GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2003

Design
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

DESIGN QUALIFIER

SPRING 03

WRITTEN EXAMINATION

We are interested in learning what you know and your ability to reason in the formulation and solution of design problems.

If you find any question or part of this exam confusing, please state your assumptions and rephrase the question and proceed.

Please read the entire exam first.

Questions 1 and 2 carry equal points.

Allocate your time carefully so that you cover all three parts that you are being examined on in these two questions, namely, Methods, Realizability and Analysis.

ORAL EXAMINATION

Please arrive half an hour before the scheduled time for the oral exam. During this period we will give you a question to think about. The scope of the oral exam is as follows:

- provide an opportunity for you to state how design fits into your research activities;
- probe your understanding of the question that we posed to you in the preceding half hour.

QUESTION 1 – METHOD & REALIZABILITY

Scenario

On 5 March 2003, several hundred thousand chickens in the Netherlands were destroyed due to the concern that they had contracted a highly contagious virus that is lethal to birds. To avoid such situations, it is desirable to vaccinate large quantities of birds such as chickens. Vaccinations can happen in a variety of manners including spraying airborne vaccinations and feed augmented with oral vaccinations. However, many vaccinations are optimally dispensed via a syringe. Currently, such vaccinations are done in a manual mode where a person holds the bird and a second person injects the vaccine. This is time consuming and expensive from a manual labor perspective.

Task

Your task is to design an automated system that is capable of vaccinating chickens without harming them. The system should be automated, easy to "load" with chickens and must guarantee that each chicken is vaccinated. Furthermore it must be able to handle a variety of birds from baby chicks to fully-grown turkeys (it turns out that turkey's can contract the same virus) at approximately 15 kg without harming them. Please remember that the system must be able to fully immobilize the bird while the vaccine is being administered.

Your boss wants you to start from scratch and document your design process thoroughly – but this is not possible for lack of time. A senior engineer has suggested that you follow the general guidelines given below and turn in a report documenting the five steps.

Deliverables

Method

1. *Clarify the Task:* State the overall function of your system. What are the most important drivers/design criteria?

2. *Conceptual Design:* State and **implement** the steps (including a specification list and functional diagrams/decomposition) for transforming the overall function that you have identified into at least three alternative design solutions. Ensure that you have identified the important sub functions for each of the five phases listed above. Sketch and describe the workings of these alternatives.

3. *Selection:* Suggest a structured approach to select one of the alternatives for further development.

Realizability

4. *Embodimet:* Further develop the alternative that you have selected.

5. *Costing:* How would you estimate the cost of your design? You may critically evaluate the design in terms of manufacturability, initial cost, maintenance cost, reliability, manipulation performance, and other criteria that you feel are important to consider in this phase of design.

6. *Pricing:* Based on the preceding analysis, how would you estimate the market size for such a system and set the price for selling such a system? Be brief.

Your Exam #: ....

You MUST write your solutions to QUESTION 2 on this exam sheet.

QUESTION 2 - ANALYSIS

II. **A.** You have been hired as a consultant for a classic car racing company. Their goal is to make some historical cars like Bert Bras' M.G. more competitive on the racing circuit. Among others, they are redesigning valve systems for the internal combustion engines and one such valve system is given in Figure 1. As shown in Figure 1, the valve is being opened by a rocker arm. The rocker arm is operated through a pushrod which is connected to a cam follower and the camshaft. The camshaft rotates between 250 and 3250 revolutions per minute, causing the valve to be opened and closed once per revolution. A helical compression spring ensures that the valve closes properly and completely.

