GWW School of Mechanical Engineering
Ph.D. Qualifier Exam, Spring 2009
Heat Transfer

Problem 1

Thermal ablation is used to treat cancerous tissue by heating it to a high enough temperature. A recent technique is to implant small metallic spheres (thermoseeds) at precise location within the cancer tumor. The metallic spheres generate volumetrically uniform heat rates when subjected to an oscillating magnetic field, while the surrounding tumor does not have a heat generation. The generated heat is conducted from the spheres into the surrounding tumor tissue. We are interested in finding the one-dimensional temperature field associated with a single thermoseed placed in an infinite medium of tissue. Interactions with neighboring thermoseeds can be neglected. Assume the thermoseed has thermal conductivity k_{ts}, radius r_{ts}, and volumetric heat generation rate of q'''. The temperature far away from the thermoseed is the body temperature T_b. The tumor tissue has a thermal conductivity of k_t.

1. Determine the steady state temperature variation in the thermoseed and the surrounding tissue.
2. Determine the location and magnitude of the maximum temperature in the tissue.
3. Assuming r_{ts} =1 mm, k_{ts} =10 W/m-K, T_b = 37°C, k_t = 0.5 W/m-K, and that 1 W of heat generation is uniformly distributed throughout the thermoseed, determine the maximum temperature in Part 2 and show the temperature variation.

__Note:__ The heat conduction in the spherical coordinates is given by:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(k\frac{\partial T}{\partial\varphi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

Problem 2

Consider flow through a pipe of diameter 0.025 m under steady-state conditions with a constant wall heat flux of 2 kW/m^2. The water entering the pipe has a uniform velocity of 0.01 m/s and a temperature of 10°C.

1. Based upon energy balance for the pipe of length, L, calculate the value of L necessary to heat the water to a mean temperature of 95°C.
2. Sketch the **velocity** and **temperature** profile as a function of radius (i) near the entrance, (ii) after the velocity profile is fully developed and (iii) far from the entry region.
3. Using energy equation, develop an expression for the fluid temperature as a function of radius. State any assumptions you make.
4. Is the flow fully thermally developed at the exit of the pipe? (Show through calculations.)

Assume fluid properties are uniform and isotropic as follows:

$\mu = 855\times10^{-6}$ Ns/m^2, Pr = 6, $\rho = 1000$ kg/m^3, Cp = 4200 J/kg-K.

For your reference, the energy equation is as follows:

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Problem 3

A thin hemispherical shell contains holes (openings) through which radiation can enter or leave the dome created by the shell, as schematically shown in the figure below. Other than the openings, the hemispherical shell is opaque. The holes are evenly distributed and the total area of the holes is half of that of the hemisphere. The dome is placed inside a large room whose walls are maintained at a temperature of $T_3 = 1200$ K. The base plate (A_1) under the dome is cooled to maintain it at a temperature of $T_1 = 300$ K. Measurements indicate that the hemispherical shell is at a uniform temperature of $T_2 = 1000$ K. The emissivities of the base plate and the inner wall of the dome are, respectively, $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.5$. Assume that all surfaces are diffuse-gray and that conduction and convection may be neglected.

1. (90% credit) Find the net radiative heat flux received by A_1.
2. (10% credit) Estimate the emissivity of the outer wall of the hemispheric dome.