# GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Fall Semester 2003**

Design
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

* Please sign your **name** on the back of this page —

GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

# DESIGN QUALIFIER

## FALL 03

## WRITTEN EXAMINATION

We are interested in learning what you know and your ability to reason in the formulation and solution of design problems.

**<u>If you find any question or part of this exam confusing, please state your assumptions and rephrase the question and proceed.</u>**

**<u>Please read the entire exam first.</u>**

**<u>Questions 1 and 2 carry equal points. Both have multiple parts.</u>**

**<u>Allocate your time carefully so that you cover all three parts that you are being examined on in these two questions, namely, Methods, Realizability and Analysis.</u>**

**<u>A document containing some formulae is available for you to use in answering Question 2</u>**

## ORAL EXAMINATION

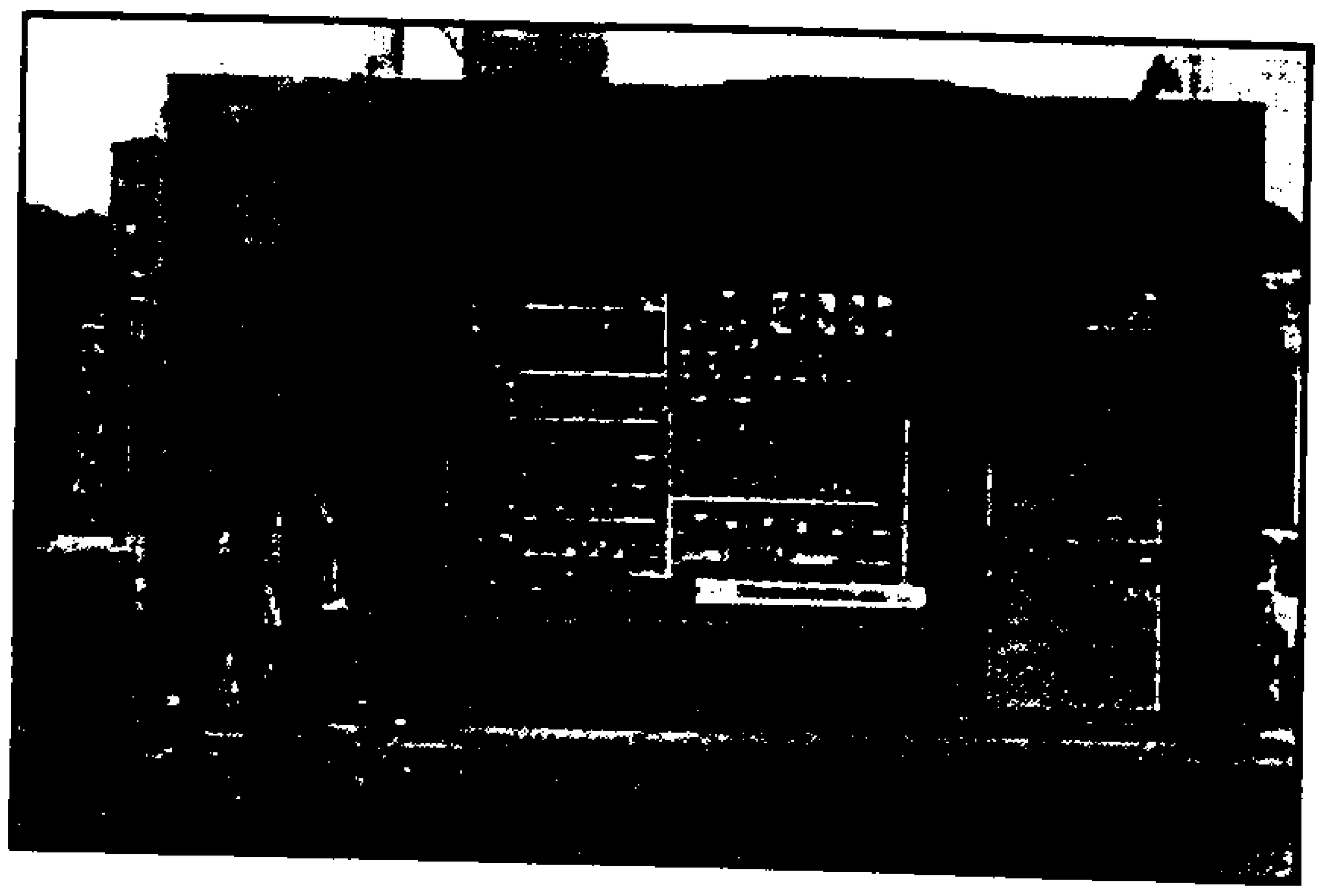
Please arrive half an hour before the scheduled time for the oral exam. During this period we will give you a question to think about. The scope of the oral exam is as follows:

* provide an opportunity for you to state how design fits into your research activities;
* probe your understanding of the question that we posed to you in the preceding half hour.

## QUESTION 1 – METHOD & REALIZABILITY

### Scenario

In today's drive-through age, Americans want to shop for anything at any time of the day. Although convenience stores provide most of the essentials, they usually have higher prices and pose a significant security risk for the attendants. In Europe and Japan, an automated alternative to the convenience store has been introduced.



These stores look like oversized vending machines and carry a variety of products ranging from a gallon of milk to a tube of tooth paste. A customer can buy up to 10 products in a single transaction and pay for the goods with cash or credit. The automated convenience store can be operated around the clock and can be restocked while in operation. Because the size is smaller than most regular convenience stores, the owner needs to determine the top selling items for a particular location and reconfigure the shelves to accommodate these products. Individual items may vary in weight ranging from a few grams (e.g. chewing gum) to several kilos (e.g. laundry detergent). Some of the products are perishable and need to be refrigerated; others may be fragile and need to be handled with care.