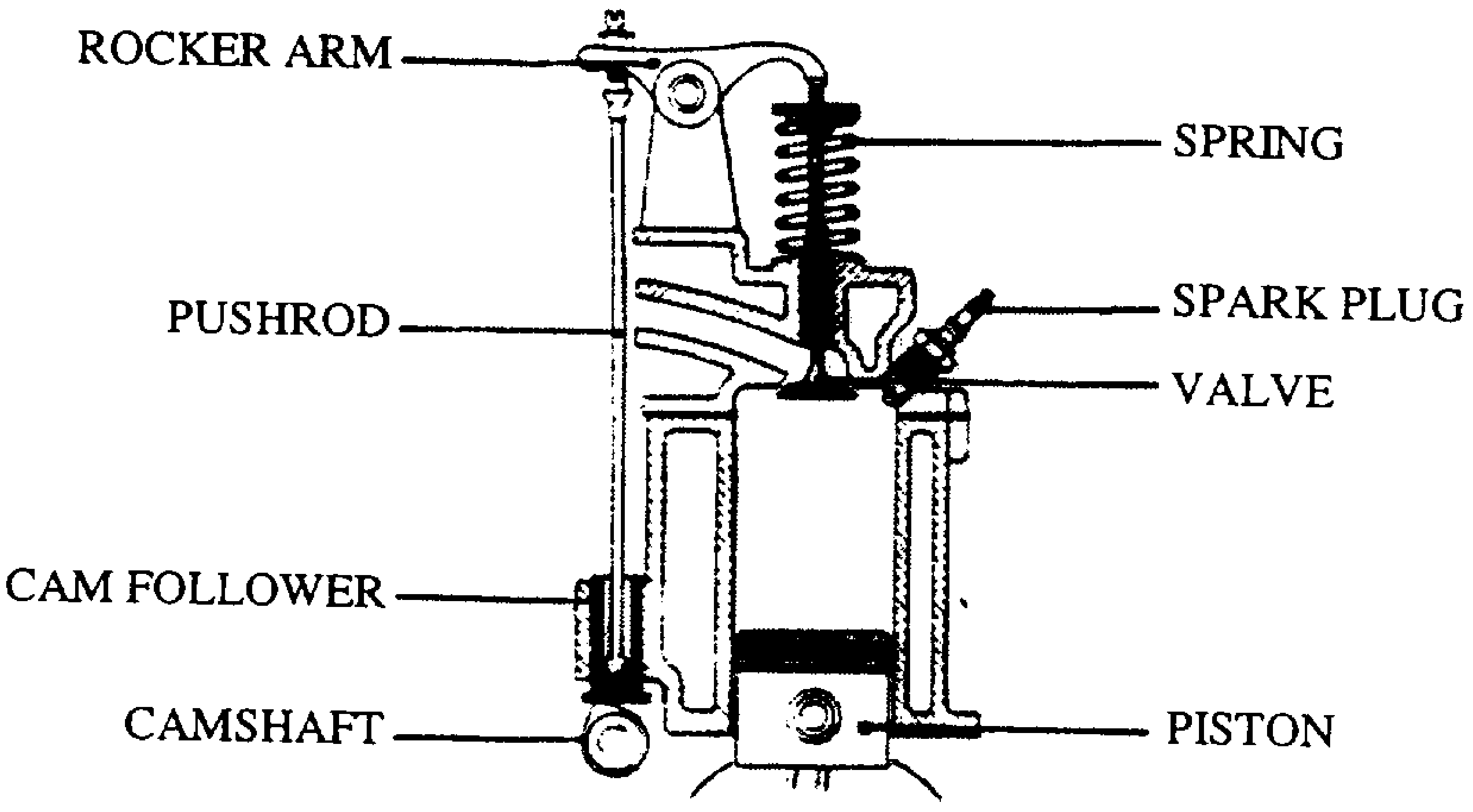


Figure 1. Valve system layout.

The free length of the spring is 4.88 cm. The fitted length (i.e., when the spring is installed) is 3.66 cm and the load on the spring at fitted length is 365 N. A maximum load of 632 N on the spring occurs when the valve is completely opened. The wire diameter d is 4 mm and the spring index C is 8. The spring material is shot-peened ASTM 232 chrome vanadium $(G = 80.8 GP_a)$ and the spring ends are squared and ground. Your job is to answer the following questions:

Data Sheets are included from Page 8 through 15.

a) How far is the valve opened at the maximum load? (1 pt.)

b) What is the number of active coils? (1 pt.)

c) What is the solid length of the spring? (1 pt.)

d) What is the factor of safety against failure due to fatigue using the Goodman fatigue failure criterion? Will the spring fail? (2 pt.)

e) What is the factor of safety against failure due to yielding? (1 pt.)

f) What is the clash allowance at the maximum working load? (1 pt.)

g) At what rpm of the camshaft would spring surge and the resonance occur? Given: γ(material's weight density) = $76.5E3\,{}^{N}\!/_{m^3}$ (2 pt.)

h) Suppose the company wants to use aluminum as the spring material in order to decrease the spring weight. If all spring dimensions remain the same and only the material is changed, what is the factor of safety against buckling? $E = 71.0$ GPa, $G = 26.2$ GPa, $\alpha = 0.5$. (1 pt.)

II B. a What is the property of the Angular Velocity Ratio (m_v) between gears in a gear set? (1 pt.)

b. What is a Pressure Line? (1 pt.)

c. What is the effect of Undercutting on gear tooth? (1 pt.)

d. Why is it desirable to have Contact Ratio, m_p greater than 1? (1 pt.)

e. Why is Backlash undesirable? (1 pt.)

DATA SHEETS (PAGE 8 THROUGH 15)

The stresses σ_a and σ_m can replace S_a and S_m in Eqs. (7-34) to (7-36) if each strength is divided by a factor of safety n. When this is done, the Soderberg equation becomes

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n}$$

The modified Goodman relation is

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

and the Gerber equation is

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

The critical deflection is given by the equation

$$y_{cr} = L_0 C_1 \left[1 - \left(1 - \frac{C_2}{\lambda_{\text{eff}}^2}\right)^{1/2}\right]$$

where y_{cr} is the deflection corresponding to the onset of instability.

TABLE 10-2

Formulas for Compression-Spring Dimensions. (N_a = Number of Active Coils)

	TYPE OF SPRING ENDS			
TERM	PLAIN	PLAIN AND GROUND	SQUARED OR CLOSED	SQUARED AND GROUND
End coils, N_e	0	1	2	2
Total coils, N_t $= N$	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0 $= L_f$	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Source: Associated Spring–Barnes Group, *Design Handbook*, Bristol, Conn., 1981, p. 32.

TABLE 10-3

End-Condition Constants α for Helical Compression Springs*

END CONDITION	CONSTANT α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

quantity λ_{eff} in Eq. (10-11) is the *effective slenderness ratio* and is given by the equation

$$\lambda_{\text{eff}} = \frac{\alpha L_0}{D} \tag{10-12}$$

C_1 and C_2 are the elastic constants and are defined by the equations

$$C_1 = \frac{E}{2(E - G)} \tag{10-13}$$

$$C_2 = \frac{2\pi^2(E - G)}{2G + E} \tag{10-14}$$

Equation (10-12) contains the *end-condition constant* α. This depends upon how the ends of the spring are supported. Table 10-3 gives values of α for usual end conditions. Note how closely these resemble the end conditions for columns.

Absolute stability occurs when, in Eq. (10-11), the term $C_2/\lambda_{\text{eff}}^2$ is less than unity. This means that the condition for absolute stability is that

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E - G)}{2G + E}\right]^{1/2} \tag{10-15}$$

