GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1998

Computer-Aided Engineering
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

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COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM - SPRING 1998

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

Bras, Fulton, and Sitaraman (Chair)

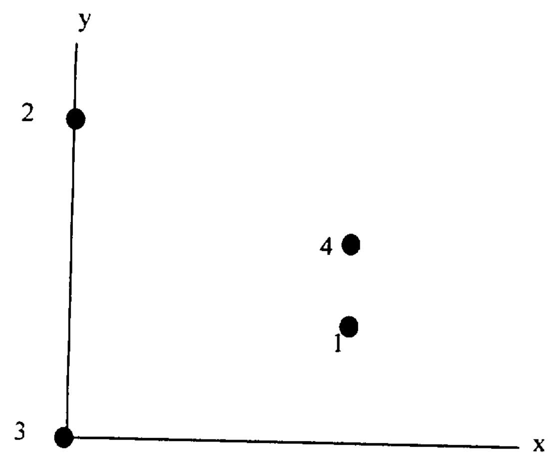
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Problem 1

a. Develop and sketch a parametric cubic spline through the following four points. Calculate at least one intermediate point between each segment to facilitate the sketch. The coordinates of the four points are:

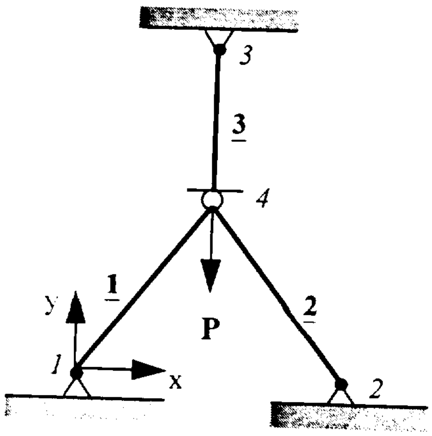
	1	2	3	4
x	5	0	0	5
y	2	5	0	3



b. Sketch a Bezier curve described by the same points given in Problem 1.a. and identify key differences between parametric and Bezier curves.

Problem 2

Figure below shows a structure consisting of three members with member numbers designated in bold, underlined text. The **bold underlined** numbers indicate member numbers and the *italicized* numbers indicate joint numbers. Joints 1, 2, and 3 are pin joints. Joint 4 is a pin joint between members 1 and 2. Member 3 has a flat bottom surface that touches the pin joint at 4.



All members are of equal length L, cross-section area A, and made of the same material. The distance between joints 1 and 2 is L.

Assume that a sinusoidally varying vertical force $P = P_0 \sin\theta$ is applied at joint 4. Use Finite-Element Formulation to analyze the structure. In particular,

- Develop individual stiffness matrix for each member
- Assemble and show the global stiffness matrix
- Show the assembled equilibrium equation in matrix form
- Identify the boundary conditions and loading conditions for $\theta = \pi/2$ and $3\pi/2$. Show the global stiffness matrix for both cases.
- Without solving for the displacements and stresses in each member, qualitatively discuss the loads on member 3.

Element Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

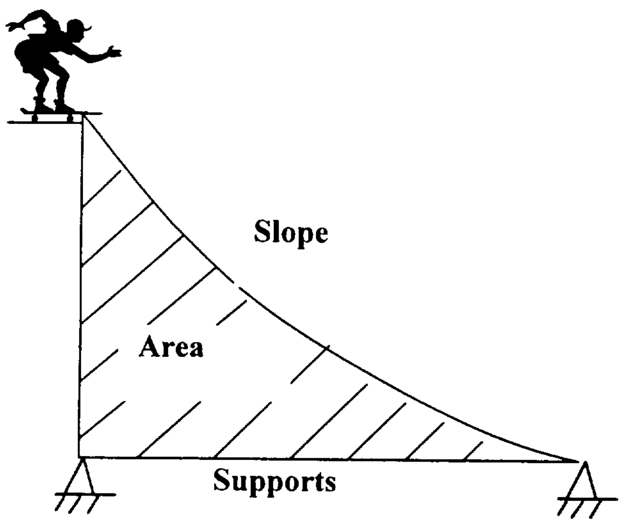
where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect to X and Y axes and are given by:

