GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1998

Computer Aided Engineering
EXAM AREA

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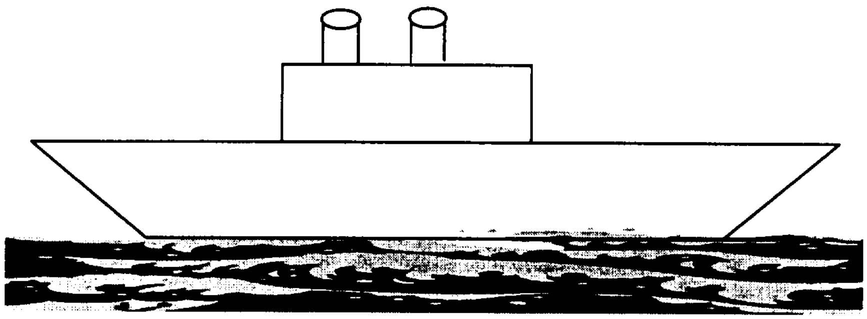
Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

GEORGIA INSTITUTE OF TECHNOLOGY
GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING

COMPUTER-AIDED ENGINEERING
Ph.D. Qualifying Exam
Fall 1998

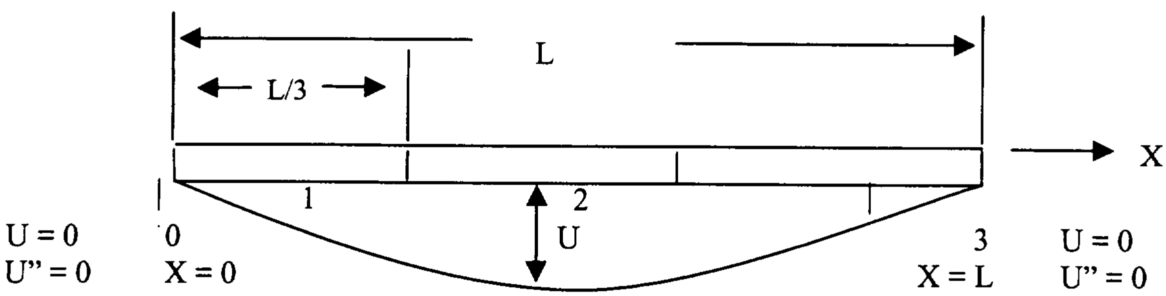
You are part of a team involved in the analysis and design of a large ship such as an ocean liner or battleship.



The following problems represent CAE based problems which are to be investigated as a part of the design.

Numerical Methods Problem

1. You are asked to estimate the first and second natural frequency of the ship sitting atop two waves located at each end. The ship is modeled as a long beam with constant bending stiffness EI, variable mass and simply supported at the ends. The governing vibration equation and boundary conditions are shown.



$EIU^{IV} - M(X)\, w^2 U = 0$ $\qquad$ $h = L/3$

Where $M(X) = M_0\,(X/L)^2\,(1 - X/L)$ is the mass per unit of length, U is the displacement of the beam,

$U'' = d^2U/dX^2$, and w is the natural frequency

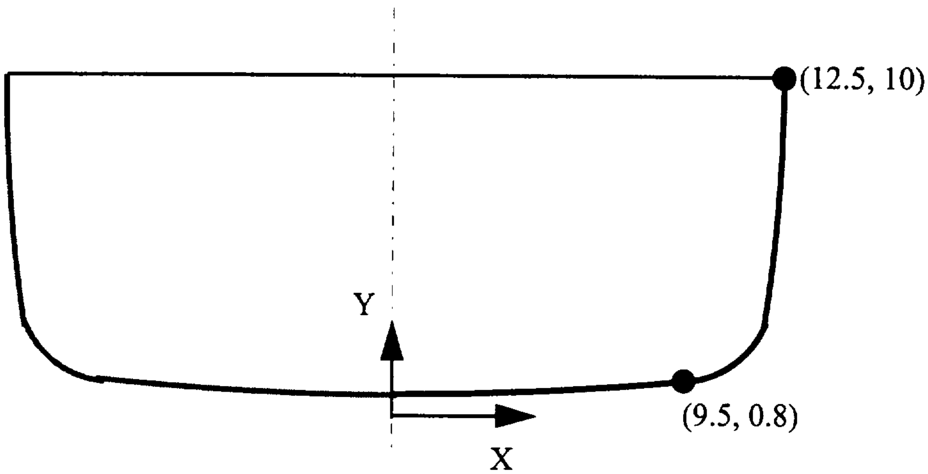
(a) Use finite differences with each mesh segment of length L/3 to approximate the differential equation where $U^{IV}_i = 1/h^4\,[U_{i-2} - 4U_{i-1} + 6U_i - 4U_{i+1} + U_{i+2}]$