### Task

Your task is to design an automated convenience store that is capable of carrying 200 products that vary in weight and are possibly perishable or fragile, as described above.

Your boss wants you to start from scratch and document your design process thoroughly – but this is not possible for lack of time. A senior engineer has suggested that you follow the general guidelines given below and turn in a report documenting each of the six steps.

### Deliverables

*Method*

1. *Clarify the Task:* State the overall function of your system. What are the most important drivers/design criteria?

2. *Conceptual Design:* State and implement the steps (including a specification list and functional diagrams/decomposition) for transforming the overall function that you have identified into at least three alternative design solutions. Ensure that you have identified the important sub functions for each of the five phases listed above. Sketch and describe the workings of these alternatives.

3. *Selection:* Suggest a structured approach to select one of the alternatives for further development.

*Realizability*

4. *Embodimet:* Further develop the alternative that you have selected.

5. *Costing:* How would you estimate the cost of your design? You may critically evaluate the design in terms of manufacturability, initial cost, maintenance cost, reliability, manipulation performance, and other criteria that you feel are important to consider in this phase of design.

6. *Pricing:* Based on the preceding analysis, how would you estimate the market size for such a system and set the price for selling such a system? Be brief.

**Your Exam #: ..............................................**
**You MUST write your solutions to QUESTION 2 on this exam sheet.**

IIA. A Helical extension spring, loaded in fatigue, has been designed for infinite life with the data given below. Find the safety factors for failure in the standard hooks. State all assumptions.

**Given:**

| | | | | |
|---|---|---|---|---|
| Minimum Force | $F_{min} = 150$ lbf | | Life | $L = \infty$ |
| Maximum Force | $F_{max} = 210$ lbf | | Shear Modulus | $G = 11.7E6$psi |
| Working Deflection | $\Delta y = 2.00$in | | Spring Index | $C = 9$ |
| Wire Endurance Limit | $S_{ew} = 45$ksi | | Wire Strength (Coefficient) | $A = 220.78$ksi |
| Wire Diameter | $D = 0.312$in | | (Chrome-silicon) | $b = -0.0934$ |
| Number of Active Coils | $N_a = 13.75$ | | Spring is unpeened. | |

a. Calculate the average initial coil stress, $\tau_i$. (1 pt.)
b. Calculate the mean stress, $\tau_m$. (0.5pt.)
c. Calculate the alternating stress, $\tau_a$. (0.5 pt.)
d. Find the fully reversed endurance limit of the wire. (2 pts.)
e. Find $\sigma_a$, $\sigma_m$, and $\sigma_{min}$. (1.5 pt.)
f. Find a fatigue safety factor for the hook in bending. (1.5 pt.)
g. Find the torsional stresses ($\tau_{Ba}$, $\tau_{Bm}$, and $\tau_{Bmin}$) in the hook using $C_2 = 5$ (1.5 pt.)
h. Find the fatigue safety factor for the hook in torsion. (1.5 pt.)

**NOTE Question IIB starts on Page 8.**

Sheet 1 for Question 2A

Sheet 2 for Question 2A

Sheet 3 for Question 2A

a. Name the two classes of fatigue (0.5 pt.)

b. What kind of steel exhibits an increasing strength when under cyclic stress reversals? (0.5 pt.)

c. What is the difference between $S_e$ and $S_e'$ ? (0.5 pt.)

d. As a mechanical designer, which of these loading conditions would you prefer and why: a load applied gradually or an equal load applied suddenly on a machine part? (0.5 pt.)

e. What is the main difference between $K_S$ and $K_W$ in a spring? (0.5 pt.)

f. Why are nuts for regular fasteners made of soft materials? (0.5 pt.)

g. List four primary reasons why bearings fail. (1 pt.)

h. What does $K_m$ stand for in gear stress calculation? Why does it need to be applied? (1 pt.)

The stresses $\sigma_a$ and $\sigma_m$ can replace $S_a$ and $S_m$ in Eqs. (7-34) to (7-36) if each strength is divided by a factor of safety $n$. When this is done, the Soderberg equation becomes

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n}$$

The modified Goodman relation is

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

and the Gerber equation is

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

The critical deflection is given by the equation

$$y_{cr} = L_0 C_1 \left[1 - \left(1 - \frac{C_2}{\lambda_{eff}^2}\right)^{1/2}\right]$$

where $y_{cr}$ is the deflection corresponding to the onset of instability.

TABLE 10-2

Formulas for Compression-Spring Dimensions. ($N_a$ = Number of Active Coils)

| | TYPE OF SPRING ENDS | | | |
|---|---|---|---|---|
| TERM | PLAIN | PLAIN AND GROUND | SQUARED OR CLOSED | SQUARED AND GROUND |
| End coils, $N_e$ | 0 | 1 | 2 | 2 |
| Total coils, $N_t$ = N | $N_a$ | $N_a + 1$ | $N_a + 2$ | $N_a + 2$ |
| Free length, $L_0$ = $L_f$ | $pN_a + d$ | $p(N_a + 1)$ | $pN_a + 3d$ | $pN_a + 2d$ |
| Solid length, $L_s$ | $d(N_t + 1)$ | $dN_t$ | $d(N_t + 1)$ | $dN_t$ |
| Pitch, $p$ | $(L_0 - d)/N_a$ | $L_0/(N_a + 1)$ | $(L_0 - 3d)/N_a$ | $(L_0 - 2d)/N_a$ |

*Source:* Associated Spring–Barnes Group, *Design Handbook*, Bristol, Conn., 1981, p. 32.

