COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2009

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- All questions have a common theme: ***Protection against Improvised Explosive Devices (IEDs)***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

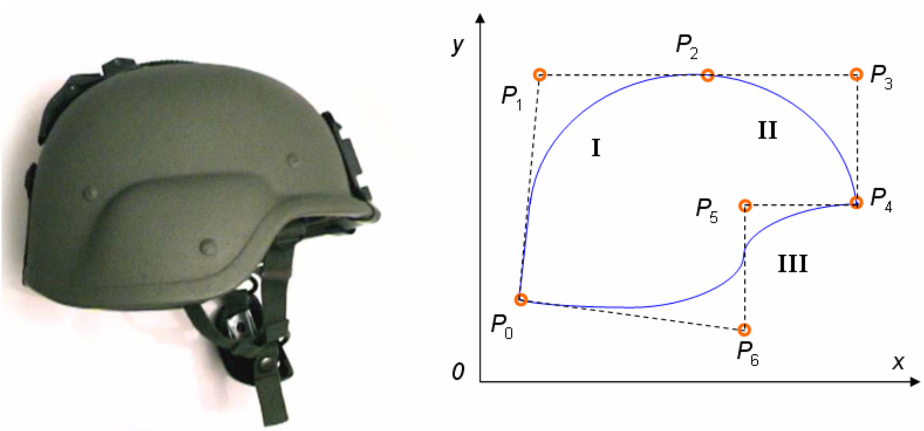
GOOD LUCK!

Question 1:
Geometric Modeling

The US Naval Research Laboratory assigned you a task to develop a robust lightweight helmet to provide protection against fragmentation from improvised explosive devices (IEDs). Assume that the shape of the helmet can be modeled using three Bezier curves, as shown in Figure 1. The coordinates of the control vertices (CVs) are:

Curve I: <P_0, P_1, P_2>, Curve II: <P_2, P_3, P_4>, Curve III: <P_4, P_5, P_6, P_0>

CV	P_o	P_1	P_2	P_3	P_4	P_5	P_6
(x, y)	(1, 2)	(a, 6)	(7, 7)	(8, b)	(8, 4)	(6, 4)	(6, 1)



Bezier curve equations:

$$b(u) = \sum_{i=0}^{n} \underset{\sim}{p}_i B_{i,n}(u)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Figure 1. Fragmentation Helmet

1) Compute the three Bezier curves.

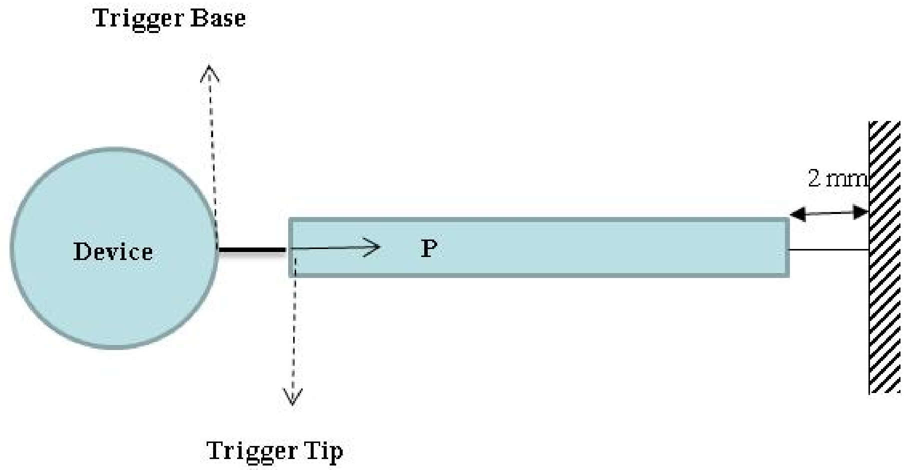
2) Derive the condition for the composite curve to have G^1 continuity at the common CV (P_2). Provide detailed procedures.

3) If we move P_1 to (5,6), what is the value of *b* (y coordinate of P_3) that is needed to satisfy G^1 continuity? If we move P_4 to (7, 4), would Curves I and II still have G^1 continuity?

4) Based on the result from a shape optimization for maximizing the protection effect, you identified that Curve III should be translate by (2,2) and then rotated by 30^o counter-clockwise about the translated coordinate system (relative transformations). Develop the transformation matrix for the translation and rotation. Apply the identified transformation matrix for the Curve III. Sketch the original and transformed Curve III.