(b) Then solve for the lowest frequency and mode shape by the inverse power method. Only go through 2-3 iterations.

(c) Finally, obtain the second mode and frequency by orthogonality.

Geometry Problem

2. For this problem, we will investigate the shape of a ship's hull using the sketch below. It shows a transverse cross-section through the hull. Ships' hulls are symmetric about their center-line. In order to save time and simplify the construction of complex shapes, ship designers design one side of the ship, then mirror that side to form the entire hull. The questions for this problem are related to the geometric modeling of this hull cross-section.



(dimensions are in meters)

a First, sketch the above cross-section. On the sketch, indicate how you would use a composite cubic Bezier curve to model the shape of the right-hand side of the hull. Use 3 cubic segments in your composite Bezier curve. Make sure the resultant composite curve passes through the 3 indicated points. Draw in the control vertices and control polygons. Label points where curves join.

b Describe how you modeled continuity between curve segments. How did you choose the location of the start/end points of each curve?

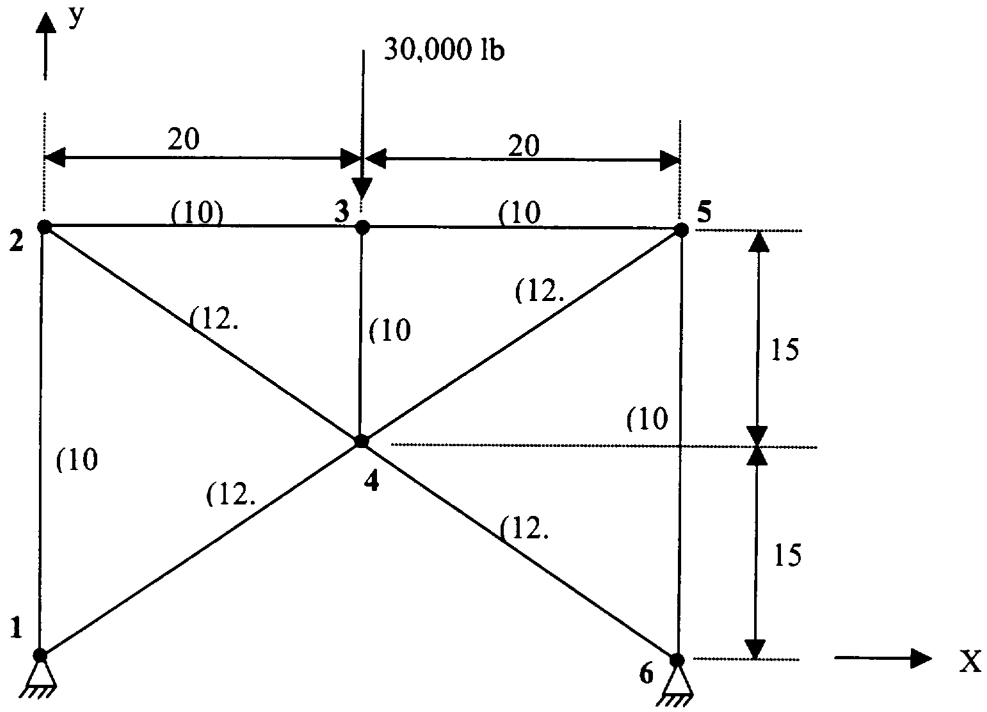
c Show mathematically how you can transform (mirror) the curves' control polygons in order to generate the left-hand side of the hull.

d Using the indicated vertex coordinates, compute the coordinates for the corresponding points on the hull's left side. Do your calculated points make sense? Why or why not?

e Discuss the advantages and disadvantages of using cubic Bezier curves, cubic splines, and B-splines for modeling this hull shape. What type of curve seems most appropriate and why?