For steels, this turns out to be

$$L_0 < 2.63\frac{D}{\alpha} \tag{10-16}$$

Table 13-2
Preferred Wire Diameters

U.S. (in)	SI (mm)	A228	A229	A227	A232	A401
0.004	0.10	✓				
0.005	0.12	✓				
0.006	0.16	✓				
0.008	0.20	✓				
0.010	0.25	✓				
0.012	0.30	✓				
0.014	0.35	✓				
0.016	0.40	✓				
0.018	0.45	✓				
0.020	0.50	✓	✓			
0.022	0.55	✓	✓			
0.024	0.60	✓	✓			
0.026	0.65	✓	✓			
0.028	0.70	✓	✓	✓		
0.030	0.80	✓	✓	✓	✓	
0.035	0.90	✓	✓	✓	✓	
0.038	1.00	✓	✓	✓	✓	
0.042	1.10	✓	✓	✓	✓	
0.045		✓	✓	✓	✓	
0.048	1.20	✓	✓	✓	✓	
0.051		✓	✓	✓	✓	
0.055	1.40	✓	✓	✓	✓	
0.059		✓	✓	✓	✓	
0.063	1.60	✓	✓	✓	✓	✓
0.067		✓	✓	✓	✓	✓
0.072	1.80	✓	✓	✓	✓	✓
0.076		✓	✓	✓	✓	✓
0.081	2.00	✓	✓	✓	✓	✓
0.085	2.20	✓	✓	✓	✓	✓
0.092		✓	✓	✓	✓	✓
0.098	2.50	✓	✓	✓	✓	✓
0.105		✓	✓	✓	✓	✓
0.112	2.80	✓	✓	✓	✓	✓
0.125	3.00	✓	✓	✓	✓	✓
0.135	3.50	✓	✓	✓	✓	✓
0.148		✓	✓	✓	✓	✓
0.162	4.00	✓	✓	✓	✓	✓
0.177	4.50	✓	✓	✓	✓	✓
0.192	5.00	✓	✓	✓	✓	✓
0.207	5.50	✓	✓	✓	✓	✓
0.225	6.00	✓	✓	✓	✓	✓
0.250	6.50	✓	✓	✓	✓	✓
0.281	7.00		✓	✓	✓	✓
0.312	8.00		✓	✓	✓	✓
0.343	9.00		✓	✓	✓	✓
0.362			✓	✓	✓	✓
0.375			✓	✓	✓	✓
0.406	10.0		✓	✓	✓	
0.437	11.0		✓	✓	✓	
0.469	12.0		✓	✓		
0.500	13.0		✓	✓		
0.531	14.0		✓	✓		
0.562	15.0		✓	✓		
0.625	16.0		✓	✓		

Active Coils in Extension Springs

All coils in the body are considered active coils, but one coil is typically added t the number of active coils to obtain the body length L_b.

$$N_t = N_a + 1 \tag{13.18}$$

$$L_b = dN_t \tag{13.19}$$

$$\tau_i \cong -4.231C^3 + 181.5C^2 - 3\,387C + 28\,640 \tag{13.21a}$$

$$\tau_i \cong -2.987C^3 + 139.7C^2 - 3\,427C + 38\,404 \tag{13.21b}$$

where τ_i is in psi. The average of the two values computed from these functions can be taken as a good starting value for initial coil stress.

