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GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Fall Quarter 1995**

COMPUTER AIDED ENGINEERING (CAE)

EXAM AREA

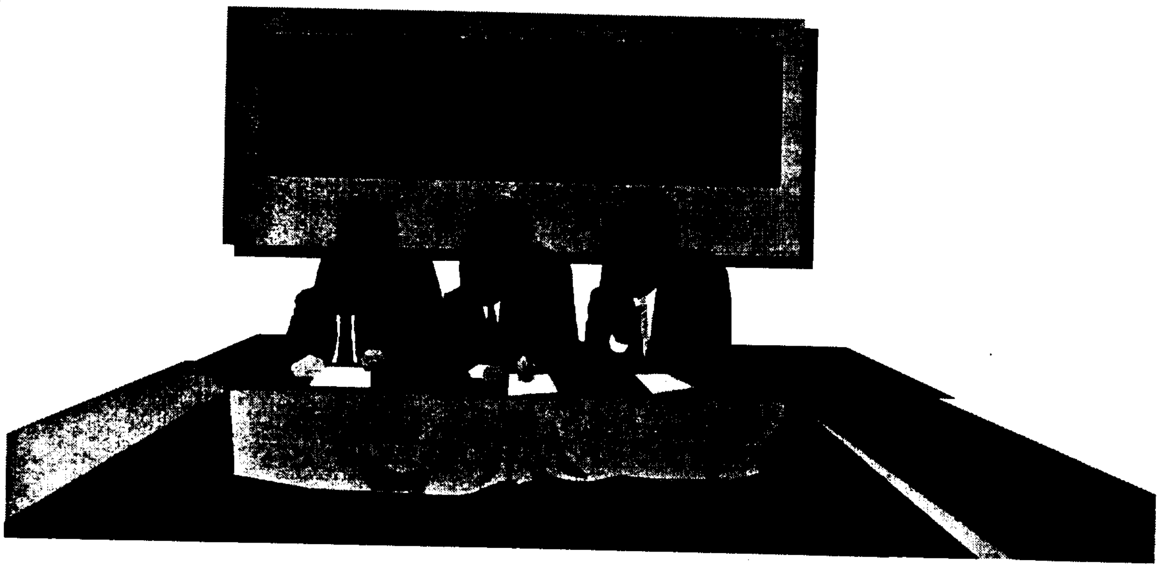
Assigned Number **(DO NOT SIGN YOUR NAME)**

**-- Please sign your name on the back of this page --**

# *COMPUTER-AIDED ENGINEERING*
## *Ph.D. QUALIFIER EXAM - FALL 1995*

***THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.***
***GEORGIA INSTITUTE OF TECHNOLOGY***
***ATLANTA, GA 30332-0405***

***Bras, Rosen, and Sitaraman (Chair)***



- All questions in this exam have a common theme: design of ***Soccer Goal Post and Net***. The questions focus on different CAE aspects, as applicable to Soccer Goal Post and Net.
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

***GOOD LUCK!***

***QUESTION #1 - AN EXTERNAL FORCE ON THE GOAL***

There is a European saying "Soccer is war" and the Olympic Committee is afraid that soccer riots may occur. Among others, there is a fear for damage to the soccer goals. Many calculations are to be made, but here we will only focus on one particular situation.

In Figure 1, a side view is shown of a soccer goal resting on its back. ACOG assumes that this can occur not only when angry fans flip the goal, but also during transport and installation. ACOG also envisions a scenario where a fan may be hanging from the lying soccer goal and exerting a force of 1000 Newton, as indicated in Figure 1. It is assumed that one of the contact points on the ground is locked in place (point 2) and that the other (point 3) allows horizontal movement.

