GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1999

Computer Aided Engineering
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

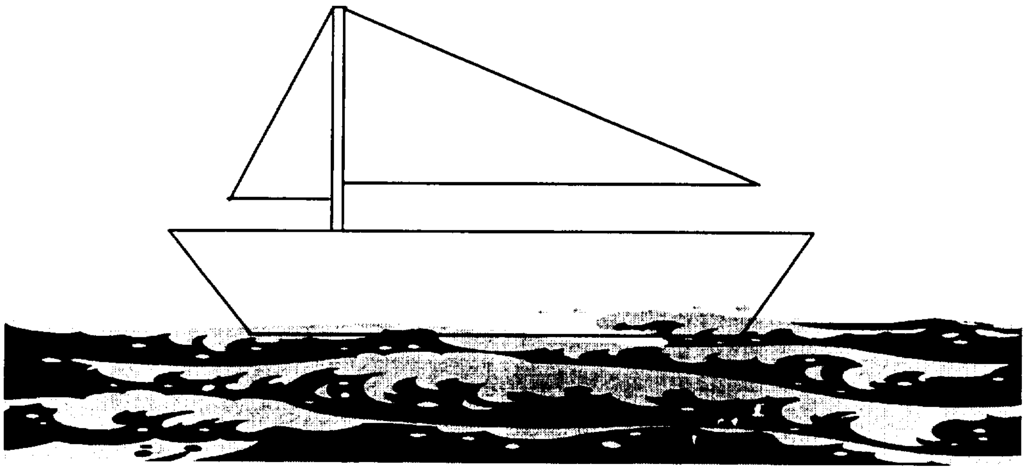
Please **p r i n t** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

GEORGIA INSTITUTE OF TECHNOLOGY
GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING

COMPUTER-AIDED DESIGN
PH.D. QUALIFYING EXAM
Spring 1999

You are a member of the design team for a sail boat for the America's Club Race. A design has been developed which must be assessed for reliability to meet the design conditions. The following problems cover some of the design concerns that must be investigated.



We are interested in learning what you know and your ability to reason. If for some reason you do not follow the question or are confused, kindly adjust the question suitably and proceed with your answer. Please structure your answers as follows:

1) Restate the problem in your own words, identifying any assumptions, judgments, and adjustments that you are making.
2) Tell us your strategy or plan for solving this problem.
3) Solve the problem.
4) Tell us about any insight you gained by solving this problem.

Oral Exam Note

When you come to the oral exam be prepared to comment briefly on your research activities and where CAE/CAD technology fits into that research.

Question 1.

One design concern is the vibration characteristics of the mast. The mast is rigidly fixed at the base and is sufficiently stabilized by guy wires at the upper end so that it can be considered simply supported. The mast has a length (L) and mass/unit length of M(x). The governing equation for the vibrating mast is:

$$EI\,U^{IV} - M(x)W^2U = 0$$

Where U is the lateral deflection of the mast, W is the frequency, and EI is the beam stiffness which is assumed constant. The mass of the mast is tapered so that $M(x) = M_0[1 - x/2L]$ where M_0 is a constant. Use the following central finite differences to approximate the derivatives.