The bending stress at point A is found from

$$\sigma_A = K_b \frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2}$$

where

$$K_b = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

and

$$C_1 = \frac{2R_1}{d}$$

The torsional stress at point B is found from

$$\tau_B = K_{w_2} \frac{8DF}{\pi d^3} \tag{13.24a}$$

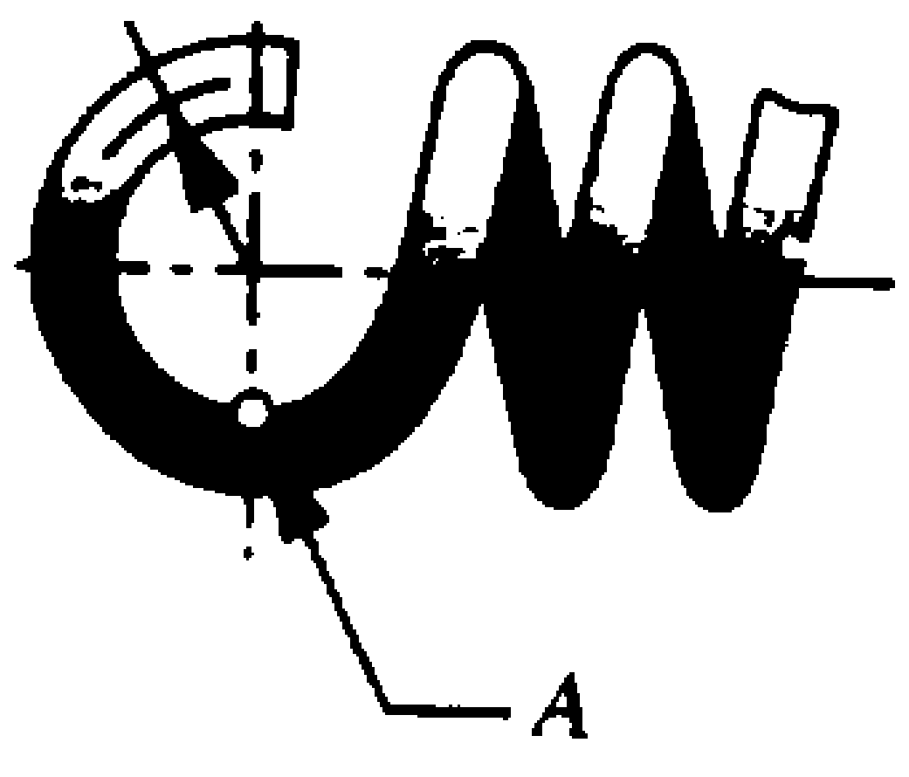
where

$$K_{w_2} = \frac{4C_2 - 1}{4C_2 - 4} \tag{13.24b}$$

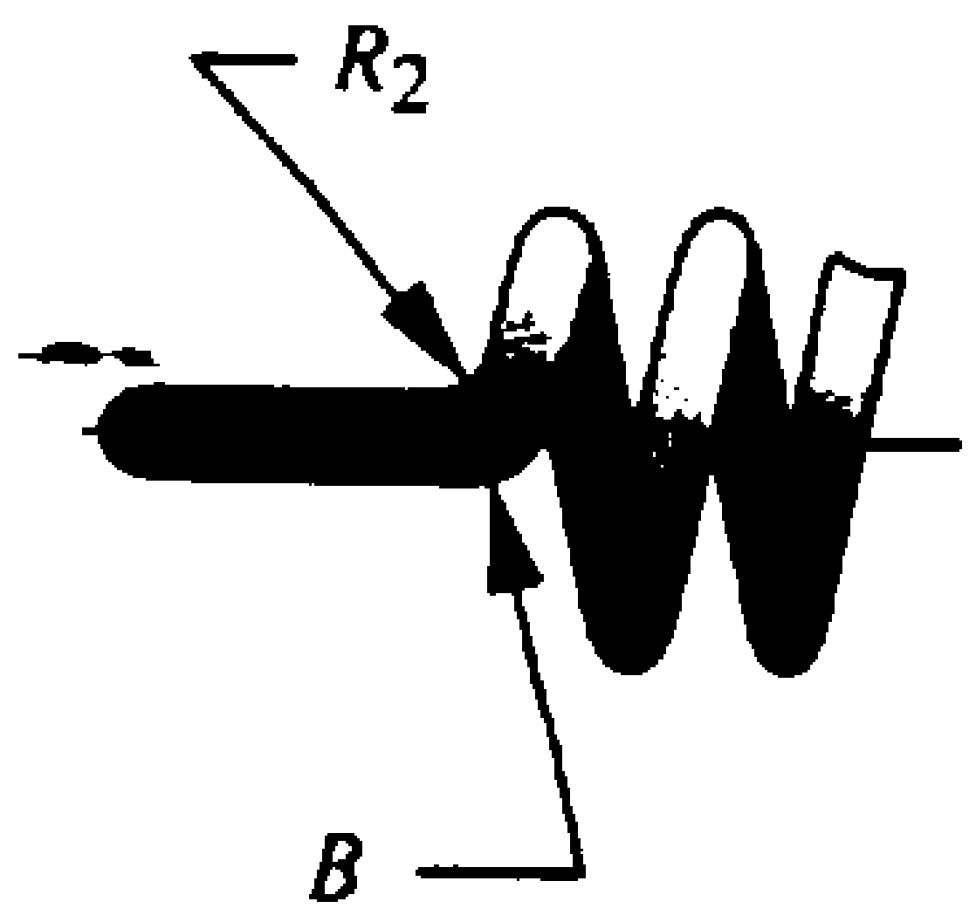
and

$$C_2 = \frac{2R_2}{d} \tag{13.24c}$$

R_2 is the side-bend radius, as shown in Figure 13-23. C_2 should be greater than 4.[1]



maximum
bending stress



maximum
torsional stress

Surging in Extension Springs

The natural frequency of a helical extension spring with both ends fixed against axial deflection is the same as that for a helical spring in compression (see equation 13.11):

$$f_n = \frac{2}{\pi N_a} \frac{d}{D^2} \sqrt{\frac{Gg}{32\gamma}} \quad \text{Hz} \tag{13.25}$$

Table 13-10 Maximum Torsional and Bending Yield Strengths S_{ys} and S_y for Helical Extension Springs in Static Applications
No Set Removal and Low-Temperature Heat Treatment Applied. Source: Ref. 1

Material	Maximum Percent of Ultimate Tensile Strength		
	S_{ys} in Torsion		S_y in Bending
	Body	End	End
Cold-drawn carbon steel (e.g., A227, A228)	45%	40%	75%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	50	40	75
Austenitic stainless steel and nonferrous alloys (e.g., A313, B134, B159, B197)	35	30	55

Compression-Spring Surge

The natural frequency ω_n or f_n of a helical compression spring depends on its boundary conditions. Fixing both ends is the more common and desirable arrangement, as its f_n will be twice that of a spring with one end fixed and the other free. For the fixed-fixed case:

$$\omega_n = \pi\sqrt{\frac{kg}{W_a}} \text{ rad/sec} \qquad f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \text{ Hz} \tag{13.11a}$$

where k is the spring rate, W_a is the weight of the spring's active coils, and g is the gravitational constant. It can be expressed either as angular frequency ω_n or linear frequency f_n. The weight of the active coils can be found from

$$W_a = \frac{\pi^2 d^2 D N_a \gamma}{4} \tag{13.11b}$$

where γ is the material's weight density. For total spring weight substitute N_t for N_a.

Substituting equations 13.7 (p. 810) and 13.11*a* into 13.11*b* gives

$$f_n = \frac{2}{\pi N_a}\frac{d}{D^2}\sqrt{\frac{Gg}{32\gamma}} \text{ Hz} \tag{13.11c}$$

for the natural frequency of a fixed-fixed helical coil spring. If one end of the spring is fixed and the other free, it acts like a fixed-fixed spring of twice its length. Its natural frequency can be found by using a number for N_a in equation 13.11*c* that is twice the actual number of active coils present in the fixed-free spring.

$$F_a = \frac{F_{max} - F_{min}}{2}$$

$$F_m = \frac{F_{max} + F_{min}}{2}$$

A **force ratio** R_F can also be defined as:

$$R_F = \frac{F_{min}}{F_{max}}$$

Table 13-5 Typical Properties of Spring Temper Alloy Strip

Source: Reference 1

Material	Sut MPa (ksi)	Rockwell Hardness	Elongation %	Bend Factor	E GPa (Mpsi)	Poisson's Ratio
Spring steel	1 700 (246)	C50	2	5	207 (30)	0.30
Stainless 301	1 300 (189)	C40	8	3	193 (28)	0.31
Stainless 302	1 300 (189)	C40	5	4	193 (28)	0.31
Monel 400	690 (100)	B95	2	5	179 (26)	0.32
Monel K500	1 200 (174)	C34	40	5	17.9 (26)	0.29
Inconel 600	1 040 (151)	C30	2	2	214 (31)	0.29
Inconel X-750	1 050 (152)	C35	20	3	214 (31)	0.29
Beryllium copper	1 300 (189)	C40	2	5	128 (18.5)	0.33
Ni-Span-C	1 400 (203)	C42	6	2	186 (27)	–
Brass CA 260	620 (90)	B90	3	3	11 (16)	0.33
Phosphor bronze	690 (100)	B90	3	2.5	103 (15	0.20
17-7PH RH950	1 450 (210)	C44	6	flat	203 (29.5)	0.34
17-7PH Cond. C	1 650 (239)	C46	1	2.5	203 (29.5)	0.34