TABLE 10-3

End-Condition Constants $\alpha$ for Helical Compression Springs*

| END CONDITION | CONSTANT $\alpha$ |
|---|---|
| Spring supported between flat parallel surfaces (fixed ends) | 0.5 |
| One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged) | 0.707 |
| Both ends pivoted (hinged) | 1 |
| One end clamped; other end free | 2 |

*Ends supported by flat surfaces must be squared and ground.

quantity $\lambda_{eff}$ in Eq. (10-11) is the *effective slenderness ratio* and is given by the equation

$$\lambda_{eff} = \frac{\alpha L_0}{D} \tag{10-12}$$

$C_1$ and $C_2$ are the elastic constants and are defined by the equations

$$C_1 = \frac{E}{2(E - G)} \tag{10-13}$$

$$C_2 = \frac{2\pi^2(E - G)}{2G + E} \tag{10-14}$$

Equation (10-12) contains the *end-condition constant* $\alpha$. This depends upon how the ends of the spring are supported. Table 10-3 gives values of $\alpha$ for usual end conditions. Note how closely these resemble the end conditions for columns.

Absolute stability occurs when, in Eq. (10-11), the term $C_2/\lambda_{eff}^2$ is less than unity. This means that the condition for absolute stability is that

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E - G)}{2G + E}\right]^{1/2} \tag{10-15}$$

For steels, this turns out to be

$$L_0 < 2.63\frac{D}{\alpha} \tag{10-16}$$

**Table 13-2**
Preferred Wire Diameters

| U.S. (in) | SI (mm) |
|---|---|
| 0.004 | 0.10 |
| 0.005 | 0.12 |
| 0.006 | 0.16 |
| 0.008 | 0.20 |
| 0.010 | 0.25 |
| 0.012 | 0.30 |
| 0.014 | 0.35 |
| 0.016 | 0.40 |
| 0.018 | 0.45 |
| 0.020 | 0.50 |
| 0.022 | 0.55 |
| 0.024 | 0.60 |
| 0.026 | 0.65 |
| 0.028 | 0.70 |
| 0.030 | 0.80 |
| 0.035 | 0.90 |
| 0.038 | 1.00 |
| 0.042 | 1.10 |
| 0.045 | |
| 0.048 | 1.20 |
| 0.051 | |
| 0.055 | 1.40 |
| 0.059 | |
| 0.063 | 1.60 |
| 0.067 | |
| 0.072 | 1.80 |
| 0.076 | |
| 0.081 | 2.00 |
| 0.085 | 2.20 |
| 0.092 | |
| 0.098 | 2.50 |
| 0.105 | |
| 0.112 | 2.80 |
| 0.125 | 3.00 |
| 0.135 | 3.50 |
| 0.148 | |
| 0.162 | 4.00 |
| 0.177 | 4.50 |
| 0.192 | 5.00 |
| 0.207 | 5.50 |
| 0.225 | 6.00 |
| 0.250 | 6.50 |
| 0.281 | 7.00 |
| 0.312 | 8.00 |
| 0.343 | 9.00 |
| 0.362 | |
| 0.375 | |
| 0.406 | 10.0 |
| 0.437 | 11.0 |
| 0.469 | 12.0 |
| 0.500 | 13.0 |
| 0.531 | 14.0 |
| 0.562 | 15.0 |
| 0.625 | 16.0 |

A229 A227 A401 A228 A232

## Active Coils in Extension Springs

All coils in the body are considered active coils, but one coil is typically added to the number of active coils to obtain the body length $L_b$.

$$N_t = N_a + 1 \tag{13.18}$$

$$L_b = dN_t \tag{13.19}$$

$$\tau_i \cong -4.231C^3 + 181.5C^2 - 3\,387C + 28\,640 \tag{13.21a}$$

$$\tau_i \cong -2.987C^3 + 139.7C^2 - 3\,427C + 38\,404 \tag{13.21b}$$

where $\tau_i$ is in psi. The average of the two values computed from these functions can be taken as a good starting value for initial coil stress.

The bending stress at point $A$ is found from

$$\sigma_A = K_b \frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2}$$

where

$$K_b = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

and

$$C_1 = \frac{2R_1}{d}$$

The torsional stress at point $B$ is found from

$$\tau_B = K_{w_2} \frac{8DF}{\pi d^3} \tag{13.24a}$$

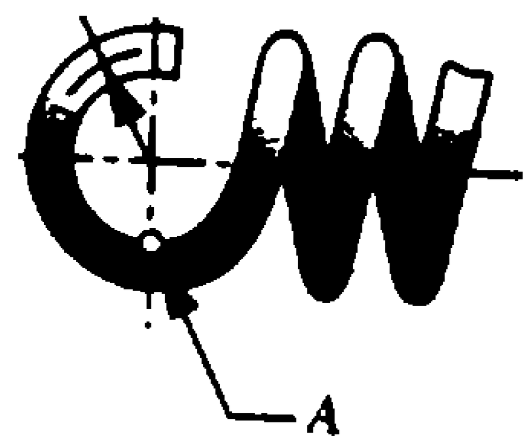
where

$$K_{w_2} = \frac{4C_2 - 1}{4C_2 - 4} \tag{13.24b}$$

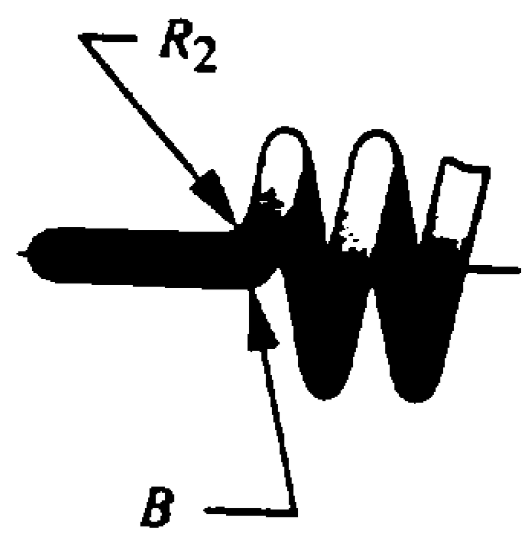
and

$$C_2 = \frac{2R_2}{d} \tag{13.24c}$$

$R_2$ is the side-bend radius, as shown in Figure 13-23. $C_2$ should be greater than 4.[1]