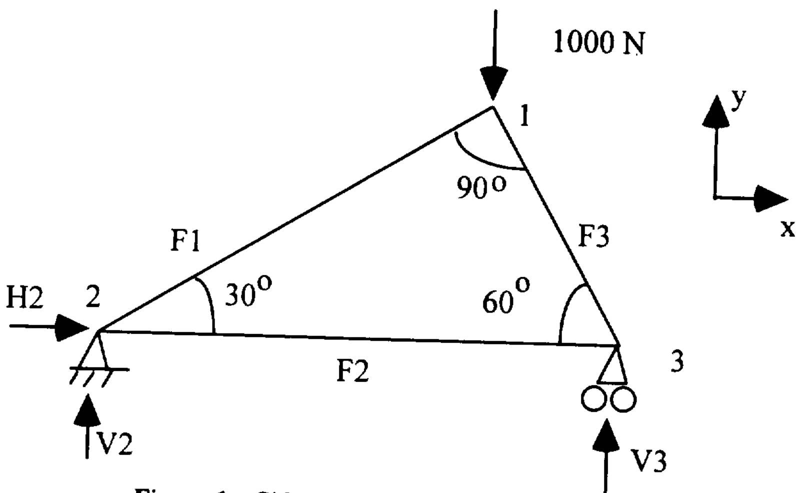



**Figure 1 – Sideview of Soccer Goal on its Back**

ACOG is interested in finding the stresses, but first the forces and reactions in the members have to be determined. H2 and V2 are the vertical forces in point 2, V3 the vertical force in point 3. F1, F2, and F3 are the forces acting in each of the three side members of the goal. Their direction is not shown in Figure 1. Finding the forces in the members and the reactions in points 2 and 3 is a problem with six equations and six unknowns. Below, this system is shown for the situation in Figure 1.

$$\begin{bmatrix} 0.866 & 0 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.866 & 0 & 0 & 0 \\ -0.866 & -1 & 0 & -1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.866 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} F1 \\ F2 \\ F3 \\ H2 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

a) Solve this system using a Gaussian elimination scheme.

b) Explain what is meant by complete and partial pivoting and why it is done.

c) What are some other methods for solving the above system of equations and discuss their pros and cons.

d) Because we are dealing with a design situation where the load of 1000 N may be changing. What method(s) would you recommend to solve the above system and at the same time minimize the number of unnecessary recalculations in case the external loads are changing?

***QUESTION #2 - FINITE-ELEMENT MODELING OF GOAL UNDER WINDY CONDITIONS***

The Figure below shows the soccer goal in its upright position. Note that all joints are pin joints in the structure.

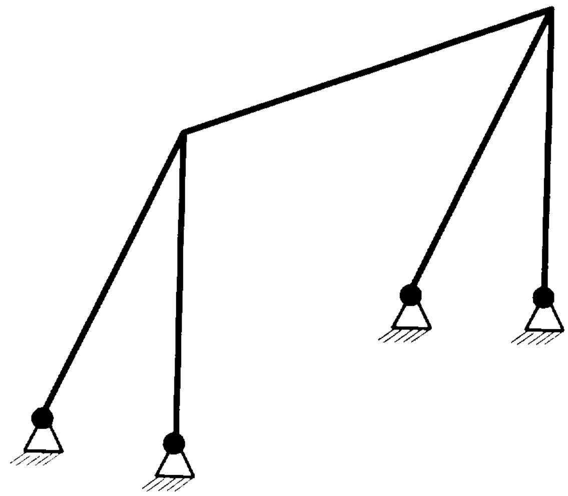


***Figure 2 - Soccer Goal Structure***

Figure below shows the end view of the goal structure. The bar that connects point 1 and 2 has a cross-section area of 12 sq. in. The bar that connects point 2 and 3 is 15 ft. long and has a cross-section area of 24 sq. in. The angle between the two bars is $30^0$.

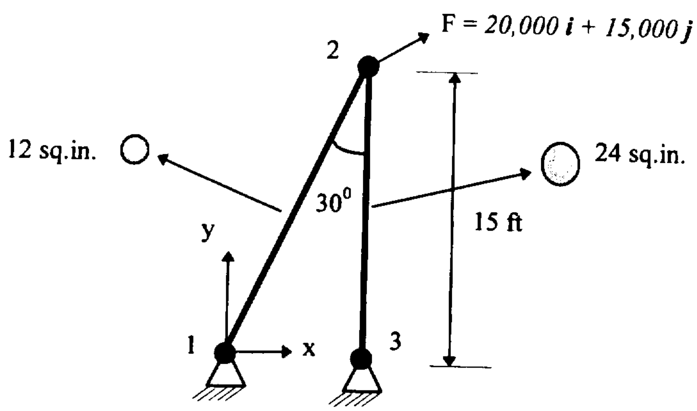



***Figure 3 - End View***

All bars are made of the same material with $E = 30X10^6$ psi. You are told that during windy days, the goal post may be pushed down to the side and may be pulled off the ground. It is estimated that such winds will exert a force whose $x$ component is *20,000 lb* and $y$ component is *15,000 lb* at point 2 as shown in Figure 3.

