GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

# Ph.D. Qualifiers Exam - Spring Quarter 1995

COMPUTER AIDED ENGINEERING
EXAM AREA

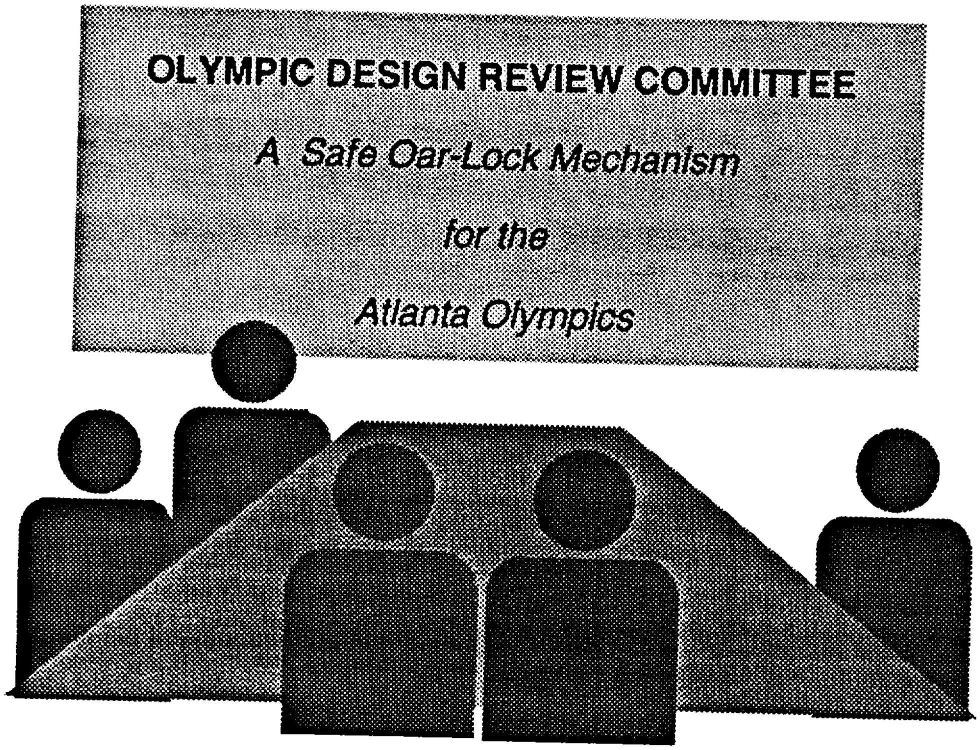
Assigned Number **(DO NOT SIGN YOUR NAME)**

**-- Please sign your name on the back of this page --**

# SCHOOL OF MECHANICAL ENGINEERING
# QUALIFIER SPRING 1995

## COMPUTER-AIDED ENGINEERING

**Bras, Fulton, Mistree and Sitaraman**



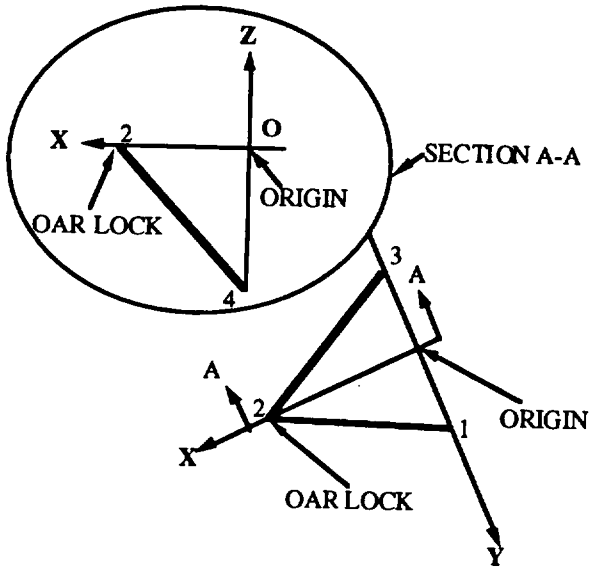

**THIS EXAM** We are interested in learning what you know and your ability to reason. If for some reason you do not follow the question or are confused kindly adjust my question suitably and proceed with your answer. Do let us know the adjustments/ assumptions you have made.