$$U^{I}_{i} = \frac{1}{2h}[-U_{i-1} + U_{i+1}]$$

$$U^{II}_{i} = \frac{1}{h^2}[U_{i-1} - 2U_i + U_{i+1}]$$

$$U^{III}_{i} = \frac{1}{2h^3}[-U_{i-2} + 2U_{i-1} - 2U_{i+1} + U_{i+2}]$$

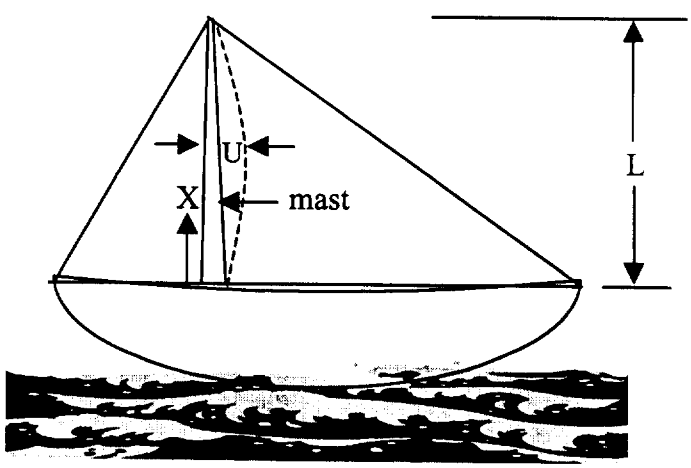
$$U^{IV}_{i} = \frac{1}{h^4}[U_{i-2} - 4U_{i-1} + 6\,U_i - 4\,U_{i+1} + U_{i+2}]$$

(a) Divide the mast length into 3 equal finite difference segments and write out the finite difference equations for the governing equation and boundary conditions.

(b) Use the inverse power method to obtain the lowest frequency of vibration and mode shape for the mast.

(c) Use the orthogonality principal and above results to estimate the next lowest frequency and mode shape.

(d) Describe how you could assess the accuracy of the finite difference results.



Question 2.

Think about the shape of the main sail of the sailboat, shown below. Main sails have two of their three edges essentially fixed by connecting them to the mast and the boom. Now, how would you model its shape geometrically?

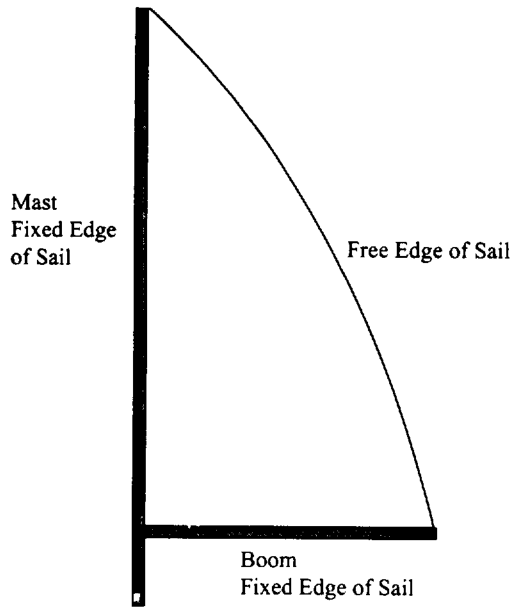
Given

Equations for **Bezier curves** are

$$b(u) = \sum_{j=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \begin{pmatrix} n \\ i \end{pmatrix} u^{i}(1-u)^{n-i}$$

where: **P** are the control vertices that define the Bezier curve.



Questions

a) Given the equation for a Bezier curve above, derive the equation of a **general** Bezier surface. Using this Bezier surface equation, derive the equation for a **bicubic** Bezier surface.

b) Propose **three different types of surfaces** (e.g., bicubic Bezier surface) that you could use to model the sail that you have in mind. Sketch three sails, one for each type of surface. Show control vertices. Indicate where each surface patch lies on the sail (if you are using more than one patch).

c) Identify at least **three criteria** that you could use to select the best geometric surface model to use in modeling the sail.

d) **Evaluate** your three surface types using your three selection criteria. Based upon your evaluations, which surface type would you select? Explain your reasoning.

Question 3.