maximum bending stress



maximum torsional stress

## Surging in Extension Springs

The natural frequency of a helical extension spring with both ends fixed against axial deflection is the same as that for a helical spring in compression (see equation 13.11):

$$f_n = \frac{2}{\pi N_a} \frac{d}{D^2} \sqrt{\frac{Gg}{32\gamma}} \quad \text{Hz} \tag{13.25}$$

**Table 13-10 Maximum Torsional and Bending Yield Strengths $S_{ys}$ and $S_y$ for Helical Extension Springs in Static Applications**
No Set Removal and Low-Temperature Heat Treatment Applied. Source: Ref. 1

| Material | Maximum Percent of Ultimate Tensile Strength | | |
|---|---|---|---|
| | $S_{ys}$ in Torsion | | $S_y$ in Bending |
| | Body | End | End |
| Cold-drawn carbon steel (e.g., A227, A228) | 45% | 40% | 75% |
| Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401) | 50 | 40 | 75 |
| Austenitic stainless steel and nonferrous alloys (e.g., A313, B134, B159, B197) | 35 | 30 | 55 |

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## Compression-Spring Surge

The natural frequency $\omega_n$ or $f_n$ of a helical compression spring depends on its boundary conditions. Fixing both ends is the more common and desirable arrangement, as its $f_n$ will be twice that of a spring with one end fixed and the other free. For the fixed-fixed case:

$$\omega_n = \pi\sqrt{\frac{kg}{W_a}} \text{ rad/sec} \qquad f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \text{ Hz} \tag{13.11a}$$

where $k$ is the spring rate, $W_a$ is the weight of the spring's active coils, and $g$ is the gravitational constant. It can be expressed either as angular frequency $\omega_n$ or linear frequency $f_n$. The weight of the active coils can be found from

$$W_a = \frac{\pi^2 d^2 D N_a \gamma}{4} \tag{13.11b}$$

where $\gamma$ is the material's weight density. For total spring weight substitute $N_t$ for $N_a$.

Substituting equations 13.7 (p. 810) and 13.11*a* into 13.11*b* gives

$$f_n = \frac{2}{\pi N_a}\frac{d}{D^2}\sqrt{\frac{Gg}{32\gamma}} \text{ Hz} \tag{13.11c}$$

for the natural frequency of a fixed-fixed helical coil spring. If one end of the spring is fixed and the other free, it acts like a fixed-fixed spring of twice its length. Its natural frequency can be found by using a number for $N_a$ in equation 13.11*c* that is twice the actual number of active coils present in the fixed-free spring.

$$F_a = \frac{F_{max} - F_{min}}{2}$$

$$F_m = \frac{F_{max} + F_{min}}{2}$$

A **force ratio** $R_F$ can also be defined as:

$$R_F = \frac{F_{min}}{F_{max}}$$

**Table 13-5 Typical Properties of Spring Temper Alloy Strip**
Source: Reference 1

| Material | Sut MPa (ksi) | Rockwell Hardness | Elongation % | Bend Factor | E GPa (Mpsi) | Poisson's Ratio |
|---|---|---|---|---|---|---|
| Spring steel | 1 700 (246) | C50 | 2 | 5 | 207 (30) | 0.30 |
| Stainless 301 | 1 300 (189) | C40 | 8 | 3 | 193 (28) | 0.31 |
| Stainless 302 | 1 300 (189) | C40 | 5 | 4 | 193 (28) | 0.31 |
| Monel 400 | 690 (100) | B95 | 2 | 5 | 179 (26) | 0.32 |
| Monel K500 | 1 200 (174) | C34 | 40 | 5 | 17.9 (26) | 0.29 |
| Inconel 600 | 1 040 (151) | C30 | 2 | 2 | 214 (31) | 0.29 |
| Inconel X-750 | 1 050 (152) | C35 | 20 | 3 | 214 (31) | 0.29 |
| Beryllium copper | 1 300 (189) | C40 | 2 | 5 | 128 (18.5) | 0.33 |
| Ni-Span-C | 1 400 (203) | C42 | 6 | 2 | 186 (27) | - |
| Brass CA 260 | 620 (90) | B90 | 3 | 3 | 11 (16) | 0.33 |
| Phosphor bronze | 690 (100) | B90 | 3 | 2.5 | 103 (15 | 0.20 |
| 17-7PH RH950 | 1 450 (210) | C44 | 6 | flat | 203 (29.5) | 0.34 |
| 17-7PH Cond. C | 1 650 (239) | C46 | 1 | 2.5 | 203 (29.5) | 0.34 |