Finite-Element Problem

3. The Figure below shows a structure within the ship. All members shown in the figure are truss or rod members. Cross-sectional areas in square inches are shown in parentheses and the node numbers are shown in bold.



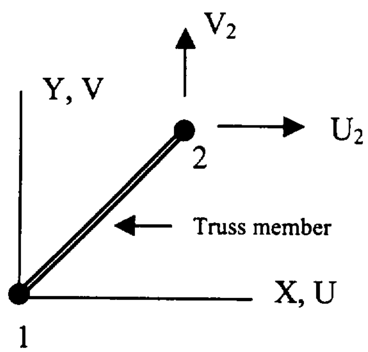
All members are made of the same material with E = $30X10^6$ psi.

Consider symmetry and analyze accordingly using finite elements.

a) Determine the stiffness matrix for individual members.
b) Assemble and show the global stiffness matrix.
c) Determine loading and boundary conditions for the nodes.
d) Solve for displacement at node 4.

Element Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$



where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect to X and Y axes and are given by:

$$l=\frac{x_2-x_1}{L}$$

$$m=\frac{y_2-y_1}{L}$$

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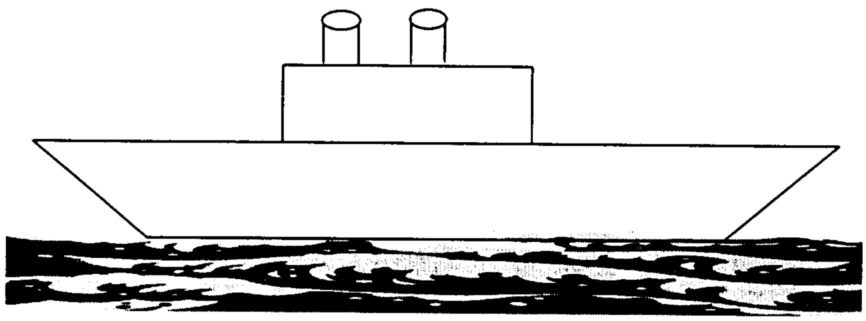

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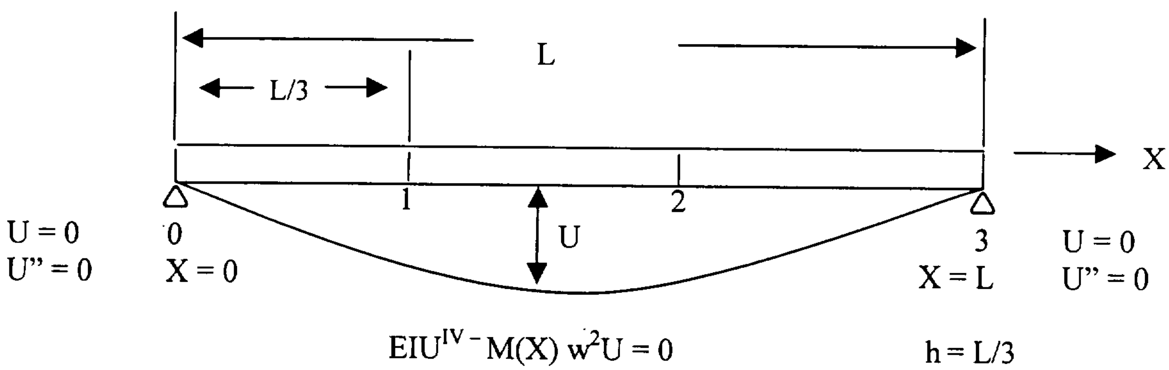