You are asked to analyze Figure 3 using Finite-Elements.

a) Select among the following finite elements, an appropriate element or elements to model the geometry

1. Contact Element
2. Rod or Truss Element
3. Beam Element
4. Triangular Shell Element
5. Brick Element

b) Assemble and show the global stiffness matrix, starting with element stiffness matrix.

c) Determine displacements at point 2.

d) Compute stresses in the bar connecting point 2 and 3.

e) If the bars have a rectangular cross-section with the same area, rather than a circular cross-section, will the displacements and stresses obtained in (c) and (d) change? Justify your answer.

***Element Stiffness Matrix for Bars Shown in Figure 3***

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l$ and $m$ are direction cosines of the element with respect to $X$ and $Y$ axes and are given by:

$$l = \frac{x_2 - x_1}{L}$$

$$m = \frac{y_2 - y_1}{L}$$

***QUESTION # 3 - MODELING THE SOCCER NET***

**Problem**

In order to make a fairly realistic CAD model of the soccer net and to support analysis of net tension, a surface model of the net is required. The soccer net is to be modeled using Bezier curves and surfaces. Your problem is to develop a (simplified) **Bezier curve model** of the net.

**Given**

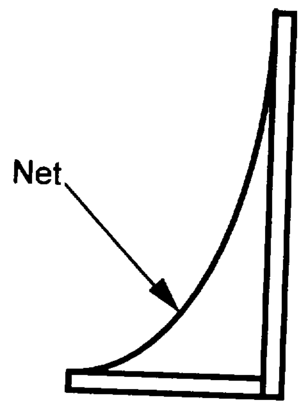
Equations for Bezier curves are

$$b(u) = \sum_{j=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \binom{n}{i} u^{i}(1-u)^{n-i}$$

where: **P** are the control vertices that define the Bezier curve.

Side view of soccer goal




The general form of the catenary equation is:

$$y = a \cdot \cosh\left(\frac{x - x_0}{a}\right) + y_0$$

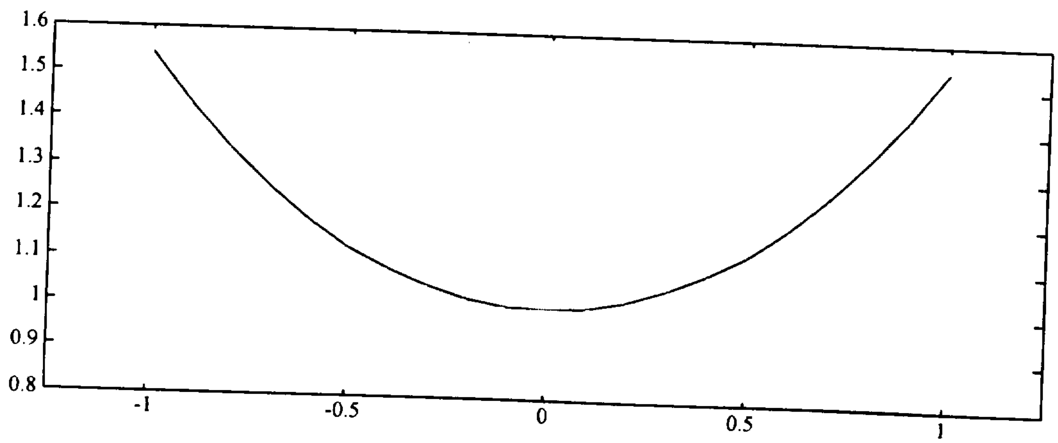
where: $a$ is a constant that controls the shape of the catenary,

$x_0$, $y_0$ are coordinates that enable the catenary to be translated.

For your reference, the derivative of cosh(x) is:

$$\frac{dy}{dx}(\cosh(x)) = \sinh(x)$$

The graph below is of $y = \cosh(x)$.