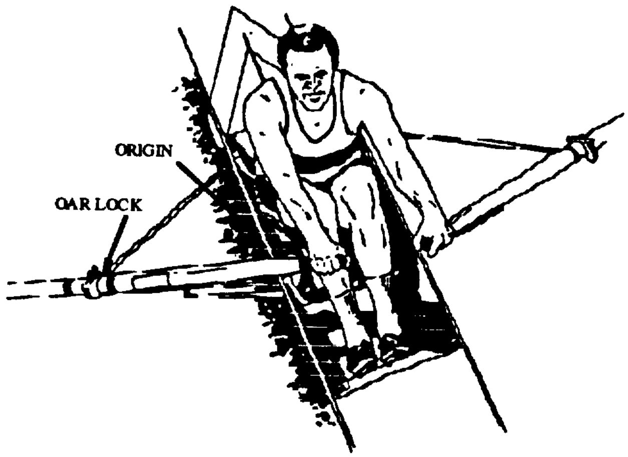
Please attempt ALL questions.

**ORALS** We start the orals by giving you the opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.

## OPENING REMARKS

*Ah-hem.* I would like the members of *The Olympic Design Review Committee* to come to order. Last week we had representatives from *Faculty and Associates* describe their design for the Oar-Lock Support Structure for the single scull to be used in the Olympics in Atlanta 1996. At that time we were concerned with three aspects of their design, namely, natural frequency of the oar, the welding cost of the oar lock, and the deflection at the oar-lock. Today we have an aspiring doctoral student from the well-known institution Georgia Tech who will provide answers to the concerns we raised. But first let us review the design submitted by *Faculty and Associates.*




The design load **P** acts at Node 2 and is given by:

$$\mathbf{P} = -4{,}000\,\hat{i} + 20{,}000\,\hat{j} - 2{,}000\,\hat{k}$$

*Important relationships for support structure....*

Length of members 12 and 23 is **L**.

123 forms an equilateral triangle.

Angles 042 and 024 are 45 degrees.

*Assumptions for support structure*

All joints are pivoted or simply-supported.

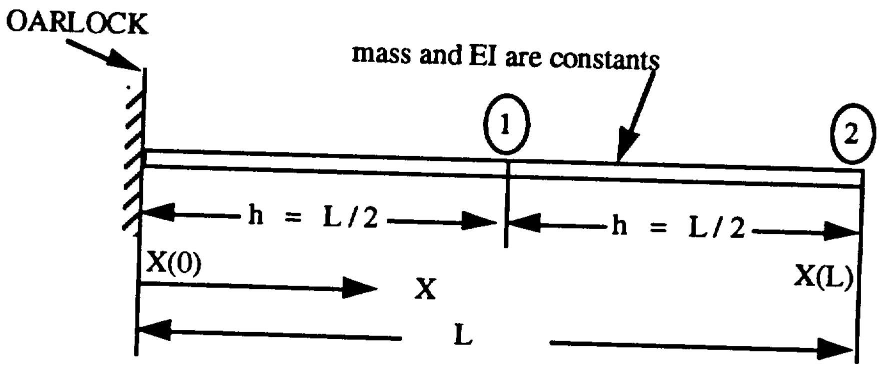
All members are made of the same material.

All members have identical cross-sections.

The base of the support structure (134) where it joins the scull is a flat surface.

**Concern 1:** ***Natural frequency of the oar must not be in phase with synchronized motion.***

*Members of the review committee thank you for giving me the opportunity to address your concern. I am going to assume the oar can be approximated as a uniform beam anchored at the oar lock as shown in the figure below. I will then use the finite difference method with h = L/2 and the inverse power method to obtain the lowest frequency of the cantilever beam approximation. This should answer your first concern.*




*Important relationships*

The free end of the beam at X = L (and at node 2) has shear V and moment M boundary conditions of:

$V(L) = -EIu''' = 0$

$M(L) = -EIu'' = 0.$

The governing equation for vibration of the beam is:

$EIu^{IV} - m\,\omega^2 u = 0$

where m is the mass / unit length, ω is the frequency, u is the beam displacement and $u' = du/dx$.