Table 13-4 Coefficients and Exponents for Equation 13.3

Source: Reference 1

ASTM #	Material	Range mm	Range in	Exponent b	Coefficient A MPa	Coefficient A psi	Correlation Factor
A227	Cold drawn	0.5–16	0.020–0.625	–0.182 2	1 753.3	141 040	0.998
A228	Music wire	0.3–6	0.010–0.250	–0.1625	2 153.5	184 649	0.9997
A229	Oil tempered	0.5–16	0.020–0.625	–0.183 3	1 831.2	146 780	0.999
A232	Chrome-v.	0.5–12	0.020–0.500	–0.145 3	1 909.9	173 128	0.998
A401	Chrome-s.	0.8–11	0.031–0.437	–0.093 4	2 059.2	220 779	0.991

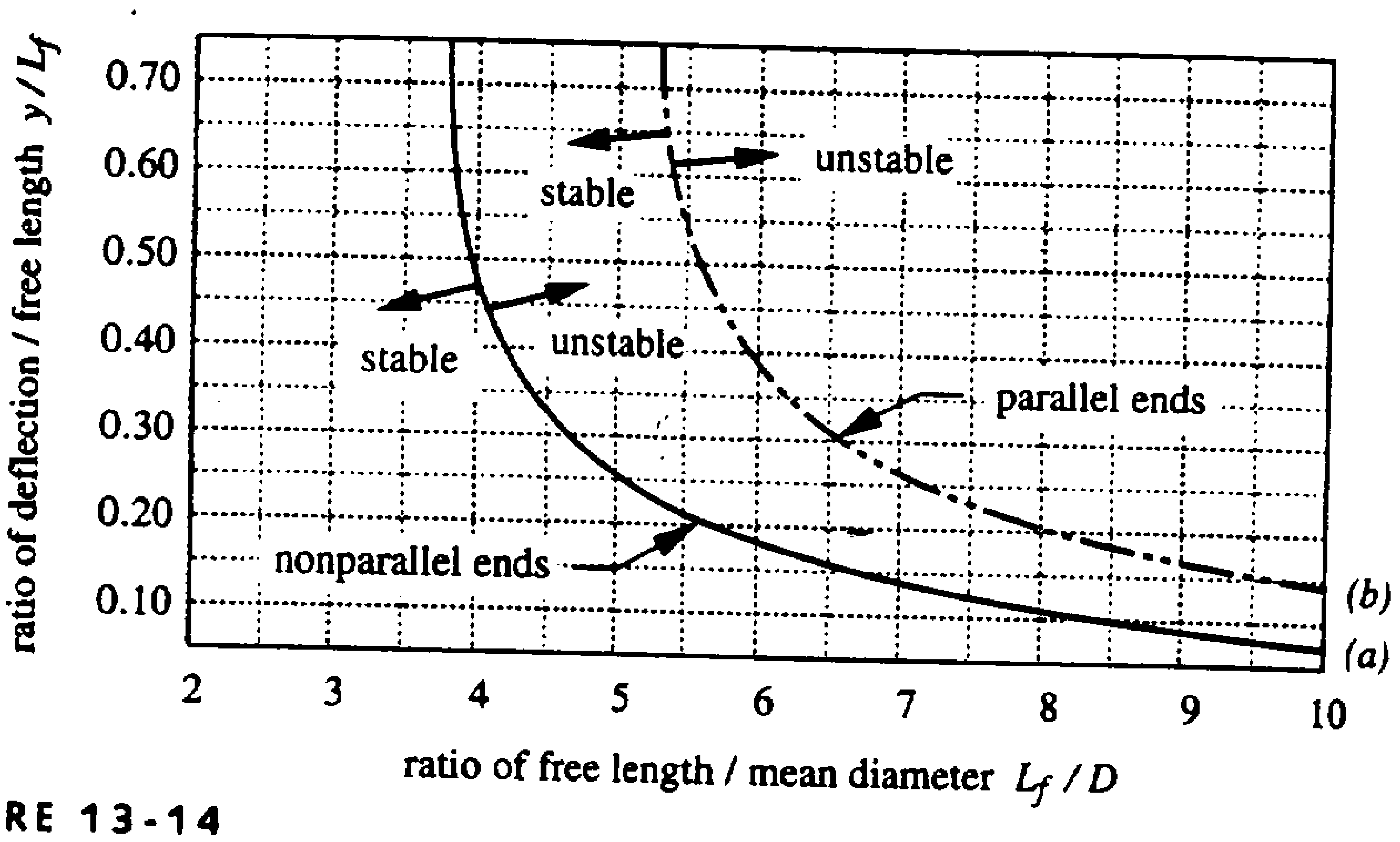


FIGURE 13-14

Critical Buckling Condition Curves Adapted from Reference 1

Table 13-6 Maximum Torsional Yield Strength S_{ys} for Helical Compression Springs in Static Applications

Bending or Buckling Stresses Not Included. Source: Adapted from Ref. 1

Material	Maximum Percent of Ultimate Tensile Strength: Before Set Removed (Use Eq 13.9b)	Maximum Percent of Ultimate Tensile Strength: After Set Removed (Use Eq. 13.8b)
Cold-drawn carbon steel (e.g., A227, A228)	45%	60–70%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	50	65–75
Austenitic stainless steel (e.g., A313)	35	55–65
Nonferrous alloys (e.g., B134, B159, B197)	35	55–65

Important Equations Used in This Chapter

Spring Rate (Section 13.1):

$$k = \frac{F}{y} \tag{13.1}$$

Combining Springs in Parallel (Section 13.1):

$$k_{total} = k_1 + k_2 + k_3 + \ldots + k_n \tag{13.2a}$$

Combining Springs in Series (Section 13.1):