$$l = \frac{x_2 - x_1}{L}$$

$$m = \frac{y_2 - y_1}{L}$$

Problem 3

In the figure below you see a skateboarder ready to go down a slope. This inclined slope is one of the attractions to be put in a new (separate) area in Piedmont Park where skate-boarders and roller-bladers can practice stunts and acrobatics. Rather than supporting the slope through trusses, the engineers decided to make the inclined slope out of solid concrete. They feel this is safer since it is now a solid "hill" and nobody can get under the slope in that way. The primary foundation of the concrete slope consists of two supports. A local committee has raised some concerns about the weight of the structure and whether the supports are sufficient. The shape of the slope is defined by the function $f(x,y) = -2x^3 + 12x^2 - 20x + 10$ and x ranges from 0 to 1.



a) Find the area (y) under the slope using the classical fourth order Runga-Kutta method for numerical integration. Assume the initial condition y = 1 at x = 0 and use a step size of 0.5.

b) Is the solution found under a) equal to the exact solution? Why or why not?

c) How many 2^{nd} order Runga-Kutta methods are there?

d) Again find the area, but now using the Simpson rule for the integration. Compare and discuss your result with the result from a).

e) Explain how the Newton-Cotes family of quadrature rules for numerical integration of a function f(x) work. Name and explain two well known methods which are part of this family.

f) What are some other methods to integrate a function f(x,y)?

Values for x and f(x,y)

x	f(x,y)
0	10
0.05	9.02975
0.1	8.118
0.15	7.26325
0.2	6.464
0.25	5.71875
0.3	5.026
0.35	4.38425
0.4	3.792
0.45	3.24775
0.5	2.75
0.55	2.29725
0.6	1.888
0.65	1.52075
0.7	1.194
0.75	0.90625
0.8	0.656
0.85	0.44175
0.9	0.262
0.95	0.11525
1	0

Classical fourth order RK method: $y_{i+1} = y_i + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$

where

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + ½ h, y_i + 2hk_1)$

$k_3 = f(x_i + ½ h, y_i + 2hk_2)$

$k_4 = f(x_i + h, y_i + hk_3)$

Plugging in the values for x=0 and h=0.5

$k_1 = f(x_i, y_i) = -2\,(0)^3 + 12\,(0)^2 - 20\,(0) + 10 = 10$

$k_2 = f(x_i + ½ h, y_i + 2hk_1) = -2\,(0.25)^3 + 12\,(0.25)^2 - 20\,(0.25) + 10 = 5.71875$

$k_3 = f(x_i + ½ h, y_i + 2hk_2) = -2\,(0.25)^3 + 12\,(0.25)^2 - 20\,(0.25) + 10 = 5.71875$

$k_4 = f(x_i + h, y_i + hk_3) = -2\,(0.5)^3 + 12\,(0.5)^2 - 20\,(0.5) + 10 = 2.75$

$y(0.5) = 1 + 1/6(10 + 2(5.71875) + 2(5.71875) + 2.75) = 6.9375$

a) This is equal to the exact solution because we are using a fourth order RK method.
b) In principle, there are an infinite number of 2nd order Runga Kutta methods.
c) Simpson rule y = (b-a)/6 (f(a) + 4 f((a+b)/2) + f(b)) thus y = 1/6 (f(0) + 4 f(0.5) + f(1)) = 2.75 This is the same as under a because for integration of a function in x alone, the fourth order RK equals the Simpson 1/3 rule.
d) Newton Cotes family of quadrature rules are derived by integrating a polynomial interpolant of the integrand f(x). Two examples are the Trapezoidal rule and Simpson Rule.
e) Euler