The relevant central finite difference approximations are:

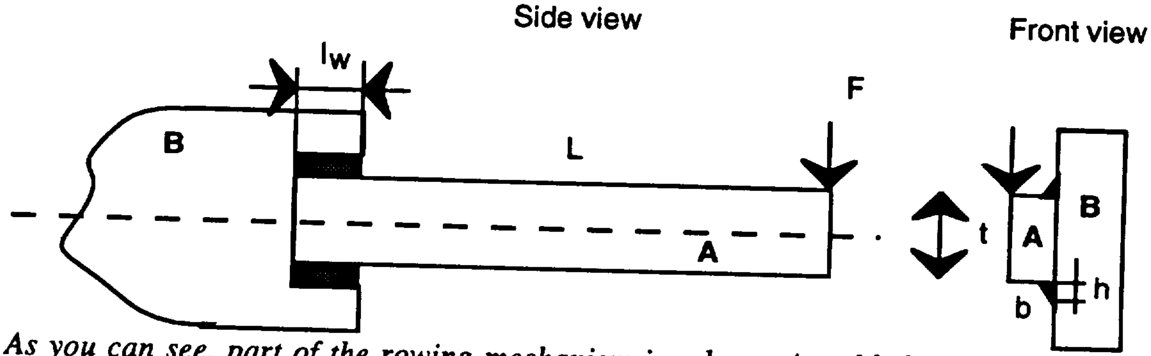
$u_i' = 1/2h\,(-u_{i-1} + u_{i+1})$

$u_i'' = 1/h^2\,(u_{i-1} - 2u_i + u_{i+1})$

$u_i''' = 1/2h^3\,(-u_{i-2} + 2u_{i-1} - 2u_{i+1} + u_{i+2})$

$u_i^{IV} = 1/h^4\,(u_{i-2} - 4u_{i-1} + 6u_i - 4u_{i+1} + u_{i+2})$

**Concern 2:** ***The welding cost of the oar lock***
*Members of the committee let me now address your second concern, namely, that of the cost of welding. The dimensions and the welds are shown in the following figure.*




*As you can see, part of the rowing mechanism is a beam A welded to a rigid support member B. The welded beam is made of 1010 steel. The length L of this beam A is 14 inches. The major cost of making such an assembly are (1) setup labor cost $c_0$, (2) welding labor cost $c_1$, and (3) material cost $c_2$. These three components are captured in the following (initial) cost function F(x) which has been derived by the Olympic Committee's Core of Engineers:*

$$F(x) = c_0 + c_1 + c_2 \qquad [\$]$$

*I will now express F(x) in terms of the dimensions of the weld and structure to get the cost equation*

$$F(x) = h^2 l_w + c_3 h^2 l_w + c_4 t b (L + l_w) \qquad [\$]$$

where

$l_w$ = weld length = 0.75 inch
h = weld height (to be determined)
t = beam width = 1.5 inch
b = beam thickness = 0.5 inch
L = length of beam = 14 inch
$c_3 = 0.1047$ \$/in$^3$
$c_4 = 0.0481$ \$/in$^3$

*To alleviate your second concern I will do the following:*

- *For the above function F(x), using the Newton-Raphson method, I will find the value for h for which the cost equals \$1. The steps will include identifying a suitable stopping criterion, using $h_0 = 0.75$ for my first iteration, and verifying the result.*
- *I will briefly describe some other methods for solving this kind of problem and highlight some advantages and disadvantages of each method.*
- *Using Newton's method I will find the value of h which minimizes the cost and comment on the number of steps I had to take.*

*I trust that this will allay your second concern.*

**Concern 3:** ***Deflection of the support structure at the oar-lock***

*And now this brings me to answer your third concern, namely, the deflection of the support structure at the oar-lock. I ask you to refer to the diagram that the Chair so kindly introduced in her opening remarks. The deflection at Node 2 appears to be the root cause of the enunciated concern. Please note that the force P acts at Node 2 and I am going to assume it is given by:*

$$\mathbf{P} = -4{,}000\,\hat{i} + 20{,}000\,\hat{j} - 2{,}000\,\hat{k}$$

*To allay the Committee's concern, I will answer the question, "How much is the deflection at Node 2?" I will be computing this deflection using the stiffness method.*

*As a bonus ,we at Faculty Associates, have also considered the consequences of member 24 breaking during the race. I will show that the remaining two truss members will NOT be sufficient to support the load P and suggest modifications to the assumptions on which this finite element model is based.*

*Important relationships ....*

The stiffness matrix for a truss element oriented arbitrarily in 3-D space is:

$$[K] = EA/L \begin{bmatrix} k_0 & -k_0 \\ -k_0 & k_0 \end{bmatrix}$$

$$k_0 = \begin{bmatrix} l^2 & lm & nl \\ lm & m^2 & mn \\ nl & mn & n^2 \end{bmatrix}$$

where $l$, $m$, and $n$ are the direction cosines between the truss member and the X, Y and Z axes and are given by: $l = (x_2 - x_1)/L$; $m = (y_2 - y_1)/L$; $n = (z_2 - z_1)/L$.