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} \tag{13.2b}$$

Spring Index (Section 13.4):

$$C = \frac{D}{d} \tag{13.5}$$

Deflection of Helical Compression Spring (Section 13.4):

$$y = \frac{8FD^3N_a}{d^4G} \tag{13.6}$$

Deflection of Helical Extension Spring (Section 13.7):

$$y = \frac{8(F - F_i)D^3N_a}{d^4G} \tag{13.22}$$

Deflection of Round-Wire Helical Torsion Spring (Section 13.8):

$$\theta_{rev} \cong 10.2\frac{MDN_a}{d^4E} \qquad \theta_{rev} \cong 10.8\frac{MDN_a}{d^4E} \tag{13.27c}$$

Spring Rate of Helical Compression Spring (Section 13.4):

$$k = \frac{F}{y} = \frac{d^4G}{8D^3N_a} \tag{13.7}$$

Spring Rate of Helical Extension Spring (Section 13.7):

$$k = \frac{F - F_i}{y} = \frac{d^4G}{8D^3N_a} \tag{13.20}$$

Spring Rate of Round-Wire Helical Torsion Spring (Section 13.8):

$$k = \frac{M}{\theta_{rev}} \cong \frac{d^4E}{10.8DN_a} \tag{13.28}$$

Static Stress in Helical Compression or Extension Spring (Section 13.7):

$$\tau_{max} = K_s\frac{8FD}{\pi d^3} \qquad \text{where} \quad K_s = \left(1 + \frac{0.5}{C}\right) \tag{13.8b}$$

Dynamic Stress in Helical Compression or Extension Spring (Section 13.7):

$$K_B = \frac{4C + 2}{4C - 3}$$

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \tag{13.9a}$$

$$\tau_{max} = K_w\frac{8FD}{\pi d^3} \tag{13.9b}$$

Spring Rate of Torsion Springs

The spring rate can always be obtained from the deflection formula:

$$k = \frac{M}{\theta_{rev}} \cong \frac{d^4E}{10.8DN_a}$$

Stress in Helical Torsion Spring at Inside Diameter (Section 13.8):

$$K_{b_i} = \frac{4C^2 - C - 1}{4C(C-1)} \qquad (13.31a)$$

$$\sigma_{i_{max}} = K_{b_i}\frac{M_{max}c}{I} = K_{b_i}\frac{M_{max}(d/2)}{\pi d^4/64} = K_{b_i}\frac{32M_{max}}{\pi d^3} \qquad (13.32a)$$

Stress in Helical Torsion Spring at Outside Diameter (Section 13.8):

$$K_{b_o} = \frac{4C^2 + C - 1}{4C(C+1)} \qquad (13.31b)$$

$$\sigma_{o_{min}} = K_{b_o}\frac{32M_{min}}{\pi d^3}; \qquad \sigma_{o_{max}} = K_{b_o}\frac{32M_{max}}{\pi d^3} \qquad (13.32b)$$

Ultimate Tensile Strength of Steel Wire—See Table 13-4 for Constants (Section 13.4):

$$S_{ut} \cong Ad^b \qquad (13.3)$$

Ultimate Shear Strength of Wire (Section 13.4):

$$S_{us} \cong 0.67S_{ut} \qquad (13.4)$$

Torsional Endurance Limits for Spring-Steel Wire for Stress Ratio R = 0 (Section 13.4):

$$S_{ew}' \cong 45.0 \text{ kpsi } (310 \text{ MPa}) \quad \text{for unpeened springs}$$
$$S_{ew}' \cong 67.5 \text{ kpsi } (465 \text{ MPa}) \quad \text{for peened springs} \qquad (13.12)$$

Torsional Endurance Limits for Spring-Steel Wire for Stress Ratio R = –1 (Section 13.4):

$$S_{es} = 0.5\frac{S_{ew}S_{us}}{S_{us} - 0.5S_{ew}} \qquad (13.17b)$$

Bending Endurance Limits for Spring-Steel Wire for Stress Ratio R = 0 (Section 13.4):

$$S_{ew_b}' = \frac{S_{ew}'}{0.577} \qquad (13.33a)$$

Bending Endurance Limits for Spring-Steel Wire for Stress Ratio R = –1 (Section 13.4):

$$S_e = 0.5\frac{S_{ew_b}S_{ut}}{S_{ut} - 0.5S_{ew_b}} \qquad (13.34c)$$

Static Safety Factor for Helical Compression or Extension Spring (Section 13.5):

$$N_s = \frac{S_{ys}}{\tau} \qquad (13.14)$$

Dynamic Safety Factor for Helical Compression or Extension Spring (Section 13.4):

$$N_{f_s} = \frac{S_{es}(S_{us} - \tau_i)}{S_{es}(\tau_m - \tau_i) + S_{us}\tau_a} \qquad (13.17a)$$

Dynamic Safety Factor for Helical Torsion Spring (Section 13.8):

$$N_{f_b} = \frac{S_e(S_{ut} - \sigma_{o_{min}})}{S_e(\sigma_{o_{mean}} - \sigma_{o_{min}}) + S_{ut}\sigma_{o_{alt}}} \qquad (13.34b)$$

$$N_y = \frac{S_y}{\sigma_{i_{max}}} \qquad (13.34a)$$

$$\sigma_{o_{mean}} = \frac{\sigma_{o_{max}} + \sigma_{o_{min}}}{2}; \qquad \sigma_{o_{alt}} = \frac{\sigma_{o_{max}} - \sigma_{o_{min}}}{2} \qquad (13.32c)$$

Solve for *d* using the static yield criterion.

$$d := \left(\frac{32 \cdot K_{bi} \cdot N_{yd} \cdot M}{\pi \cdot K_s \cdot A \cdot mm^3}\right)^{\frac{1}{3+b}} \cdot mm$$