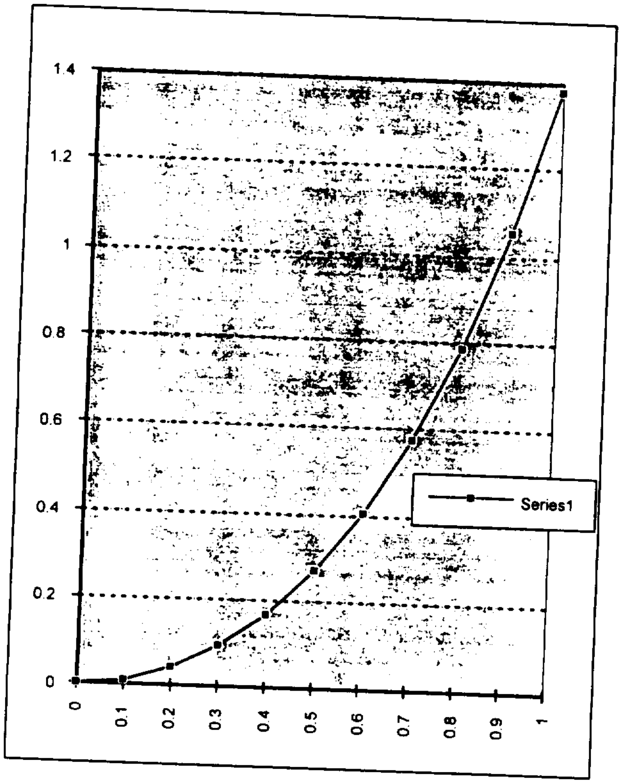


**Questions**

a) Derive the equation of a cubic Bezier curve that matches as closely as possible the catenary curve shown below. The graph is to scale; plotted points are also given. Derive an expression for the accuracy of your Bezier curve. Plot your Bezier curve. The equation of the catenary is:

$$y = 0.5\cosh\left(\frac{x}{0.5}\right) - 0.5$$

Describe the procedure that you are taking to derive your Bezier curve. Please **describe** and **explain** any problems that you may encounter.




| X | Y |
|---|---|
| 0 | 0 |
| 0.1 | 0.01003338 |
| 0.2 | 0.04053619 |
| 0.3 | 0.09273261 |
| 0.4 | 0.16871747 |
| 0.5 | 0.27154032 |
| 0.6 | 0.40532778 |
| 0.7 | 0.57544923 |
| 0.8 | 0.78873224 |
| 0.9 | 1.05373659 |
| 1 | 1.38109785 |

b) **List** and **explain** at least **two alternative approaches** to more accurately model the catenary curve.

c) Describe the pros and cons of basing modern CAD systems on Bezier curves and surfaces.

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**Ph.D. Qualifiers Exam - Fall Quarter 1995**

Design
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Assigned Number **(DO NOT SIGN YOUR NAME)**

**-- Please sign your name on the back of this page --**

# Ph.D. Qualifying Examination in Design

## Fall 1995

George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

Written Examination:

Attempt all parts of the question. Be sure to list and justify all of your assumptions explicitly. Read the entire exam first.

The Olympics will be held in Atlanta next year. Bicycle racing will be one of the events, held on the streets of Atlanta and in the Stone Mountain velodrome, and will be very widely followed. In the past, bicycles have been defined as having a tubular frame structure (see figure 1). A new monocoque structure has been developed that greatly reduces weight and wind resistance (see figure 2). Both of these designs still rely on chain drives. Chain drives are reliable, easy to manufacture and to repair. But they are dirty, messy, increase wind resistance and are prone to catch clothing and bodies.

Your job is to develop an alternative means of converting the rotational motion that the rider develops at the pedals into rotational motion of the rear wheel that does NOT utilize a chain. For the sake of this exercise, assume that you are designing for a bicycle with a fixed ratio between the angular velocity of the pedals and the rear wheel. Your design should be light, reduce wind resistance, and improve the performance of the cyclist.

Your design must withstand the rigors of Olympic training schedules and events. As national pride is of paramount concern, cost is no object, but the final design must be realizable (manufacturable and assemblable). As repairs are not feasible in the time frame of an Olympic event, the mechanism you design should be replaceable with no tools in a minimum amount of time (i.e., a time goal of zero seconds).

*QUESTIONS:*

This qualifier is aimed at assessing what you know about design methods, physical realizability and analysis. The questions have been designed to give you an opportunity to show what you know about these issues. Please attempt to put your answers in the context of material covered in ME3110, ME4180 and (ME6170). ME6170 is in parentheses because it is a graduate class, so it is optional.

a) First, we would like you to develop no less than five and no more than ten concepts for a mechanism that converts pedal power to wheel motion. Spend no more than 15 minutes on this phase. (Place a star next to each of the five to ten ideas you want us to evaluate in our grading.)

b) Describe the process you would have followed if you had sufficient time to identify the "most likely to succeed" candidate designs. Identify the "most-likely-to-succeed" candidate designs. To save time, of these five to ten candidate designs, pick the three which appear most promising to you and present a brief (less than one page) feasibility and/or performance evaluation of each. Justify your choices.

c) Choose one of the three candidates and perform a more detailed feasibility study. Present a model and a function structure of the candidate. Determine design specifications for the components, including those based on performance requirements (which you MUST set and express CLEARLY on your answer sheet). Indicate the importance or weight you attach to each specification.

d) Based on your feasibility study, present an overall evaluation of the candidate design chosen for part c.

e) As you have done such a good job on your design, the Olympic committee wishes to sell it to the weekend enthusiast to pay off the debt from staging the games. Discuss the changes in your design necessary to mass produce it cheaply.

f) Critically evaluate the design process embodied in steps "a" through "e".