**Table 13-4 Coefficients and Exponents for Equation 13.3**
Source: Reference 1

| ASTM # | Material | Range mm | Range in | Exponent $b$ | Coefficient $A$ MPa | Coefficient $A$ psi | Correlation Factor |
|---|---|---|---|---|---|---|---|
| A227 | Cold drawn | 0.5–16 | 0.020–0.625 | –0.182 2 | 1 753.3 | 141 040 | 0.998 |
| A228 | Music wire | 0.3–6 | 0.010–0.250 | –0.1625 | 2 153.5 | 184 649 | 0.9997 |
| A229 | Oil tempered | 0.5–16 | 0.020–0.625 | –0.183 3 | 1 831.2 | 146 780 | 0.999 |
| A232 | Chrome-v. | 0.5–12 | 0.020–0.500 | –0.145 3 | 1 909.9 | 173 128 | 0.998 |
| A401 | Chrome-s. | 0.8–11 | 0.031–0.437 | –0.093 4 | 2 059.2 | 220 779 | 0.991 |

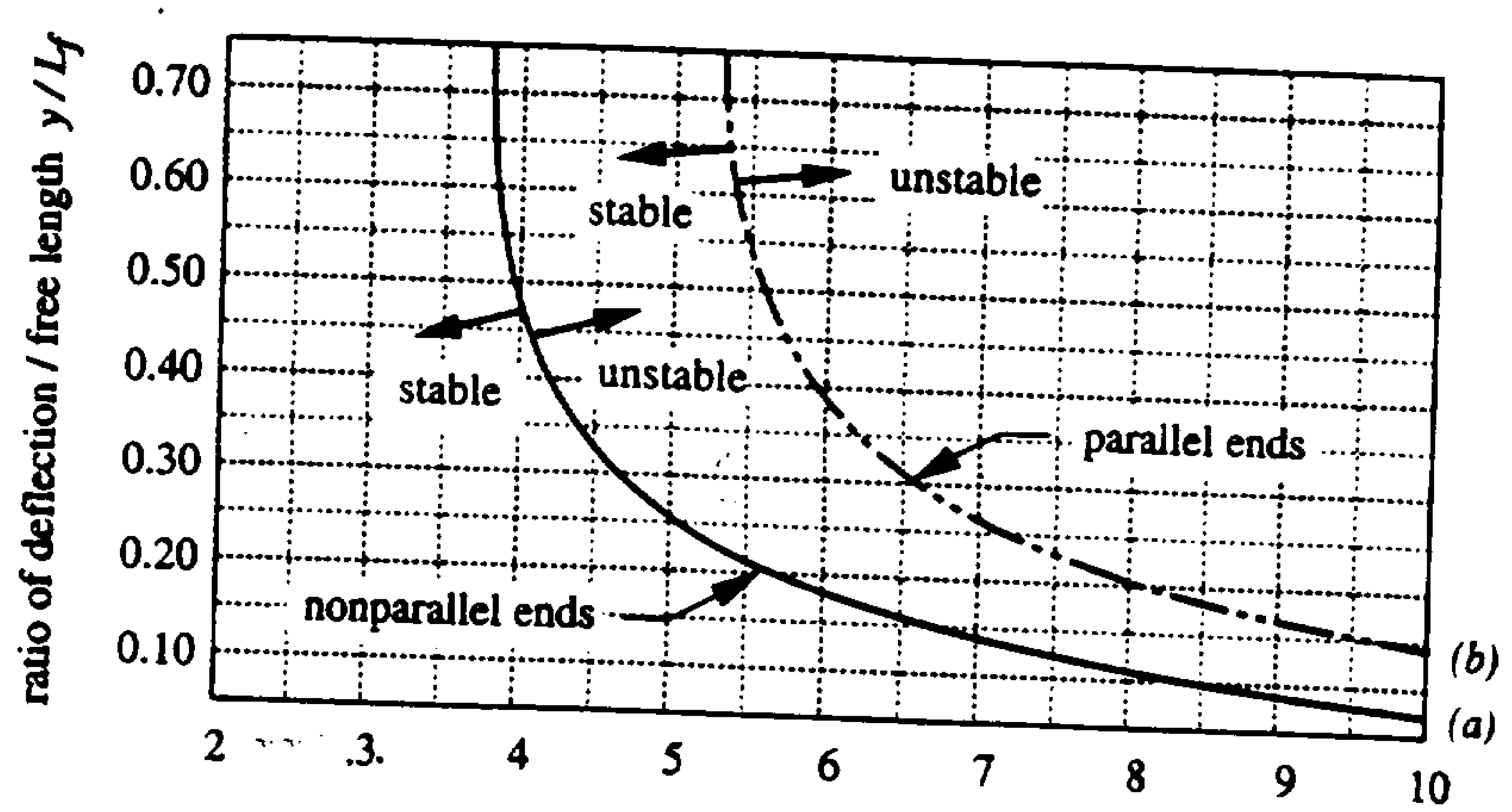



**FIGURE 13-14**
Critical Buckling Condition Curves Adapted from Reference 1

**Table 13-6 Maximum Torsional Yield Strength $S_{ys}$ for Helical Compression Springs in Static Applications**
Bending or Buckling Stresses Not Included. Source: Adapted from Ref. 1

| Material | Maximum Percent of Ultimate Tensile Strength: Before Set Removed (Use Eq. 13.9b) | After Set Removed (Use Eq. 13.8b) |
|---|---|---|
| Cold-drawn carbon steel (e.g., A227, A228) | 45% | 60–70% |
| Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401) | 50 | 65–75 |
| Austenitic stainless steel (e.g., A313) | 35 | 55–65 |
| Nonferrous alloys (e.g., B134, B159, B197) | 35 | 55–65 |