Question 2 – Finite Element Analysis

An improvised spherical explosive device has a pull-out trigger. The trigger has an axial stiffness of 50 N/mm. An axial rod with a length of 30 mm and a cross-section area of 25 mm^2 is attached to the tip of the trigger as shown in the figure. The rod is suspended at the other end with a string of length 2 mm attached to a rigid surface. The rod is made of material with a modulus of elasticity of 200 GPa. A remotely activated force P = 300 sin $\pi t/720$ N where t is the time in seconds is applied on the rod. For the sake of simplicity, assume that the force is applied at the tip of the trigger, as shown in the figure. An axial pulling force of 150 N is required at the base of the trigger to detonate the device. Using finite-element calculations, determine at what time the device will explode.



1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down element stiffness matrix and assembly stiffness matrix.
5. Determine how the reaction force at the trigger base will change, when the time t varies from 0 to 360 s – in particular, when the applied force is 150 N and the when the applied force is 300 N. Thus determine at what time the device will explode.

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

$$[K] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

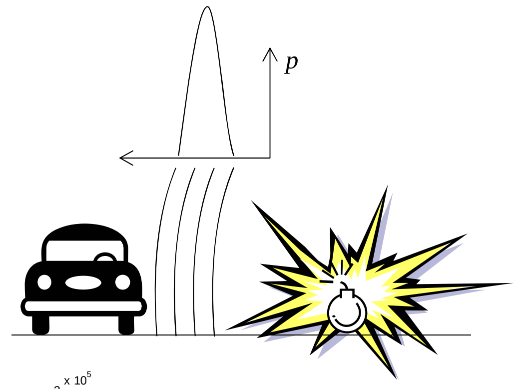
where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect to X and Y axes.

Element B - Stiffness Matrix

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where E, I, and h are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Problem 3: Numerical Methods



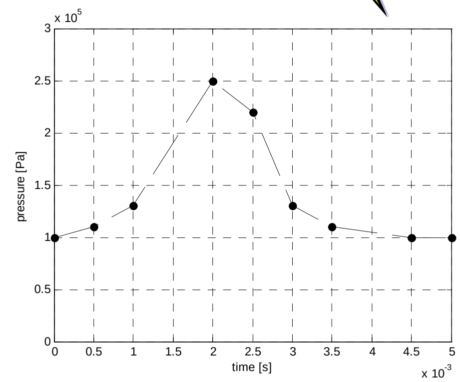
To assess the impact of an IED explosion on nearby vehicles, the pressure wave of a typical IED explosion has been measured; the data are shown in the graph on the right and in the table below. As a first, simplified approximation, you have decided to apply the impulse-momentum theorem to determine the speed of a parked car after the shock wave has passed. You can use the following equations:

$$I = \int_{t_1}^{t_2} F(t)dt \qquad (1)$$

$$m\Delta v = I \qquad (2)$$

$$F(t) = (p(t) - p_{atm})A \qquad (3)$$

where I is the impulse, F is the force acting on the vehicle, p is the pressure in the shock wave as experienced by the car, A is the cross-sectional area of the car, m is the mass of the car, and Δv is the change in velocity of the car. Given the data in the table below, compute the velocity of the car after the shockwave has passed it. Additional data: *m = 1200kg*, p_{atm} *=100kPa, A=4.357m²*



Time [ms]	0	0.5	1	2	2.5	3	3.5	4.5	5
Pressure [MPa]	0.1	0.11	0.13	0.25	0.22	0.13	0.11	0.1	0.1

3.1) Compute the final velocity of the car as accurately as possible with the given information.

3.2) What are the mathematical approximations you made in the answer to the previous question? Are these approximations reasonable?

3.3) Given that there are 9 data points, one could fit a 8th order polynomial exactly, and then integrate the polynomial analytically to obtain the impulse. Would this method be more or less accurate than the method you used in your answer to question 3.1? Explain.

Equations:

$$I = \int_a^b f(x)dx \cong \int_a^b f_n(x)dx$$

$$I \cong (b-a)\frac{f(a)+f(b)}{2} \qquad E_t = -\frac{(b-a)^3}{12} f''(\xi)$$

$$I \cong (b-a)\frac{f(x_0)+2\sum_{i=1}^{n-1} f(x_i)+f(x_n)}{2n} \qquad E_t = -\frac{(b-a)^3}{12n^2}\bar{f}''$$

$$I \cong (b-a)\frac{f(x_0)+4f(x_1)+f(x_2)}{6} \qquad E_t = -\frac{(b-a)^5}{2880} f^{(4)}(\xi)$$

$$I \cong (b-a)\frac{f(x_0)+4\sum_{i=1,3,5}^{n-1} f(x_i)+2\sum_{j=2,4,6}^{n-2} f(x_j)+f(x_n)}{3n} \qquad E_t = -\frac{(b-a)^5}{180n^4}\bar{f}^{(4)}$$

$$I \cong (b-a)\frac{f(x_0)+3f(x_1)+3f(x_2)+f(x_3)}{8} \qquad E_t = -\frac{(b-a)^5}{6480} f^{(4)}(\xi)$$