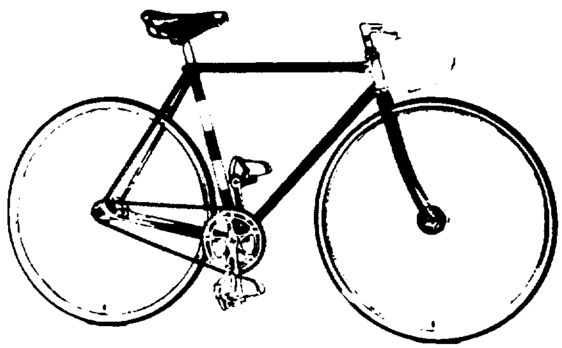


FIG 1

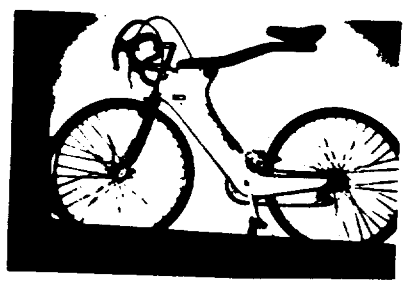
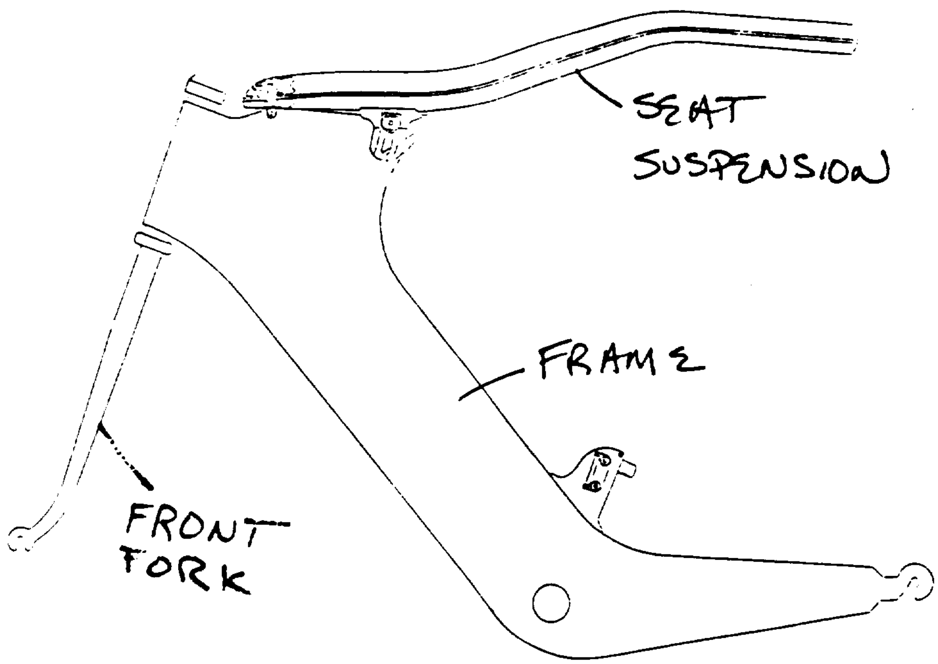
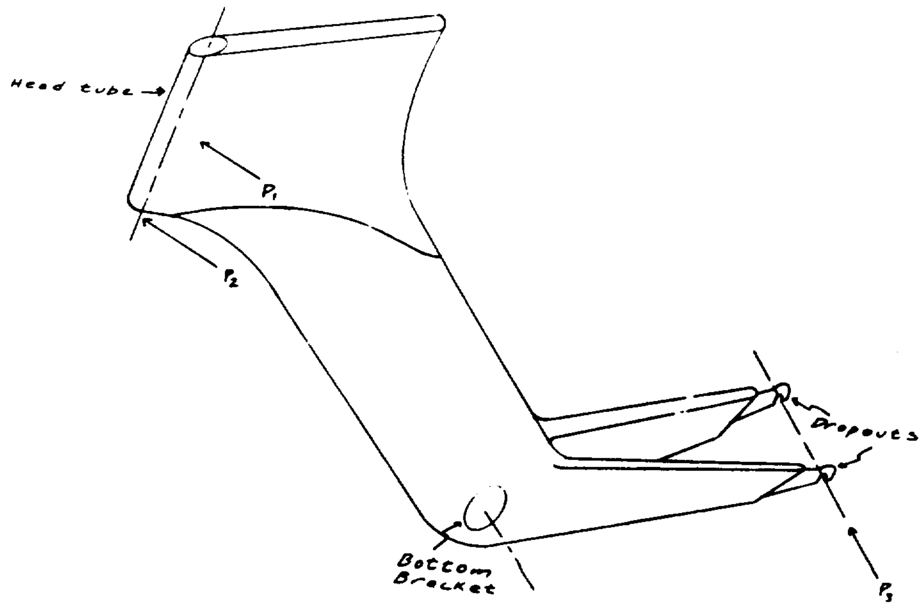


