# GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Spring Semester 2004**

## Computer-Aided Engineering
EXAM AREA

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Assigned Number **(DO NOT SIGN YOUR NAME)**

* Please sign your **name** on the back of this page —

Please **p r i n t** your name here.

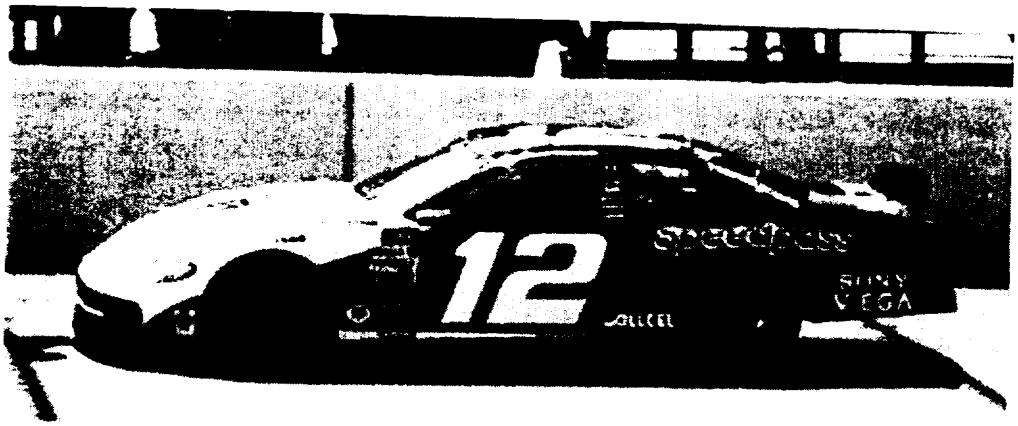
**The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.**

# COMPUTER-AIDED ENGINEERING

## *Ph.D. QUALIFIER EXAM – Spring 2004*


*GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.*
*GEORGIA INSTITUTE OF TECHNOLOGY*
*ATLANTA, GA 30332-0405*

***Bras, Rosen (Chair), and Sitaraman***


- All questions in this exam have a common theme: ***Racing Cars***

- Answer **all** questions.

- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.

- Show all steps and calculations.

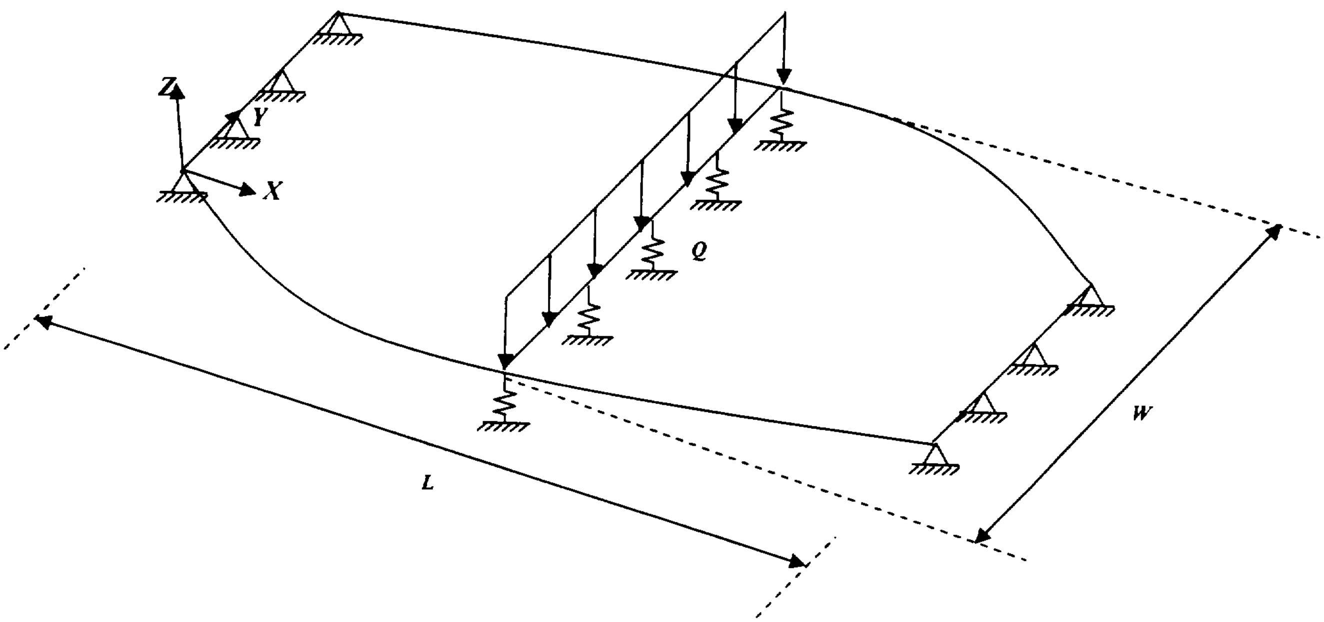
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