Number of Coils in Torsion Springs

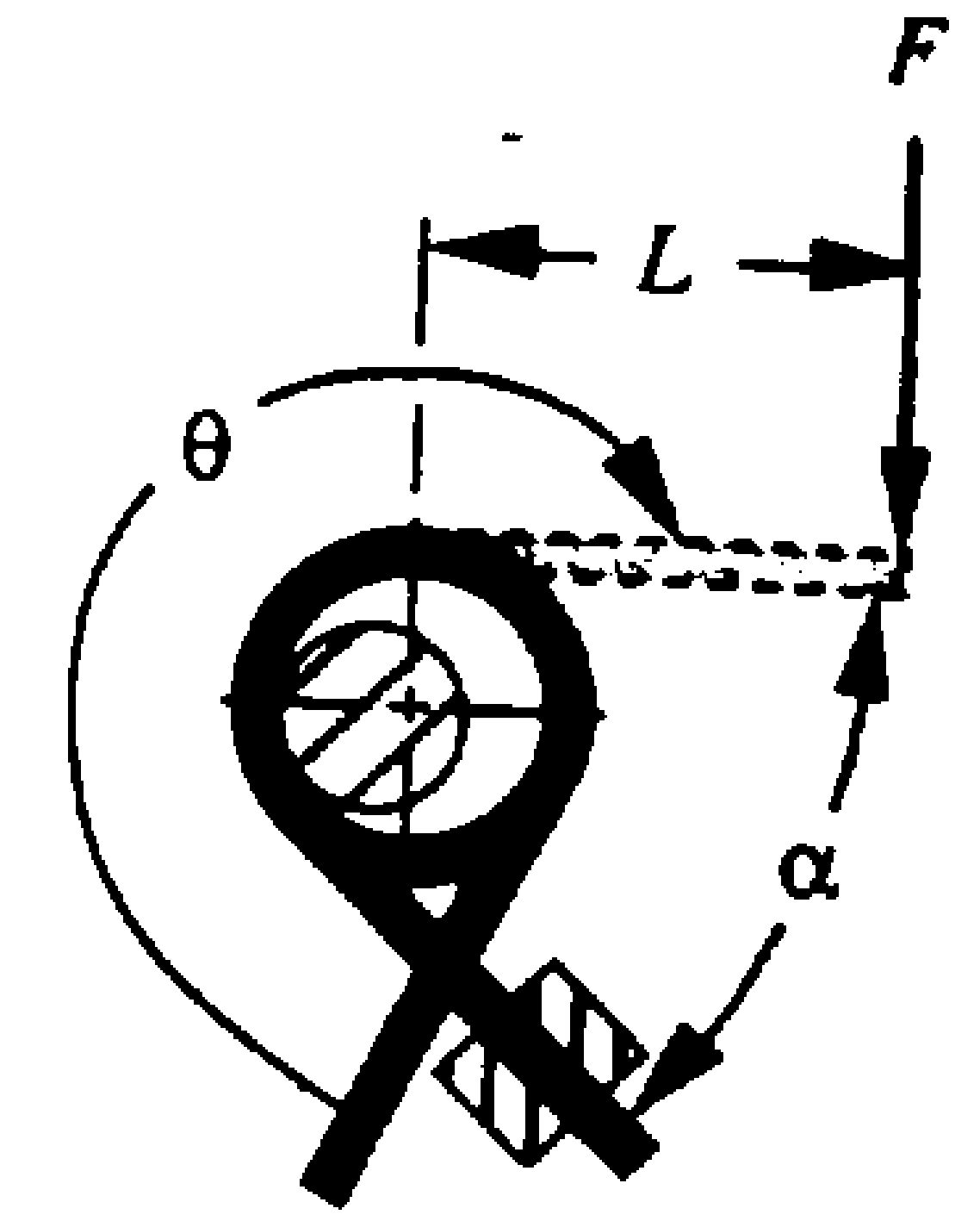
The active coils are equal to the number of body turns N_b plus some contribution from the ends, which also bend. For straight ends, the contribution can be expressed as an equivalent number of coils N_e:

$$N_e = \frac{L_1 + L_2}{3\pi D} \tag{13.26a}$$

where L_1 and L_2 are the respective lengths of the tangent-ends of the coil. The number of active coils is then

$$N_a = N_b + N_e \tag{13.26b}$$

where N_b is the number of coils in the spring body.



free position

specify:
α—angle between ends
F—load on ends at α
L—moment arm
θ—angular deflection from free position

Deflection of Torsion Springs

The angular deflection of the coil-end is normally expressed in radians but is often converted to revolutions. We will use revolutions. Since it is essentially a beam in bending, the (angular) deflection can be expressed as

$$\theta_{rev} = \frac{1}{2\pi}\theta_{rad} = \frac{1}{2\pi}\frac{ML_w}{EI} \tag{13.27a}$$

Coil Closure

When a torsion spring is loaded to close the coils (as it should be), [redacted] The minimum inside coil diameter at full deflection is

$$D_{i_{min}} = \frac{DN_b}{N_b + \theta_{rev}} - d \tag{13.29}$$

where D is the unloaded mean coil diameter. Any pin that the coil works over should be limited to about 90% of this minimum inside diameter.

The maximum coil-body length at full "windup" is:

$$L_{max} = d(N_b + 1 + \theta) \tag{13.30}$$

Table 13-13 Maximum Recommended Bending Yield Strength S_y for Helical Torsion Springs in Static Applications

Source: Adapted from Reference 1

Material	Maximum Percent of Ultimate Tensile Strength Stress Relieved	 Favorable Residual Stress
Cold-drawn carbon steel (e.g., A227, A228)	80%	100%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	85	100
Austenitic stainless steel and nonferrous alloys (e.g., A313, B134, B159, B197)	60	80

The torsional-endurance limit data for helical compression springs shown in equ tion 13.12 (p. 816) can be adapted for bending by using the von Mises relationship b tween torsion and tension loading.

$$S_{ew_b}' = \frac{S_{ew}'}{0.577} \qquad \text{(13.33}$$

which gives

$$S_{ew_b}' \cong \frac{45.0}{0.577} = 78 \text{ kpsi} \quad (537 \text{ MPa}) \quad \text{for unpeened springs}$$

$$S_{ew_b}' \cong \frac{67.5}{0.577} = 117 \text{ kpsi} \quad (806 \text{ MPa}) \quad \text{for peened springs} \qquad \text{(13.33}$$