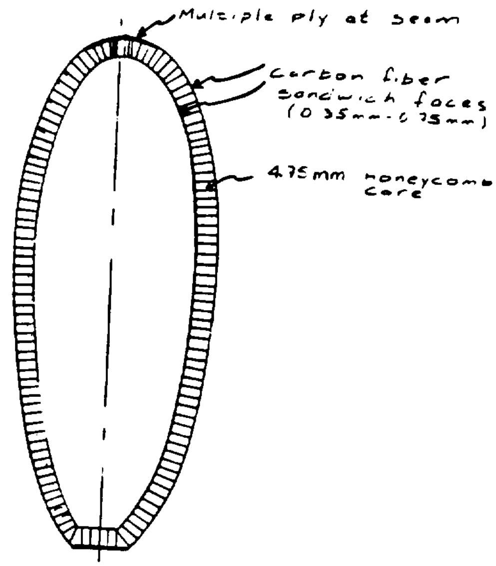
FIG 2



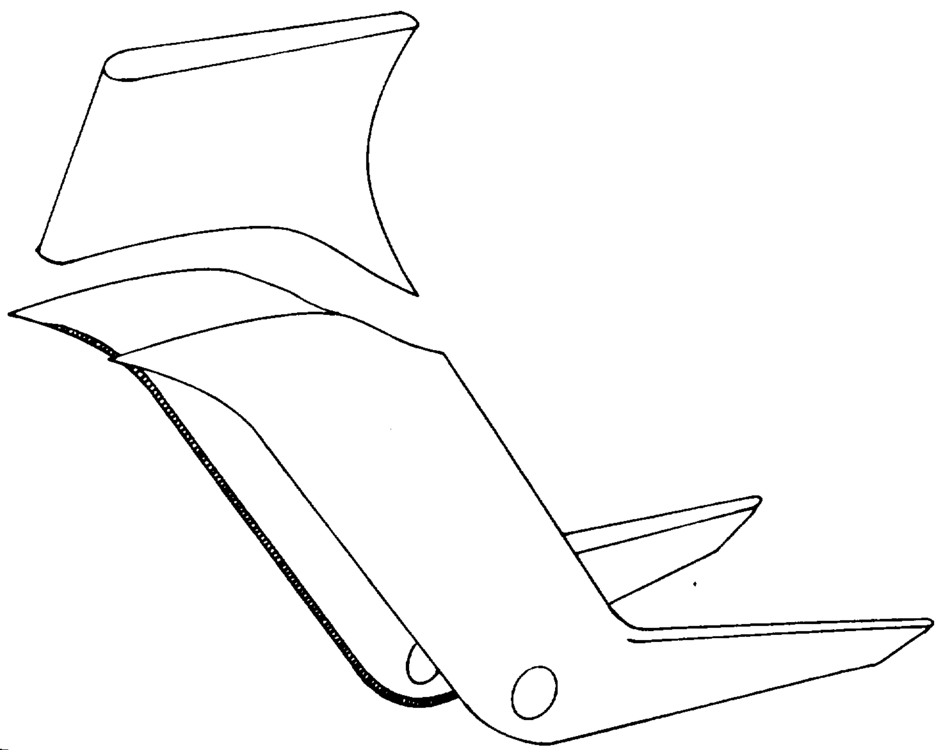

SIDE VIEW




OBLIQUE VIEW




**Frame X-Section**



EXPLODED VIEW