## Important Equations Used in This Chapter

**Spring Rate (Section 13.1):**

$$k = \frac{F}{y} \qquad (13.1)$$

**Combining Springs in Parallel (Section 13.1):**

$$k_{total} = k_1 + k_2 + k_3 + \ldots + k_n \qquad (13.2a)$$

**Combining Springs in Series (Section 13.1):**

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} \qquad (13.2b)$$

**Spring Index (Section 13.4):**

$$C = \frac{D}{d} \qquad (13.5)$$

**Deflection of Helical Compression Spring (Section 13.4):**

$$y = \frac{8FD^3N_a}{d^4G} \qquad (13.6)$$

**Deflection of Helical Extension Spring (Section 13.7):**

$$y = \frac{8(F - F_i)D^3N_a}{d^4G} \qquad (13.22)$$

**Deflection of Round-Wire Helical Torsion Spring (Section 13.8):**

$$\theta_{rev} \cong 10.2\frac{MDN_a}{d^4E} \qquad \theta_{rev} \cong 10.8\frac{MDN_a}{d^4E} \qquad (13.27c)$$

**Spring Rate of Helical Compression Spring (Section 13.4):**

$$k = \frac{F}{y} = \frac{d^4G}{8D^3N_a} \qquad (13.7)$$

**Spring Rate of Helical Extension Spring (Section 13.7):**

$$k = \frac{F - F_i}{y} = \frac{d^4G}{8D^3N_a} \qquad (13.20)$$

**Spring Rate of Round-Wire Helical Torsion Spring (Section 13.8):**

$$k = \frac{M}{\theta_{rev}} \cong \frac{d^4E}{10.8DN_a} \qquad (13.28)$$

**Static Stress in Helical Compression or Extension Spring (Section 13.7):**

$$\tau_{max} = K_s\frac{8FD}{\pi d^3} \qquad \text{where} \quad K_s = \left(1 + \frac{0.5}{C}\right) \qquad (13.8b)$$

**Dynamic Stress in Helical Compression or Extension Spring (Section 13.7):**

$$K_B = \frac{4C + 2}{4C - 3}$$

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \qquad (13.9a)$$

$$\tau_{max} = K_w\frac{8FD}{\pi d^3} \qquad (13.9b)$$

**Spring Rate of Torsion Springs . .**

The spring rate can always be obtained from the deflection formula:

$$k = \frac{M}{\theta_{rev}} \cong \frac{d^4E}{10.8DN_a}$$

**Stress in Helical Torsion Spring at Inside Diameter (Section 13.8):**

$$K_{b_i} = \frac{4C^2 - C - 1}{4C(C-1)} \tag{13.31a}$$

$$\sigma_{i_{max}} = K_{b_i}\frac{M_{max}c}{I} = K_{b_i}\frac{M_{max}(d/2)}{\pi d^4/64} = K_{b_i}\frac{32M_{max}}{\pi d^3} \tag{13.32a}$$

**Stress in Helical Torsion Spring at Outside Diameter (Section 13.8):**

$$K_{b_o} = \frac{4C^2 + C - 1}{4C(C+1)} \tag{13.31b}$$

$$\sigma_{o_{min}} = K_{b_o}\frac{32M_{min}}{\pi d^3}; \qquad \sigma_{o_{max}} = K_{b_o}\frac{32M_{max}}{\pi d^3} \tag{13.32b}$$

$$\sigma_{o_{alt}} = \frac{\sigma_{o_{max}} - \sigma_{o_{min}}}{2}; \qquad \sigma_{o_{mean}} = \frac{\sigma_{o_{max}} + \sigma_{o_{min}}}{2} \tag{13.32c}$$

**Ultimate Tensile Strength of Steel Wire—See Table 13-4 for Constants (Section 13.4):**

$$S_{ut} \cong Ad^b \tag{13.3}$$

**Ultimate Shear Strength of Wire (Section 13.4):**

$$S_{us} \cong 0.67S_{ut} \tag{13.4}$$

**Torsional Endurance Limits for Spring-Steel Wire for Stress Ratio R = 0 (Section 13.4):**

$S_{ew}' \cong 45.0$ kpsi (310 MPa) for unpeened springs

$S_{ew}' \cong 67.5$ kpsi (465 MPa) for peened springs (13.12)

**Torsional Endurance Limits for Spring-Steel Wire for Stress Ratio R = −1 (Section 13.4):**

$$S_{es} = 0.5\frac{S_{ew}S_{us}}{S_{us} - 0.5S_{ew}} \tag{13.17b}$$

**Bending Endurance Limits for Spring-Steel Wire for Stress Ratio R = 0 (Section 13.4):**

$$S_{ew_b}' = \frac{S_{ew}'}{0.577} \tag{13.33a}$$

**Bending Endurance Limits for Spring-Steel Wire for Stress Ratio R = −1 (Section 13.4):**

$$S_e = 0.5\frac{S_{ew_b}S_{ut}}{S_{ut} - 0.5S_{ew_b}} \tag{13.34c}$$

**Static Safety Factor for Helical Compression or Extension Spring (Section 13.5):**

$$N_s = \frac{S_{ys}}{\tau} \tag{13.14}$$

**Dynamic Safety Factor for Helical Compression or Extension Spring (Section 13.4):**

$$N_{f_s} = \frac{S_{es}(S_{us} - \tau_i)}{S_{es}(\tau_m - \tau_i) + S_{us}\tau_a} \tag{13.17a}$$