***GOOD LUCK!***

## Question 1 - Finite Element Modeling

Figure below shows a flat automotive body panel lying on the $X$-$Y$ plane. When assembled, the body panel can be assumed to be simply supported on two opposite edges as shown. A uniform distributed force with a magnitude of $p$ acts along the width of the panel as shown, and the panel is supported by a bed of springs with a spring constant of $k$. Assume that the distributed force and the springs act vertically, along the $Z$-axis. The length of the panel along the $X$-axis is $L$, the width of the panel along the $Y$-axis, where the distributed force and the springs are positioned, is $W$, and the thickness of the panel is $T$. Assume that the thickness of the panel is one to two orders of magnitude small compared to the width and the length of the panel. Assume also that the distributed force and the bed of springs are positioned at a distance L/2 from the supported edges of the panel.

- You are asked by the design team to use finite-element method to determine the **vertical deflection** of the panel at location $Q$. $Q$ is situated at the center of the line where the distributed force is applied. The design team is NOT INTERESTED in the stress/strain distribution in the panel.
- Stiffness matrices for selected finite elements are given below. Choose an appropriate element.
- You are asked to use minimum number of elements, preferably ONE element, to determine the vertical deflection at $Q$.
- State all your assumptions clearly and show all your steps.




***Element 1 - Stiffness Matrix***

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

$$l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

where $E$, $A$, and $L$ are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l$ and $m$ are direction cosines of the element with respect to $X$ and $Y$ axes.

***Element 2 - Stiffness Matrix***

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where $E$, $I$, and $h$ are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

**Question 2**
**Numerical Methods**

A Nascar race car laps a racing circuit in 84 seconds. The speed of the car at each 6 second interval is determined using a radar gun and given from the beginning of the lap, in meters/second, by the entries in the following table.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 |
| Speed | 41 | 45 | 50 | 52 | 49 | 44 | 40 | 36 | 33 | 28 | 26 | 30 | 35 | 39 | 41 |

a) Calculate the length of the track using the trapezoidal rule. Use 3 segments.

b) How many $2^{nd}$ order Runga-Kutta methods are there?

c) Again find the track length, but now using a Simpson's rule method for the integration. Please identify an appropriate Simpson's rule, such as Simpson's 1/3 or Simpson's 3/8 rule, and tell us why you selected that method. Compare and discuss your result with the result from a).

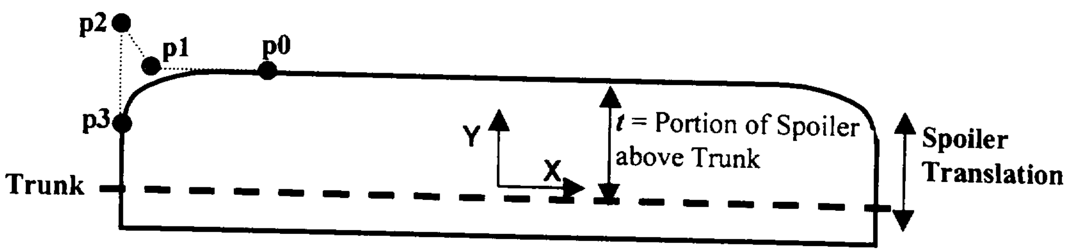
d) What are some numerical methods to find the magnitudes of the accelerations and decelerations of the race car during its lap? Discuss their pros and cons.

## Question 3

### Geometric Modeling

The rear spoiler at the back of the car must be adjustable to enable the race crew to adjust the car's aerodynamics before and during a race. The front view of the adjustable rear spoiler is shown below. For simplicity, assume that the top surface of the trunk is planar and is indicated by the dashed line labeled 'Trunk.' Assume that the central portion of the spoiler is linear, while each end is modeled by a cubic Bezier curve with the control vertices as shown.

**p0** = (-20, $t$, 0), **p1** = (-30, $t$, 0), **p2** = (-32, $t$+3, 0), **p3** = (-32, $t$-4, 0). Assume that the spoiler height above the trunk is $t$ = 6.




The equation for a Bezier curve is included at right.

$$b(u) = \sum_{i=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Given this, answer the following questions:

**a.** Derive the equation of the cubic Bezier curve that models the curved part of the spoiler profile using the given control vertex values.

**b.** Compute the coordinates of the point on the Bezier curve at u = 0.6.

**c.** Compute the area of the spoiler that is exposed to the air; i.e., the area of the spoiler between the trunk line and the top profile described by the two cubic Bezier curves and the line. Assume that the spoiler profile is mirror symmetric about the Y axis.

**d.** Describe how you would compute the spoiler's area if the cubic Bezier curves were actually cubic B-spline curves with the same 4 control vertices. What is the area?

**e.** Describe how you would compute the spoiler's area if the trunk line was actually a curve, specifically two cubic B-spline curves (mirror symmetric) with C1 continuity.