to a uniform heat flux q''_w. In the *fully developed temperature profile region,* with negligible viscous dissipation,

a) Use the energy equation with the appropriate simplifications to develop an expression for the temperature profile of the fluid in terms of the axial temperature gradient.

b) Then obtain expressions for the bulk (T_b) and surface (T_s) temperatures.

c) Show that

$$Nu = \frac{h\times a}{k} = \frac{35}{17} \quad where \quad h = \frac{q''_w}{T_s - T_b}$$

Note that for this case,

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_b}{dx} = constant$$

3. Consider the heat transfer from a spacecraft to the outer space. The surface of the spacecraft has an emissivity 0.8 and is maintained at a temperature of 140 K. Assume that the exposed surface area is 30 m^2, and the outer space temperature is 2.7 K. [Be sure to list all assumptions that you make.]

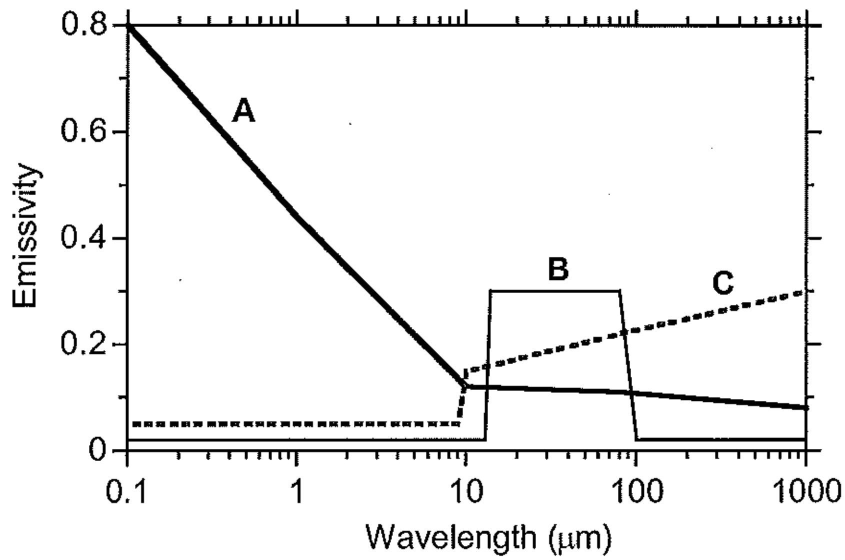
(i) Determine the heat transfer rate from the spacecraft to the outer space.

(ii) Suppose the spacecraft surface is surrounded by a thin layer of radiation shield. For simplicity, the surface area of the shield may be assumed the same as that of the spacecraft. The emissivity of the radiation shield is 0.1 on either side. Determine the rate of heat loss from the spacecraft with the shield.

(iii) Find the temperature of the radiation shield.

(iv) Suppose there are *N*-layers of radiation shields with the same emissivity. Derive an expression for the rate of heat loss from the surface of the spacecraft.

(v) In practice, the emissivity of any material is a function of wavelength. There are three materials whose spectral emissivity is shown in the figure below. Which of the materials (A, B, or C) will you recommend for use as the radiation shield? Briefly explain your reasoning.



0.8
0.6
0.4
0.2
0
Emissivity
A
B
C
0.1
1
10
100
1000
Wavelength (μm)