**Dynamic Safety Factor for Helical Torsion Spring (Section 13.8):**

$$N_{f_b} = \frac{S_e\left(S_{ut} - \sigma_{o_{min}}\right)}{S_e\left(\sigma_{o_{mean}} - \sigma_{o_{min}}\right) + S_{ut}\sigma_{o_{alt}}} \tag{13.34b}$$

$$N_y = \frac{S_y}{\sigma_{i_{max}}} \tag{13.34a}$$

Solve for $d$ using the static yield criterion.

$$d := \left(\frac{32 \cdot K_{bi} \cdot N_{yd} \cdot M}{\pi \cdot K_s \cdot A \cdot mm^3}\right)^{\frac{1}{3+b}} \cdot mm$$

### Number of Coils in Torsion Springs

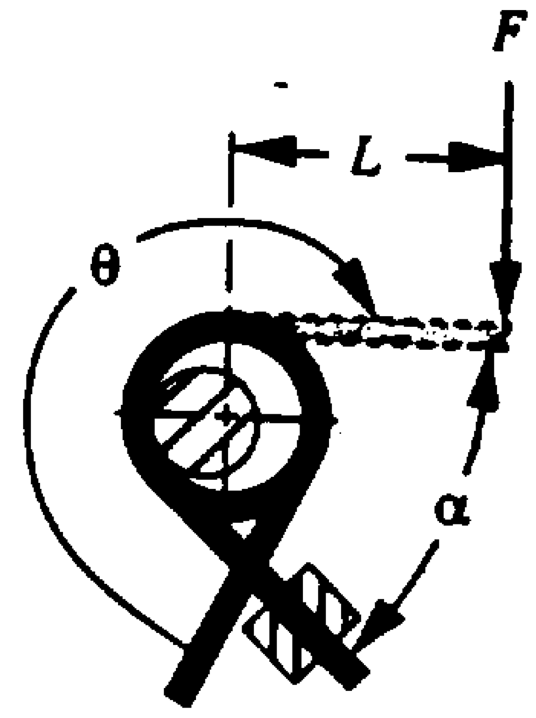
The active coils are equal to the number of body turns $N_b$ plus some contribution from the ends, which also bend. For straight ends, the contribution can be expressed as an equivalent number of coils $N_e$:

$$N_e = \frac{L_1 + L_2}{3\pi D} \tag{13.26a}$$

where $L_1$ and $L_2$ are the respective lengths of the tangent-ends of the coil. The number of active coils is then

$$N_a = N_b + N_e \tag{13.26b}$$

where $N_b$ is the number of coils in the spring body.




specify:
α—angle between ends
F—load on ends at α
L—moment arm
θ—angular deflection from free position

### Deflection of Torsion Springs

The angular deflection of the coil-end is normally expressed in radians but is often converted to revolutions. We will use revolutions. Since it is essentially a beam in bending, the (angular) deflection can be expressed as

$$\theta_{rev} = \frac{1}{2\pi}\theta_{rad} = \frac{1}{2\pi}\frac{ML_w}{EI} \tag{13.27a}$$

### Coil Closure

When a torsion spring is loaded to close the coils (as it should be), [redacted] The minimum inside coil diameter at full deflection is

$$D_{i_{min}} = \frac{DN_b}{N_b + \theta_{rev}} - d \tag{13.29}$$

where $D$ is the unloaded mean coil diameter. Any pin that the coil works over should be limited to about 90% of this minimum inside diameter.

The maximum coil-body length at full "windup" is:

$$L_{max} = d(N_b + 1 + \theta) \tag{13.30}$$

The torsional-endurance limit data for helical compression springs shown in equation 13.12 (p. 816) can be adapted for bending by using the von Mises relationship between torsion and tension loading.

$$S_{ew_b}' = \frac{S_{ew}'}{0.577} \tag{13.33a}$$

which gives

$$S_{ew_b}' \cong \frac{45.0}{0.577} = 78 \text{ kpsi } (537 \text{ MPa}) \quad \text{for unpeened springs}$$

$$S_{ew_b}' \cong \frac{67.5}{0.577} = 117 \text{ kpsi } (806 \text{ MPa}) \quad \text{for peened springs} \tag{13.33b}$$

**Table 13-13 Maximum Recommended Bending Yield Strength $S_y$ for Helical Torsion Springs in Static Applications**
Source: Adapted from Reference 1

| Material | Maximum Percent of Ultimate Tensile Strength | |
|---|---|---|
| | Stress Relieved | Favorable Residual Stress |
| Cold-drawn carbon steel (e.g., A227, A228) | 80% | 100% |
| Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401) | 85 | 100 |
| Austenitic stainless steel and nonferrous alloys (e.g., A313, B134, B159, B197) | 60 | 80 |

**Table C-1 Physical Properties of Some Engineering Materials**

Data from Various Sources.* These Properties are Essentially Similar for All Alloys of the Particular Material