Table C-1 **Physical Properties of Some Engineering Materials**

Data from Various Sources.* These Properties are Essentially Similar for All Alloys of the Particular Material

Material	Modulus of Elasticity E		Modulus of Rigidity G		Poisson's Ratio ν	Weight Density γ	Mass Density ρ	Specific Gravity
	Mpsi	GPa	Mpsi	GPa		lb/in³	Mg/m³	
Aluminum Alloys	10.4	71.7	3.9	26.8	0.34	0.10	2.8	2.8
Beryllium Copper	18.5	127.6	7.2	49.4	0.29	0.30	8.3	8.3
Brass, Bronze	16.0	110.3	6.0	41.5	0.33	0.31	8.6	8.6
Copper	17.5	120.7	6.5	44.7	0.35	0.32	8.9	8.9
Iron, Cast, Gray	15.0	103.4	5.9	40.4	0.28	0.26	7.2	7.2
Iron, Cast, Ductile	24.5	168.9	9.4	65.0	0.30	0.25	6.9	6.9
Iron, Cast, Malleable	25.0	172.4	9.6	66.3	0.30	0.26	7.3	7.3
Magnesium Alloys	6.5	44.8	2.4	16.8	0.33	0.07	1.8	1.8
Nickel Alloys	30.0	206.8	11.5	79.6	0.30	0.30	8.3	8.3
Steel, Carbon	30.0	206.8	11.7	80.8	0.28	0.28	7.8	7.8
Steel, Alloys	30.0	206.8	11.7	80.8	0.28	0.28	7.8	7.8
Steel, Stainless	27.5	189.6	10.7	74.1	0.28	0.28	7.8	7.8
Titanium Alloys	16.5	113.8	6.2	42.4	0.34	0.16	4.4	4.4
Zinc Alloys	12.0	82.7	4.5	31.1	0.33	0.24	6.6	6.6

* *Properties of Some Metals and Alloys*, International Nickel Co., Inc., N.Y.; *Metals Handbook*, American Society for Metals, Materials Park, Ohio.

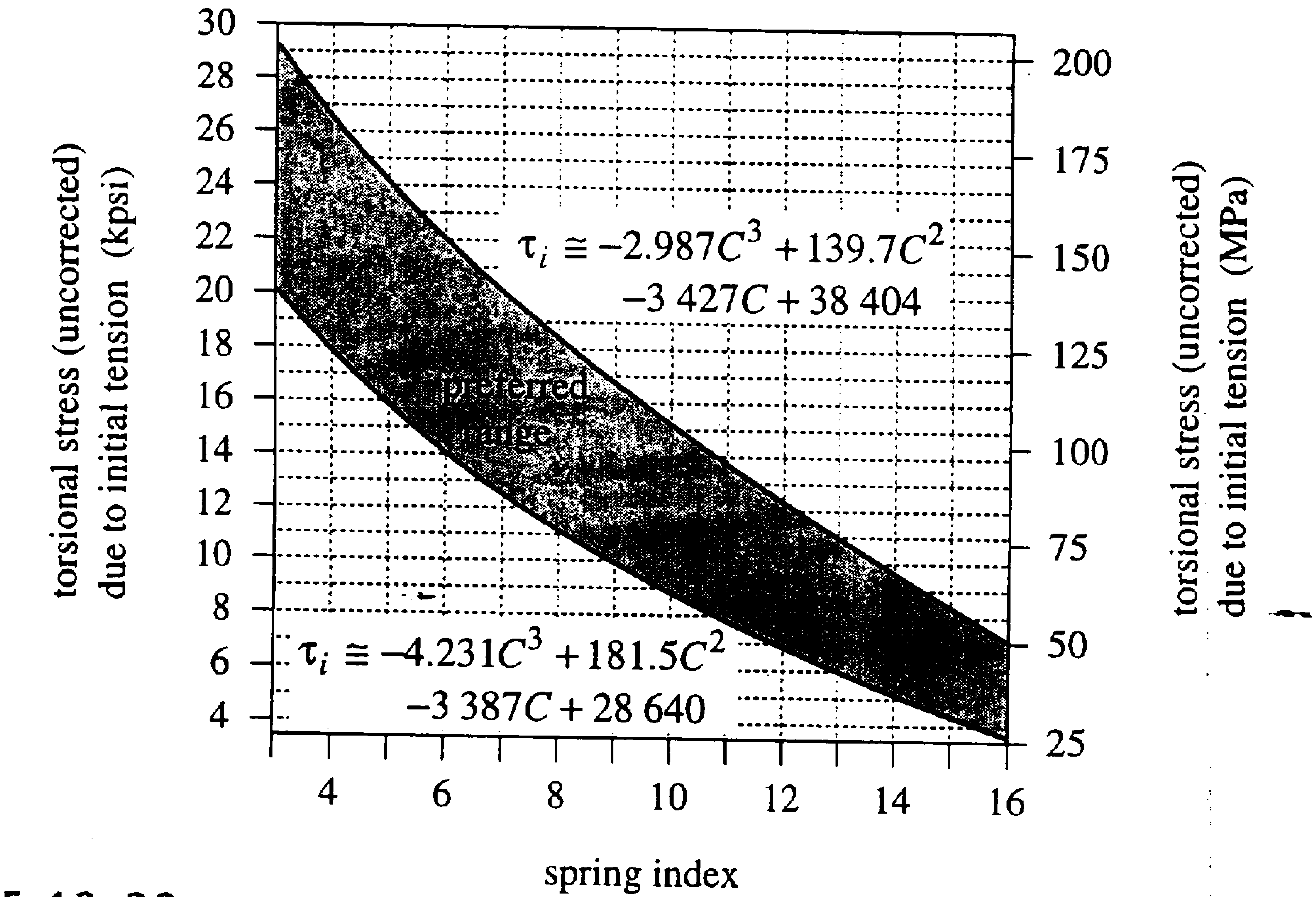


FIGURE 13-22

Preferred Range of Initial Stress in Extension Springs as a Function of Spring Index