A structural unit in the boat is constrained at two ends by rigid supports as shown in Figure A below. The structural unit consists of four members with properties as shown in the figure. E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of a member respectively. Assume that members 2 and 3 are identical and are symmetrically located. Assume that an axial force of F is applied at the center of member 1 as shown in the figure. Using finite-element formulation,

- write down the stiffness matrix for each member
- assemble and write down the assembly stiffness matrix
- identify the boundary conditions
- determine the displacements at points P and Q
- determine the reaction forces at the two rigid end-supports

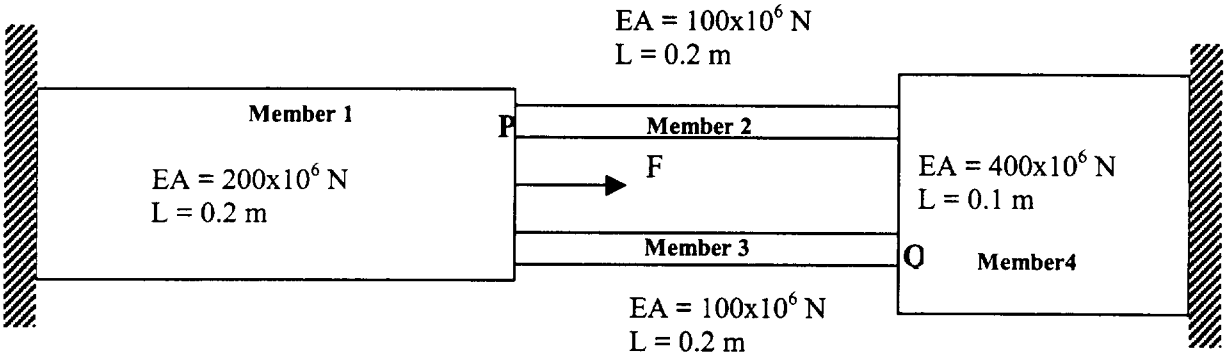


Figure A

Suppose the right-side rigid support is replaced by a spring-like support as shown in Figure B, with a spring constant of 1×10^9 N/m. Using finite-element formulation,

- determine the assembly matrix of the new structure
- identify the boundary conditions
- outline the procedure to solve for displacements and reaction forces. You need not solve.

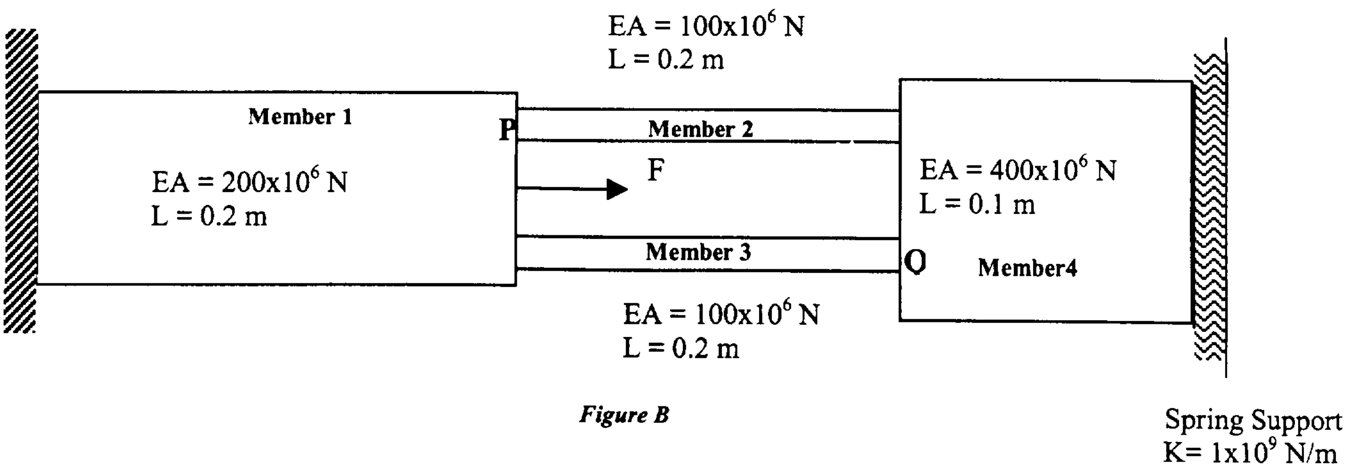


Figure B