| Material | Modulus of Elasticity $E$ | | Modulus of Rigidity $G$ | | Poisson's Ratio $\nu$ | Weight Density $\gamma$ | Mass Density $\rho$ | Specific Gravity |
|---|---|---|---|---|---|---|---|---|
| | Mpsi | GPa | Mpsi | GPa | | lb/in$^3$ | Mg/m$^3$ | |
| Aluminum Alloys | 10.4 | 71.7 | 3.9 | 26.8 | 0.34 | 0.10 | 2.8 | 2.8 |
| Beryllium Copper | 18.5 | 127.6 | 7.2 | 49.4 | 0.29 | 0.30 | 8.3 | 8.3 |
| Brass, Bronze | 16.0 | 110.3 | 6.0 | 41.5 | 0.33 | 0.31 | 8.6 | 8.6 |
| Copper | 17.5 | 120.7 | 6.5 | 44.7 | 0.35 | 0.32 | 8.9 | 8.9 |
| Iron, Cast, Gray | 15.0 | 103.4 | 5.9 | 40.4 | 0.28 | 0.26 | 7.2 | 7.2 |
| Iron, Cast, Ductile | 24.5 | 168.9 | 9.4 | 65.0 | 0.30 | 0.25 | 6.9 | 6.9 |
| Iron, Cast, Malleable | 25.0 | 172.4 | 9.6 | 66.3 | 0.30 | 0.26 | 7.3 | 7.3 |
| Magnesium Alloys | 6.5 | 44.8 | 2.4 | 16.8 | 0.33 | 0.07 | 1.8 | 1.8 |
| Nickel Alloys | 30.0 | 206.8 | 11.5 | 79.6 | 0.30 | 0.30 | 8.3 | 8.3 |
| Steel, Carbon | 30.0 | 206.8 | 11.7 | 80.8 | 0.28 | 0.28 | 7.8 | 7.8 |
| Steel, Alloys | 30.0 | 206.8 | 11.7 | 80.8 | 0.28 | 0.28 | 7.8 | 7.8 |
| Steel, Stainless | 27.5 | 189.6 | 10.7 | 74.1 | 0.28 | 0.28 | 7.8 | 7.8 |
| Titanium Alloys | 16.5 | 113.8 | 6.2 | 42.4 | 0.34 | 0.16 | 4.4 | 4.4 |
| Zinc Alloys | 12.0 | 82.7 | 4.5 | 31.1 | 0.33 | 0.24 | 6.6 | 6.6 |

* *Properties of Some Metals and Alloys*, International Nickel Co., Inc., N.Y.; *Metals Handbook*, American Society for Metals, Materials Park, Ohio.

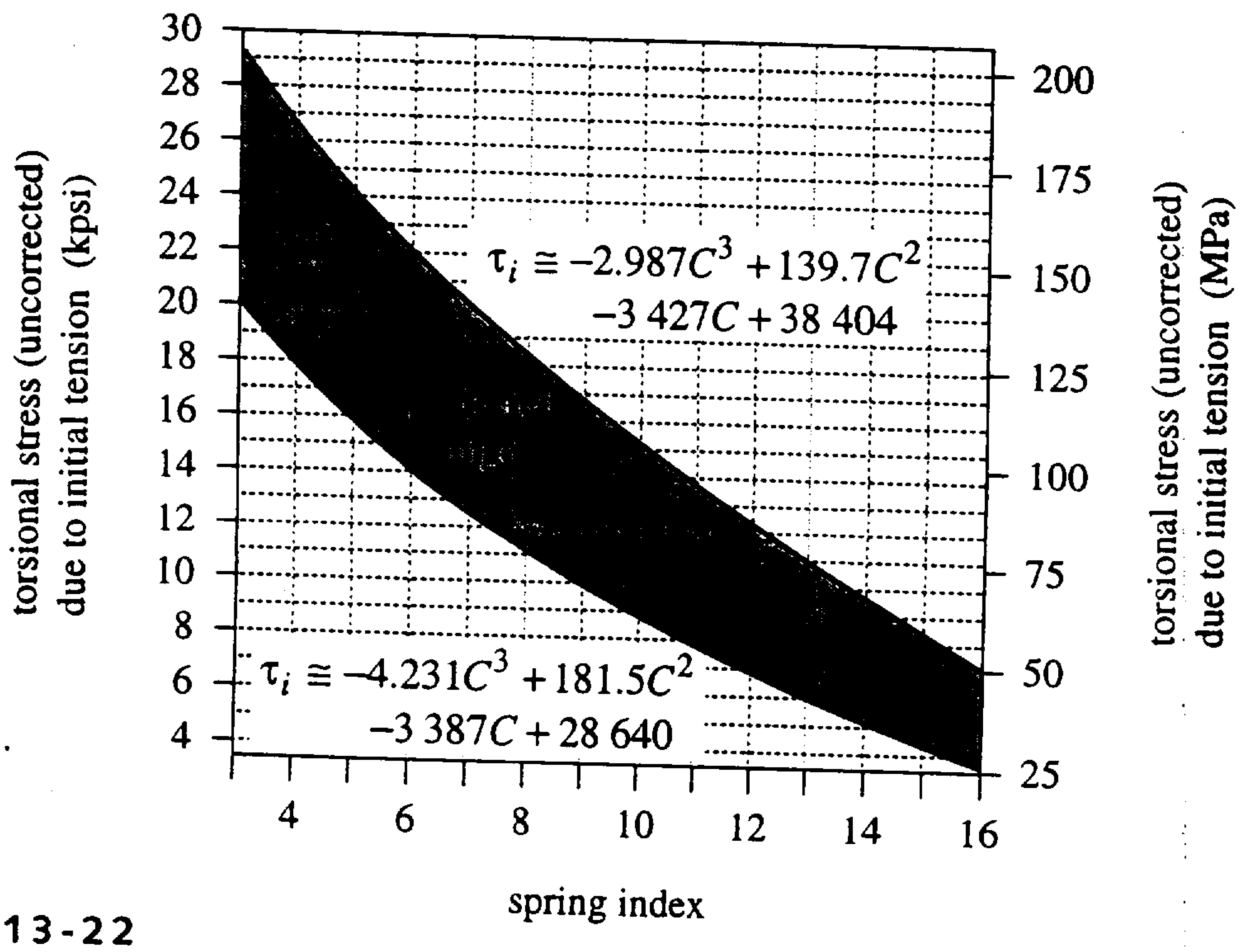



**FIGURE 13-22**

Preferred Range of Initial Stress in Extension Springs as a Function of Spring Index