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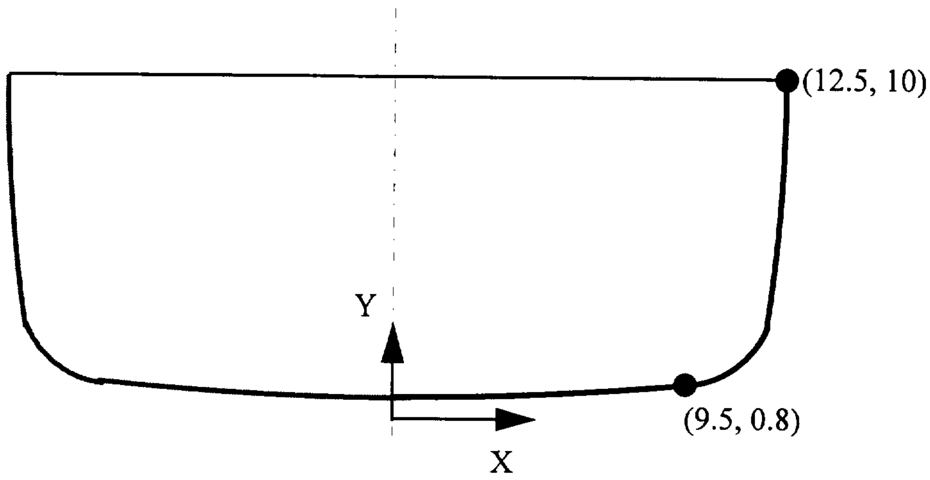

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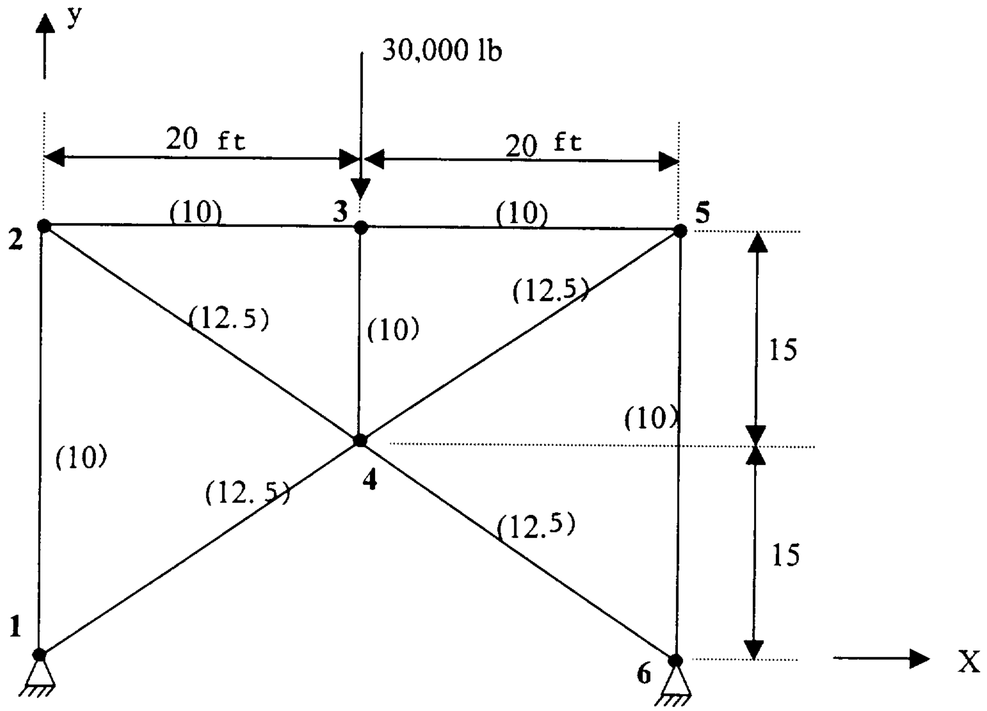

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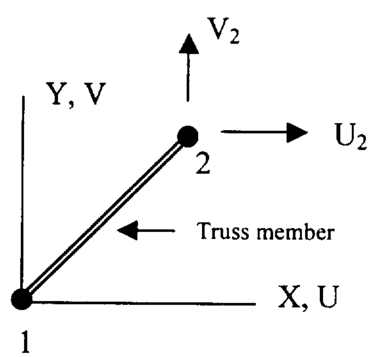

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